This book will address the discussion on online distance education, teacher education, and how the mathematics is transformed with the Internet, based on examples that illustrate the possibilities of different course models and on the theoretical construct humans-with-media. We will attempt to give the reader the sensation of experiencing one of the various distance courses in which we have participated, or a virtual community that does not have the structure of a course. And if the reader has not yet participated in any of these possibilities, we believe that the book may help, but not substitute, the experience of participating in a discussion list, a course, or a virtual community constituted by a specific interest.

This book is part of a collection of books called Trends in Mathematics Education, originally published in Brazil. This collection began being published in 2001 and currently has 21 titles published by more than 30 different authors. It is designed to present research to a broader audience that extends beyond academia. The books have been widely used in graduate courses, research groups and in some undergraduate classes. About 60,000 copies of the Portuguese edition have been sold. Some titles have been translated into Spanish and English.
ONLINE DISTANCE EDUCATION
Online Distance Education

By

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Note from the Coordinator of the collection of books entitled Tendências em Educação Matemática (Trends in Mathematics Education):

This book is part of a collection of books called Trends in Mathematics Education. This collection began being published in 2001 and currently has 21 titles published by more than 30 different authors. It is designed to present research to a broader audience that extends beyond academia. The books have been widely used in graduate courses, research groups and in some undergraduate classes. About 60,000 copies of the Portuguese edition have been sold. Some titles have been translated into Spanish and English.

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INTRODUCTION

Distance education, the Internet, and “downloading files” are expressions that have invaded our lives in recent years. For some, distance education is seen as something pernicious that should be banned to avoid affecting the quality of teaching. For others, however, it may be seen as a salvation or even a way to finally democratize student access to public universities. In this book, we attempt to escape this dualistic trap and present examples and research results of online distance education in action.

Online distance education can be understood as an educational modality that mainly takes place mediated by interactions via the Internet and associated technologies. Courses where the interaction occurs using interfaces such as chat rooms, videoconferences, forums, etc., are included in this modality. We will thus discuss how this form of education takes on its own shape and, in particular, how it molds mathematics education. We present accounts of what can be called online mathematics education. Using specific examples to illustrate, we will show how mathematics can be transformed when working with it in virtual environments.

Although the authors are all mathematics education researchers, and the large majority of examples are taken from courses for teachers in which the focus is mathematics or mathematics education, the reader will observe that the discussion can be adapted, in a “situated” manner, to other fields of knowledge.

We believe that education in virtual environments shapes the participation of students and teachers in a way that is analogous to how the classroom shapes this participation. Like Castells (2003), we see the Internet as being impregnated with social relations, not only with respect to the interactions among the humans who use it, but in the very way the Internet has distributed itself. It is no coincidence that broad-band Internet has arrived forcefully in economic centers while often failing to reach the pockets of the “Fourth World” in the poorer neighborhoods of large and medium-sized cities, as well as many rural areas where purchasing power and the “logic” of the market do not justify the installation of fiber optic cables.

The implementation of public policies that make Online Distance Education viable is fundamental to fill the immense gaps left by the (lack of) logic of the market. In this sense, tax exemptions for the purchase of computers are welcome, but the money from the Fund for the Universalization of Telecommunications Services (FUST) should truly be put to use to create fiber optic networks dedicated to education, connecting schools and universities in the country.
INTRODUCTION

Years ago, Borba and Penteado (2001) proposed that these funds be used to democratize Internet access. Its growth in education and for other ends indicates that we will soon be facing bottlenecks in access. A network for Internet only needs to be created with FUST funds, paid for by everyone who uses the telephone, as well as state and municipal governments. It is also necessary to criticize the so-called “right of passage”, according to which companies responsible for managing the highways do not allow fibers purchased with public funds to pass by them. When more highways are privatized, it is fundamental, in the transportation sector, that the “right of passage” remain with the government so that companies do not charge exorbitant fees for the right, which should belong to everyone, to have access to the Internet.

Access to computer technology in general, and the Internet in particular, has become as important as guaranteeing that every child have paper, pencil, and books. Castells has proposed that the division between the First, Second, and Third World will be replace by another that creates the “Fourth World” in all regions of the planet. The immense accumulation of capital by some companies, and the lack of access to the Internet and to the wealth in general, which still persists, are only some of the ingredients of the “civil war” that is taking over so many Brazilian cities.

We are not proposing that Internet access will resolve the problems of inequality that have accumulated over the centuries – or decades, depending on how you look at it – in countries like Brazil, but rather understanding that it is analogous to what access to school represented in the past, and still represents today if you consider access to good schools.

However, access alone is not enough, unless we also understand how educational processes take place when the Internet becomes more than a mere supporting actor. Thus, we need to understand how to deal with it in an educational context. For example, it is necessary to recognize that interactions such as the multi-logue, proposed by Borba and Penteado (2001) and Gracias (2003), change the “etiquette” with respect to what is seen as correct in the interaction between teachers and students in a virtual classroom based on chat, since many people can express themselves simultaneously, in this case. These ideas, based on initial reflections on our experiences and research in online distance education, gather strength in this book as we analyze how written text, which characterizes chat, modifies the mathematics produced by participants in distance courses who use this interface as their main protagonist. Is it possible that it also modifies the geography, physics, biology, and arts learned by students, among other fields of knowledge?

The answer to this question lies beyond this book. On the other hand, we will show data and analyses regarding how those who engage in new forms of communication are experiencing continuing teacher education. We will show how distance courses can point to paths that approximate the daily practice of
teachers in the classroom, as well as reflection on this practice, through the
discussion of problems proposed to or by the teachers in courses that value the
voices of those engaged in practice.

Of course there are different models of practice in virtual environments.
The Internet is a symbol of diversity, and almost anything can be found there,
including attempts to reproduce activities in this environment that are similar
to those developed in traditional educational environments. Thus, just as there
are those who “copy” the book onto the blackboard, there are those who see
online distance education as merely “downloading files” from a web page. In
this type of model, interaction with the teacher is not given priority, except in
the form of prepared questions and answers stored in a data base of Frequently
Asked Questions (FAQs). This main attraction of this model is the cost, but
rarely does it adapt to learning in mathematics or other fields in which
discussion is essential, although it is certainly possible to list the necessary
steps to build an apiary, for example. Collaboration also stands out as a
fundamental element of online distance education, to the degree to which we
believe that learning is multidirectional, and not only from teacher to student,
or student to student.

This book will thus address the discussion on online distance education,
teacher education, and how the mathematics is transformed with the Internet,
based on examples that illustrate the possibilities of different course models.
Furthermore, we will attempt to give the reader the sensation of experiencing
one of the various distance courses in which we have participated, or a virtual
community that does not have the structure of a course. And if the reader has
not yet participated in any of these possibilities, we believe that the book may
help, but not substitute, the experience of participating in a discussion list, a
course, or a virtual community constituted by a specific interest.
CHAPTER I

SOME ELEMENTS OF ONLINE DISTANCE EDUCATION

As we reflected on online distance education in our research group, GPIMEM\(^1\), certain questions emerged: “How does the teaching-learning process take place? What is the teacher’s role in this new educational modality? How do different interfaces change the interactions among the participants?” These questions will be answered throughout this book, but we believe it is important to first present a brief historical retrospective on distance education in Brazil as well as some of the perspectives of those who, like ourselves, have been conducting research in this area.

We base our classification on that elaborated by Vianney et al. (2003) who carried out a longitudinal study of distance education in Brazil in which they identified three generations. The first emerged in 1904 with correspondence teaching, which emphasized professional education in technical fields, such as carpentry, trading, and others.

The second generation of distance education was defined by the emergence of high school equivalency courses for adults in the 1970s and 1980s which were usually carried out via satellite while students received printed material to study. Among the communications resources used were radio, television, audio tapes, and even videotapes on occasion.

In 1996, following two years of expansion of the Internet into the university environment, the first official legislation on distance education at the university level was passed. This expansion marked the beginning of the third generation of distance education, “which has been structuring itself based on advanced technology” (TORRES, 2004, p.31).

This generation of distance education has gained strength with the legislation. Fragale Filho (2003, p.13) explains:

> Viewed with mistrust, treated as a supplementary or complementary form of face-to-face teaching, it was practically ignored in legislative concerns related to regulation of education in Brazil. However, with the emergence of new technologies, the barriers were broken, making its spread possible, and leading to an unprecedented increase in demand and the introduction of its regulation into the legislative agenda.

Thus, addressing issues involving regulation, such as the distinction between distance and face-to-face education and the procedures needed to define and evaluate practices, became a challenge for educational policymakers “when they find themselves compelled to elaborate, approve, and
Implement legislative proposals for the sector” (Fragale Filho, 2003, p.13).

Law No. 9.394, from December 20, 1996 (LDB – Basic Directives for Education in Brazil), presented quantitative and qualitative goals to be achieved in the sphere of distance education, no longer treating it as an experimental project (Lobo, 2000). Of the few articles that referred to distance education, paragraph 4 of article 80 assured that

[...] distance education will enjoy differential treatment which will include: I) reduced transmission costs in commercial radio-dissemination, sound and image channels; II) concession of channels with exclusively educational purposes; III) reservation of a minimum time, without charge to the government, by the owners of commercial networks.

This article also states that distance education can only be offered by institutions credentialed by the Federal Government, whose responsibility it is to regulate the necessary requisites for examinations and diplomas. Thus, until regulation of these aspects had been achieved, the other regulations remained ineffective (Lobo, 2000).

Decree 2.494, on February 10, 1998, advanced a bit further, regulating article 80 of the LDB and defining distance education as

[...] a form of teaching that enables self-learning, mediated by systematically organized didactic materials presented in different informational supports, used alone or in combination, and transmitted via diverse means of communication.

What one notes, as Fragale Filho (2003) pointed out, was that, in fact, article 80 of the LDB did not clearly explain the legislative concept of distance education, but sought to indicate who could offer it and the way mechanisms of control should be structured. Along these lines, we question what is understood as “self-learning”. We believe that the student, upon choosing distance education, will have to assume great responsibility for what they learn, characterized by autonomy and discipline by some authors, especially when the time is flexible. Nevertheless, we find it relevant to emphasize that following the student, especially in formal educational processes, is fundamental for their development.

Still with respect to the decree, it was determined that all institutions credentialed to offer distance education could do so according to the criteria established two months later, in Article 2 of Decree 301, on April 7, 1998.

On October 18, 2001, Decree 2.253 was granted, which allowed the development of non-face-to-face courses in recognized face-to-face undergraduate programs, even if the institution is not credentialed to offer distance education. According to this decree, courses can be conducted in part or entirely using non-face-to-face resources, composing up to 20% of total course hours needed to fulfill requirements for completion of the program.
According to Fragale Filho (2003, p.20), this “decree ended up creating a numerical level which, once passed, transforms a face-to-face course into a non-face-to-face course, i.e. a distance course.” Analyzing this possibility, he notes that

[...] this means that we dealing with the partial offer of non-face-to-face contents under the name of experimentalism, not including, in a strict sense, the universe relative to distance education, which is unfortunate, since the PNE itself recommends striving for a clearer articulation between face-to-face and non-face-to-face teaching. (FRAGALE FILHO, 2003, p.20)

On December 19, 2005, Decree 5.622 was passed which introduced, in Article 1, a new concept of distance education:

Distance education is characterized as an educational modality in which the didactic-pedagogical mediation in the teaching-learning processes occurs with the use of information and communication means and technologies, with students and teachers developing educational activities in diverse times and places.

In addition, the following are still required: students’ occasional on-site presence for examinations; obligatory internships and defenses of end-of-course projects, when described in the pertinent legislation; and activities related to teaching laboratories. Levels of teaching at which distance education can be offered are also defined, highlighting that “distance courses and programs should be projected to have the same duration defined for the respective face-to-face courses”.

Other aspects were also addressed in this decree, showing a normative concern regarding questions related to distance education. Some points still remain to be addressed, but these will appear based experiences realized in this modality.

SOME CONCEPTIONS OF DISTANCE EDUCATION

Face-to-face teaching is rooted in our lives. Associated with it is the practice developed based solely on physical presence and meetings of the people involved in the process. Therefore, regular days, locales, and times are usually established (MORAN, 2002).

On the other hand, what are the principal characteristics of distance education? For Torres (2004, p.60), it is a

[...] systematic form of education that uses technical means and bidirectional/multidirectional communication technologies with the purpose of promoting autonomous learning through a dialogical and collaborative relation between equidistant students and teachers.
For Gonzalez (2005), principal characteristics of distance education include the separation between teacher and student in space and/or time; greater student control over their own learning; and communication mediated by printed documents or some form of technology.

Moran (2002, p.1) describes it as follows: “distance education may or may not include face-to-face encounters, but it takes place fundamentally with teachers and students physically separated in space and/or time, but able to be together through communication technology”. This is the conception that we have adopted. Thus, the focus is not on the number of face-to-face hours, but on the possibility for interaction at a distance among the actors in the process, mediated by technology. Bringing together people who are geographically distant, possibly creating a space for exchange between different cultures, is a central factor that defines this modality of teaching.

The Internet opened up a range of possibilities for courses offered via distance, changing ways of thinking about and doing distance education, and according to Valente (2003a, 2003b), current models are differentiated by the degree of interaction between teachers and students.

There are proposals denominated “one-to-one” in which the material is provided in a format similar to a book for the student to study individually with little or no contact with the teacher. Student evaluation is carried out afterwards based on a standardized test. In this case, the Internet is the source of information, and it is up to the student to transform it into knowledge. Courses of this type serve a large number of students and tend to generate large profits for their organizers. If we think of continuing teacher education, we can affirm that this type of course does not privilege the role of interaction in the professional development of teachers. It is similar to what Valente calls the broadcast approach.

Other proposals are defined by interactions that occur in a manner similar to traditional face-to-face classes, in which the teacher assigns activities that are developed and handed in by the students. Much of the interaction is limited to the exchange of questions and answers, in a relation known as “one-to-many”. Some experiences in distance education have been developed in this way, as an adaptation of face-to-face classes, with “new clothes”, but can be considered obsolete by current day standards. Valente calls this approach the virtualization of traditional school.

In the “many-to-many” approach, interaction is more intense, with the possibility for rapid feedback via the Internet, in synchronous and asynchronous activities that allow communication between the student and teacher as well as among students. In this scenario, the teacher follows the students closely, challenging them and encouraging their participation in the group, which Valente calls being together virtually³.

Before these classifications became known, authors such as Borba and Penteado (2001) were already warning against “domesticating” a medium like
the Internet, saying that the new possibilities provided by such technologies should be taken advantage of. Based on previous experiences with the use of software in face-to-face environments (Borba, 1999a), Borba and Penteado proposed distance courses that included a strong emphasis on interaction, exploiting all the possibilities in this sense. They pointed to a model for distance courses, developed since 2000, based on chat rooms, e-mail, and traditional mail. This model has been expanded and transformed since that time, incorporating other interfaces like forums and videoconferences with the aid of instant communicators and mobile phones for emergencies, and always emphasizing communication among course participants.

Regardless of the model adopted, technological media are necessary to enable communication. These are commonly referred to as Virtual Learning Environments (VLE) and constitute a scenario in which teaching and learning can take place in qualitatively different ways depending on the resources at hand. When using a VLE with audio and video resources, for example, the possibilities are very different from one in which the interaction takes place through writing only, by means of chat.

Resources of asynchronous communication include discussion lists, portfolios, and forums, which enable students to express their questions and ideas and share their solutions to problems each in their own time. With synchronous interaction tools, like chat and videoconference, it is possible to share ideas in real time even when people are not in the same physical location.

We believe that synchronous and asynchronous interactions are important in online distance education as long as there is collaboration among the participants. The nature of the learning is qualitatively different when there is interaction, depending on its intensity and quality, and the curriculum should be organized taking into consideration the possibilities offered by the media used.

According to Belloni’s (2003) characterization, the concept of interaction originates from sociology, and is a process in which at least two human actors are present who relate to each other simultaneously (i.e. synchronously) or at different times (asynchronously). It is an elementary phenomenon of human relations, which include educational relations. Thus, it differs from interactivity, as the latter is associated with the possibility of interacting with a machine.

Considering the concepts interaction and interactivity, we note that Information and Communication Technologies (ICTs) have broadened the possibilities in the sphere of online distance education. With increasingly advanced programs, modern interfaces, and possibilities for rapid feedback, as well as the range of hypertexts available on the Internet, interactivity has been intensified. However, this interactivity is often limited to the relation between the student and the informational content accessed via CD-ROMS or websites, for example.
Interaction via the Internet, in turn, makes it possible to combine various possibilities of human interaction, through the use of different software and interfaces, with freedom in relation to time and space. The relations between students and the various elements that make up the educational scenario, such as content, the teacher, other students, the teaching institution, etc., are situated in this context.

Thus, the physical absence of the teacher is compensated for by intense communication which limits the possibility that the student will feel alone or isolated. To avoid this, their questions are answered in a timely manner and their participation is constantly encouraged.

For Silva (2003a), the possibility of interaction is not simply yet another product of the digital age, but rather a new communication paradigm that is gradually substituting the transmission paradigm that characterizes mass media communication. Interaction demands that we re-think the traditional media and the role of the actors involved in the process.

COLLABORATION IN ONLINE EDUCATION

With the advance of the Internet, online distance education proposals have emphasized the dialogical process made possible by the tools available on the web that enable communication in real or deferred time. Silva (2003b) calls our attention to the fact that, in order for interaction in a distance course to be effective, at least three fundamental aspects must be satisfied. One is collaborative participation, understood as participation that is not limited to “yes” or “no”, but which aims to intervene in the communication process, making one a co-creator of the emission and reception. Another refers to bi-directionality and the dialogic relationship itself, since the communication that develops in a course should be produced jointly by the students and the teacher who participate in the emission and reception, and who are poles who codify and de-codify. The third aspect is the existence of connections in open webs, which emphasizes that communication assumes multiple articulated networks that enable free exchange, association, and meaning-making.

We think these aspects are important, and similar to Valente’s “being together virtually”, our proposals for online distance education have been structured, since 2000, on the conception that interaction, dialogue, and collaboration are the factors that condition the nature of the learning. It is our belief that the quality of online distance education is directly related to these aspects, as they determine the quality of the participation of the people involved during the process of knowledge production.

When the focus is mathematical learning, interaction is a necessary condition in the process. Exchanging ideas, sharing possible solutions for a problem, and presenting one’s rationale are actions that make up mathematical “doing”. And to develop the process at a distance, models that enable the
involvement of many people have gained ground in relation to those that focus in individuality.

In this sense, dialogue is seen as a process of discovery influenced by collective action and sharing. Thus, it is not constituted merely of the act of people communicating, but of the depth and richness of this act. Dialogue is a process that goes beyond simple conversation (Alrø; Skovsmose, 2006). To produce knowledge, it is important to perceive the importance of people expressing their opinions, sharing their experiences and feelings such as insecurity, fear, and doubt. In the same way, one must know how to value the participation of the other, listening respectfully to what is shared.

Freire (2005) points out that it is not through silence that people constitute themselves, but among other factors, through words. For him, dialogue is based on the encounter between humans for the common task of knowing how to act, mediated by the world, and it imposes itself as the path by which they acquire meaning as people. Thus, it cannot be reduced to “the act of depositing the ideas of one subject in another, nor become the simple act of exchanging ideas to be consumed by the exchangers” (p. 93).

Nor can it be a war between people who want to impose their truths rather than seek them together. “The conquest implicit in dialogue is of the world by the dialogical subjects, and not of one over the other. […] Authentic education is not done by ‘A’ for ‘B’, or by ‘A’ over ‘B’, but by ‘A’ with ‘B’” (p. 93, authors’ emphasis). Thus, Freire emphasizes that the dialogical act cannot have arrogant positions, but requires humility. Also, “there is no true dialogue if there is, in its subjects, no true thinking. Critical thinking. […] Without dialogue, there is no communication, and without this, there is no true education” (p.97-98)

Alrø and Skovsmose (2006) state that the quality of learning is intimately linked to the quality of communication. The relations between people are crucial to facilitating learning, as learning is a personal act but is shaped in a context of interpersonal relations, and dialogue, as a means if interaction, enables the mutual enrichment of people.

Ideas like these regarding the importance of dialogical relations have inhabited education and mathematics education for some time, as can be seen in the seminal works of authors such as Paulo Freire and Bicudo (1979), respectively. This makes them highly relevant for the online distance education scenario, since communication, synchronous as well as asynchronous, must be permeated with this deeper notion of dialogue, in which participants open up to one another using the interfaces available in a given virtual environment.

Considering collaboration as part of the interactive process, teacher and students should act as partners in the mathematical learning process. It differs from cooperation in that it goes beyond simply helping one’s classmate to carry out a task. Authors such as Fiorentini (2004), Hargreaves (2001), Kenski (2003), Miskulin et al. (2005), Guérios (2005), and Nacarato (2005) point to
issues involving different aspects of collaboration, and emphasize that, in a collaborative process, everyone actively participates. Activities are carried out collectively in such a way that the task of one complements the task of the other, since in collaboration, all are aiming to achieve common objectives by working together and mutually supporting one another.

When a group develops collaboratively, its members are not interested in doing tasks and engaging in activities for their own benefit, but rather establish common goals permeated by reciprocity. Ferreira and Miorim (2003, p.17) note that “collaboration means assuming joint responsibility for the process. It means taking turns, having a voice, and being listened to; it means feeling one is a member of something that only works because everyone works and builds the road collectively to achieve the objectives”.

The option to belong to a group is influenced by the identification of a person with other members of the group, as well as the possibility of sharing problems, experiences, and common objectives. Trust is a basic ingredient to building a group in which the creation of collaborative work relations is meaningful, and this trust is based on dialogue, loyalty, and reciprocity in moments of decision making.

Collaboration is determined by the internal will of each individual to work together with the other and to be part of a given group. In this way, relations tend to be spontaneous, voluntary, guided by development, spread over time and space, and unpredictable.

However, exchanging experiences, sharing solutions to problems proposed, and joint action do not imply thinking in a uniform fashion. It is an environment of contribution in which individuals join forces in the search for collective benefit. The collective is not necessarily synonymous with solid and uniform, since as a group, the individuality of its members is respected in such a way that, based on their differences, they produce and grow together.

In heterogeneity, different forms of relations among equals are established which, in the process of working as a group, are needed to manage conflicts, propose alternatives, review concepts, take positions, divide up the work, re-think ideas, etc. Thus, a collaborative group can promote exchange and learning without losing the individuality of each member and without having to arrive at a single, uniform perspective.

This process does not prevent each member from having their own point of view and distinct interests, but rather benefits from different contributions based on different levels of participation. Collaboration also does not imply that everyone participates in the same way. Each makes their own voice heard from where they are at, but everyone works together. We know that teacher and student have distinct roles in the learning process, and what we want to emphasize is that each can participate actively in their own way throughout the process.
From the perspective of teacher education, experiences with distance courses like those analyzed by Gracias (2003), Bairral (2005), and Zulatto and Borba (2006) have shown that commitment and collaboration tend to flow when individual interests are respected and valued, since these factors significantly influence the quality of discussion in a VLE.

Mutual support among group members is a fundamental factor to survival in a collaborative environment. Respect for different conceptual knowledge and the experiences of every future teacher, as well as their difficulties, is indispensable to the learning process. They need to feel that their practice is valued and that they have the support of the others as well as the teacher as they try, collaboratively, to find a solution to a problem or question.

In this context, members of a collaborative group assume the role of protagonists as they become actors in the production of knowledge, learning as well as teaching, without being limited to being mere providers of information and materials. They are different voices, positions, and shared experiences that can contribute to the improvement of teaching practice. Collaboration among teachers demands a group synergy that allows knowledge production to take place simultaneously with the personal and professional development of the group members.

We consider education, particularly of future teachers, to be a movement of process, which is justified by our understanding that movements of formal education occur at precise moments, whereas the reaction does not, since formal moments fertilize the teacher’s practice, compelling them to new ways of doing things. Thus, it is as though each immediate action corresponds to a reaction that is not only immediate. The effects of formal education are reflected in the teacher’s entire professional process, intermingled with other reactions provoked by other experiences, formal or otherwise, that acquire significance when they are reflected in teaching practice.

Knowledge produced in formal educational moments interacts with the teacher’s life, in the professional and personal dimensions, and should produce an internal movement that provokes a process of transformations in the teacher. We understand that continuing education is a course that can be interpreted “as an unique and continuous path, which leads us to the conjecture that in this path, transformations occur, provoked by the interaction between formal stages of education and experientiality, in the dynamic of the collective everyday” (Guéríos, 2005, p.136).

Thus, continuing teacher education should consider relevant aspects of teachers’ professional experience and provide them with the opportunity to reflect constantly and critically on their practice. The teaching process can constitute spaces for this reflection, as well as compel the teacher to develop their capacity for intuition, raising hypotheses, reflection, analysis, organization, etc.
From this perspective, collaborative work can become an important scenario for teachers’ professional development, in a way that the group assumes a fundamental role in processes of producing knowledge and reflection. This appropriation or internalization, however, is an individual process which does not depend only on shared moments, but on the professional development of each person, as well.

In this sense, like Perez et al. (2002), we believe in continuing education in which reflection on teaching practice, collaboration, and discussion are crucial elements, and teachers are provided with the conditions to face, individually and collectively, new and different learning situations. As we intend to show, it is our belief that online distance education brings new possibilities to continuing education, whether in the form of courses or of virtual communities oriented toward exchange. Consequently, the practicing teacher can interact with colleagues and specialists throughout Brazil and the world without leaving their work environment.

Therefore, teachers’ professional development and reflexive practice become the principal elements guiding continuing education in a way that considers them to be subjects in their own education and leads them to realize the importance of their teaching practice in the professional development process.

Continuing education can also constitute a process that broadens conditions for exchanging experiences and seeking innovations and solutions to problems that emerge in the everyday life of the school. Using teachers’ experience as the starting point for continuing education does not imply denial of the knowledge produced by educational sciences, but rather to consider practice as the starting point and finish line of the educational process. In this way, formal educational experiences become a space for reflection.

In the continuing education experiences that we organized in GPIMEM, face-to-face as well as distance, we were attentive to these considerations and sought to provide an environment of exchange that led to a collaborative educational process and learning. We worked within this perspective influenced by our conception of the role of the teacher in education processes, especially in online distance education.

Silva calls our attention to the fact that teachers need to prepare themselves for the role of teacher in an online distance education environment:

Rather than merely teaching, the teacher will have to learn to make multiple experimentations and expressions available, as well as creating networks that allow multiple occurrences. Rather than merely transmitting, he will be a formulator of problems, provoker of situations, architect of routes, mobilizer of the experience of knowledge.

(SILVA, 2003b, p.12)

In this quote, one observes that many of the characteristics described are also desirable for teachers in a face-to-face class. For authors like Maia
(2002), there is no difference between the teacher who works in a face-to-face situation and one who teaches distance courses. Both should have the basic characteristics needed to carry out the role of teacher, beginning with the premise that they are willing to share specific knowledge with a group of students, and therefore, their attention should be centered on learning, with a pedagogical proposal that includes relevant aspects such as the means of communication, the methodology, and others.

One could say, therefore, that the professional is the same; that a teacher who teaches face-to-face courses can also teach distance courses, and vice versa. However, the teacher needs to pay attention to her teaching practice which, focused on learning, must be adapted to a new environment and new pedagogical proposal that requires a different methodology from that used in a face-to-face class. For some authors, these changes demand a characteristic approach that gives life to a “new professional” who, according to Kenski (2003, p.143), “must act and be different in the virtual environment. This necessity is due to the very specificity of cyberspace, which makes possible new forms, new spaces, and new times for teaching, interaction, and communication among everyone”. He emphasizes that the teacher’s competence should shift toward encouraging learning and thinking, becoming the animator who incites students to exchange ways of knowing and to guide, in a personalized way, the course of the learning. It is important to propose tasks, establish reading lists, etc., so that the student feels the teacher’s presence even in a virtual learning environment.

Thus, it continues to be the teacher who defines the contents of the course and conducts it or, in some cases, the company administering the course (face-to-face or online) may define the contents, taking this right away from the teacher. In a differentiated pedagogical perspective, in which students have the possibility to explore the content collaboratively, or to pursue their own interests, the curricular structure and the teacher cannot be rigid. Thus, the communication cannot be one-way, from teacher to student, but in various directions, from student to student, student to students, teacher to student, and teacher to students.

Prado and Almeida (2003) point out that the role of the teacher is not specifically that of the source of information, but mainly of adviser and partner in learning, considering the ideas and particularities of the students. The teacher therefore needs to assume different roles, as mediator, observer, articulator: “His main function is to guide students’ learning – learning that develops through collaborative interaction […], facilitating the creation of a network of communication and collaboration in which everyone interrelates” (p. 72, author’s emphasis).

Some authors still emphasize that the administration of an online course, at first glance, seems easy, however most demand more preparation time and involvement than conventional courses. As an example, one can think of the
need to visit the virtual environment or website of the course daily, more than once a day if possible, so that students feel supported by the teacher. This demands a considerable amount of time. Based on the examples given, we will be specifying other competencies that online teachers should have, which are conditioned by the interface being used, and differ depending on the main means of communication – chat or videoconference, for example.

Therefore, it is essential to highlight that the use of computer technology demands, at least at first, a considerable time commitment from the teacher to prepare activities, plan, and attend to students, which must be constant so as to avoid discouraging students. And participation in courses for professional development and to remain up to date also demands time. The teacher must be familiar with the technological tool being used, which does not necessarily dispense with the need for a technician who can provide support to resolve problems with equipment, if necessary.

In this context, like Zulatto (2007), we understand that student and teacher, together with the technologies they use, walk hand-in-hand in the production of knowledge, considering that

\textbf{collaborative online learning} is a process in which students, teachers, and technology participate actively and interact at a distance to produce meanings collectively, raising uncertainties that encourage the search for understandings and raising new uncertainties. In this way, humans and media plan and develop actions that are of interest to a group, respecting the individualities, to produce knowledge collaboratively in cyberspace (p. 70, author’s emphasis)

Using a camera as a metaphor, we can consider the above quote to be a provisory synthesis when we zoom in with the camera, and when we zoom out, it transforms into an entire community that is interested in transforming itself as new technological possibilities are offered.

We believe new technologies are not to be feared or idolatized. ICTs transform our lives and change collaborative thinking and practice in continuing education courses and in other practices involving educators in the virtual world. On the other hand, we can shape the way online education becomes consolidated in Brazil and in the world, in the particular context of mathematics education. In this sense, we are fighting so that models based on interaction among participants prevail over mass models that are mainly profit-oriented and based on a vision of education as a commodity.
In the preceding chapter, we presented some elements related to online distance education, such as interaction, collaboration, dialogue, continuing education, and the role of the teacher, and we discussed how they are important for knowledge production in distance courses. In the context of mathematics, in particular, we showed how different media can transform the nature of this field of knowledge, based on examples from research.

In mathematics education, various studies have explored the possibilities of using chat as an interface, including Bairral (2002; 2004; 2005), Bello (2004), Lopes (2004), and others. In all of them, courses conducted via chat rooms and other interfaces were proffered to high school and university students as well as practicing teachers with the objective of investigating issues related to student assessment in online distance education, construction of mathematical knowledge on a given content, and others. For this, course models were elaborated based on perspectives related to the objectives of the studies as well as the view of knowledge of the course organizers.

GPIMEM, as described by Borba and Penteado (2001), began researching models for online distance education courses some time ago that converged with the group’s conception regarding teaching and learning based on aspects such as dialogue and the premise that knowledge is produced by collectives of human and non-human actors. In addition to pedagogical issues, the available technology was also taken into consideration due to the dependence on existing resources on the UNESP-Rio Claro campus to carry out the educational model adopted. The first university extension course was offered in 2000, entitled Trends in Mathematics Education, proffered entirely via distance by professors and researchers in the group. Since then, new versions of the course have been developed and offered annually for teachers of mathematics and related fields. Changes have been made over the years related to technical as well as pedagogical issues.

Based on our experiences and research, we will present a portrait in this chapter of some of our experiences in these courses, discussing the educational model we adopted and highlighting aspects related to chat and the mathematical discussions that took place using chat.
CHAPTER II

IN SEARCH OF A MODEL

Among various courses offered by GPIMEM, the Trends in Mathematics Education course can be considered to be a different kind of practice, as new versions of the courses were created and transformed in accordance with the new interests and experiences of the professors and researchers involved.

In the initial version offered in 2000, a chat resource was used that was available for free on a web page, in which the moderator could control access to the discussions through a registration system. A homepage was created as a complement to function as a bulletin board for posting information such as course syllabus and bibliographic references for the teacher-students. In addition, a discussion list, via e-mail, was also used for asynchronous communication among participants. We did not have access to virtual learning environments at that time, which later became available for free.

Versions of the Trends course, structured in this way, were successfully carried out and were the subject of study for researchers such as Gracias (2003), who addressed the nature of the reorganization of thinking based on the technologies employed as well as the pedagogical model adopted. Nevertheless, we sought different alternatives, including options for simultaneous interaction and information. Based on studies and pilot tests, we opted initially for TelEduc, a VLE that offers a range of options, with tools like forum, portfolio, discussion lists, bulletin board, chat, and others. With TelEduc, the nature of the course was changed based on the technological possibilities it provided. Research was carried out based on the versions of the Trends course that were offered in this VLE, including Borba (2004) and Santos (2006).

Nevertheless, in 2006, we migrated to another environment, TIDIA-Ae. The decision to change emerged as a result of our group’s collaboration in the development of this platform, which we saw as an opportunity to participate in the design of the environment. Taking advantage of our specialized knowledge, we have contributed suggestions to the developers with the aim of building a free environment that can be used for different activities in various fields of knowledge. In addition, the version-in-progress of TIDIA-Ae has some innovative tools, such as hypertext, which is a collaborative, asynchronous text editor.

In our view, there is no ideal model of environment for conducting courses of this nature, since the resources offered by each platform are distinct, making different options available to users. We believe that there is a virtual environment that adapts most coherently to the context according to the objectives established in advance. It is up to the organizer of the activities to analyze the pros and cons of each of the interfaces available. Issues related to the technology used should also be considered as one thinks about a course or the actions to be carried out using these platforms.
Based on our experiences as educators and organizers of distance courses, we were faced with the possibilities and the limitations of the platforms used (BORBA et al., 2005). In the context of mathematics education, these difficulties are strongly related to the very nature of the mathematical language, with its characterized by particularities that often make discussion difficult. For example, if we have a given problem whose sentence would be

$$\int_2^4 \left( \frac{1}{x^2} + x \right) dx$$

we would have to write “the integral defined in the interval of two to four of the function one over x squared plus x”, or “integral of 2 to 4 of 1 over x squared + x dx”. As we write the sentence, regardless of the manner chosen, in addition taking longer for the participant to interpret it and translate it into mathematical symbols, it can lead to mistakes, as it is well known that people often write informally in chat and use abbreviations to save time.

Initially, the objective of the Trends course was to present and discuss some of the existing areas of research in mathematics education, such as teacher education, ICTs, and mathematical modeling. The intention was to provide the teacher-students with an initial understanding of what research in mathematics education is and its different areas of study. In the process, we also began to ask what would happen when mathematical topics were discussed via chat. What would a mathematical discussion be like in a chat? Would mathematics be transformed in a virtual environment? Thus, beginning in 2002 when the course was being offered for the third time, we began to discuss mathematics in chat rooms based on activities developed within an experimental-with-technologies perspective, in which students (teacher-students, in our case) act together with software to generate, conjecture, and present solutions to a given problem.

The dynamic of these courses consists mainly of the teacher-students reading texts assigned by the professor prior to each synchronous meeting selected from a list of required as well as optional readings on the themes to be discussed in class. These synchronous meetings, which last an average of approximately three hours, are conducted via chat. Two debaters are chosen prior to each class to stimulate discussion and are responsible for presenting questions to their colleagues. The presence of the debaters does not prevent the other participants of the professors from raising issues during the class; rather, their role is to stimulate discussion, providing them with the opportunity to practice “leadership” in activities of this nature. In this way, we believe we are contributing to the educational experience of teachers who may be proffering distance courses themselves in the future, an issue pointed out by Borba (2004) as being important. At the end of each session, one of the teacher-students is given responsibility for writing a summary of the class and making it available to the others. Due to the course structure described here,
the number of participants is limited to 25 so that everyone can “speak” and “be listened to”. On occasion, when possible, we invite the authors of the texts being debated to participate in the discussions. Another practice we have adopted is to occasionally ask specialists on the themes being discussed to take part in the class as invited guests, enriching the debate.

For those classes in which the theme to be discussed was a specific mathematical content, activities were planned and made available ahead of time to the teacher-students. Activities involving the use of software generally made use of open-source software; in the case of Geometricks\(^7\), which was used in one of the versions of the course, participants had the option of purchasing the software or using a free demo version.

As the reader can see, the model adopted for the course is in synergy with our proposals as well as intimately related with our view of knowledge, as we believe that the act of learning is not passive, and that interaction between professors and students in debates is fundamental. It is thus up to the professors to identify those who are not engaging in discussions and question them. This has been a constant practice in the courses offered by GPIMEM since 2000.

Over the course of the meetings, the teacher-students come to realize that the prior readings are fundamental to be able to participate in the discussions, and that due to the dynamic of the course as well as the nature of chat, there are no long explanations or talks on given themes. The reader may be asking at this point, “Why due to the nature of chat?” Discussions in chat rooms have qualitatively different characteristics than those that take place in other learning environments, virtual or otherwise. As mentioned in the introduction, there is a tendency in chat for multilogue to occur, i.e. simultaneous conversations on subjects directly or indirectly related to the main focus of the meeting, with participants sometimes involved in more than one discussion, or “jumping” from one to another. We know that the word “dialogue” characterizes a conversation between two or more people; thus, the idea of multilogue is related to the multiplicity of dialogues taking place at the same time in a chat session. In addition, they are not linear, in the sense that questions and responses do not appear sequentially on the screen. Borba and Penteado (2001), for example, to illustrate a multilogue, presented an excerpt from a chat session using different fonts to highlight the different dialogues going on in order to help the reader.

With this multiplicity of simultaneous conversations, it is often difficult for the professor, as well as the teacher-students, to follow them all and give feedback. This dynamic changes the very nature of the production of knowledge, which is conditioned by the interaction that takes place in a chat room. Those who have never taken part in this type of discussion feel lost and confused at first; however, participants appeared to adapt “naturally”, since after a few meetings, they no longer mentioned having problems with the
“avalanche” of information and questions occurring simultaneously.

Up to this point we have been discussing, in some detail, issues related to the model we adopted for the course and some of the consequences. We would still like to highlight how discussion occurs via chat during activities related to a specific field of knowledge, mathematics. In the section that follows, we will provide examples that illustrate some specific aspects of the discussion and production of mathematics using chat.

First, however, we would like to emphasize that professors who teach using chat rooms must cope with new demands. They must be prepared to deal with various questions at the same time, referring to distinct aspects of the theme under debate. In our courses, professors commonly need to handle different questions at the same time during a mathematics education class, since the teacher-students ask questions without having read the questions of the others. For example, texts A and B are being debated during a given meeting. Even when the professor decides, after dialoguing with students, to initiate the discussion with text B, it is not uncommon for questions to be raised about text A. Gracias (2003) illustrates this well. In mathematics classes, there is a need to follow different lines of reasoning on a given problem simultaneously; and when the professor, for some reason, allows a question to go by unnoticed, participants protest emphatically. Knowing how to handle these demands, as well as having the ability to type quickly and read the messages on the screen at the same time, appear to be important skills for this type of teaching. As we will see, special demands also emerge when dealing with mathematics, especially geometry, when participants do not share a common screen, as in one of the examples that follows.

MATHEMATICAL PROBLEMS IN CHAT

Examples of Functions

When we consider virtual discussions about mathematics, questions arise regarding the type of activities we should propose to the teacher-students. Borba (2004; 2005) offered some reflections in this regard, providing evidence that there are pedagogical approaches that may be more appropriate and adapted to the possibilities provided by the Internet. Since, as outlined in Chapter I, our view is that knowledge is the product of a collective and dialogue, interaction and collaboration, and that these are factors that condition learning, we believe that open, exploratory activities are in synergy with virtual educational practice.

Based on these premises, activities were elaborated in which teacher-students were encouraged to use mathematical software, like Winplot®, for example, to explore conjectures based on plotted functions.

Activities with specific mathematical contents were introduced in the third version of the Trends course, with discussions involving functions and
Euclidean geometry. Other mathematical topics, like fractals, spatial Euclidean geometry, and mathematical models, were explored in later versions of the course. In the case of fractals, the debates followed a model similar to the readings-based mathematics education classes, as it was a new theme for most participants, and the class was based on another book from this series written by Barbosa (2002).

In one class in which the theme was functions, one of the proposed activities was based on the graph below (FIG. 1). Participants received only a Word file containing the graph and were asked to find the algebraic expression that represents it using graphing software to “experiment” and deduce.

Figure 1

The professor, who is the first author of this book, presented one of the objectives of the activity: “The [...] proposal of the activity is to be able to present a problem originating from the graph and not the algebra”, i.e., the activity consisted of finding the algebraic expression for the graph. At the very beginning of the discussion, one of the teacher-students asked, “… how do I enter functions defined by multiple expressions in Winplot?” The professor then provided a clue, saying that the law of function is not composed of various expressions, but rather a product of functions. The discussion that ensued is presented below. Note that the professor is identified as mborba in some excerpts and as MarceloBorba in others, and the others are teacher-students.

The time the comments were written appear in parentheses.

(20:30:53) Nilceia: It’s the product of a quadratic and a trigonometric?

(20:31:35) Luiza: This one is really difficult. You can see is that we have one curve for x < 0 and another for x > 0, and that in one of the
In the above excerpt, we can notice that the teacher-students were using visual aspects of the graph to conjecture about the possible law of the function. However, one of the participants, after the professor’s “clue”, felt the need to do a test, experimenting with software, saying that observation alone was insufficient. For this reason, she opened Winplot and drew a graph of the function $f(x) = x \cdot \sin(x) \cdot x^2$, illustrating that experimentation also takes place simultaneously with the chat discussion. We can also note that the way teacher-students wrote the laws of the functions mathematically using chat is the same used to insert a function in Winplot, making it one of the mathematical languages used in chat. In this sense, it appears to be natural for teacher-students to use the software’s language in the chat session.

Since there was a pause in the discussion at this point, the professor suggested presenting the answer to the problem. The reactions to this varied:

(20:50:32) Eliane: Is there any way you could give the answer and wait just a minute to test it in Winplot?

(20:50:44) Nilceia: Yes, but please explain how we arrived at this conclusion.

(20:50:53) mborba: Of course, Eliane. And I will give it to you little by
Based on what the teacher-students say, one can note different positions. Eliane, for example, wanted the answer so she could use it to get a glimpse of a possible solution based on experimentation with software. Nilcêia was also less worried about the answer itself than understanding the solution found by the professor. It is worth noting that, as the above excerpt shows, the teacher-students were willing to think about the activity, and the mere presentation of the solution did not satisfy them, whereas exploring did. After a few comments had been posted, the professor presented the solution to the problem:

(20:52:07) mborba: The “little belly” is because the parabola, which I already said is part of the product, interacts with the other function in the interval [-4,0] in a way that the former “still has power”; later it no longer does.

(20:53:38) mborba: In other words, the function “defeats” the second degree function in the rest of the negative interval, and “coincides”, it has “synergy” with the parabola and the exponential e^x, or with another base greater than one . . . so then the size of the “little belly” will vary.

(20:54:42) mborba: [...] but here I thought I could say something right away so you guys could evaluate this type of activity and test the function y=e^x(x^2).

(20:54:58) Eliane: Now I understand, cuz it begins to increase after the axis and keeps going and accelerates the growth of the parabola even more. Cool . . .

(20:55:08) mborba: Note that this informal way of talking about the graph should only be at first.

(20:55:47) mborba: What “it” are you referring to, Eliane? The product, for sure, because the exponential is always increasing, with a base greater than one.

(20:56:54) Nilceia: To arrive at this conclusion, do I need to test various graphs?

(20:57:48) Eliane: Sorry, it becomes greater than one, so the product becomes increasingly greater than the value of the parabola itself, which does not happen before the y-axis, because the exponential is less than one, and the product, although increasing, makes the parabola go down.

(20:57:54) mborba: Nilceia, I think that it could be one way for us to learn about functions, about their behavior. In fact, the second graph is
At this point, the professor asked the teacher-students how the questions presented up until that moment could be formalized, which led to other discussions about aspects related to the software and its use in the classroom, how to make the “bridge” between exploration and formalization, etc.

We interpret the law of this function as being the product of the two functions when $x$ tends toward $+\infty$ or $-\infty$. In the first case, the two functions are increasing and positive, and $\lim_{x \to +\infty} e^x = \lim_{x \to +\infty} x^2 = +\infty$. The product of these limits is always increasing, therefore $\lim_{x \to +\infty} x^2 \cdot e^x = +\infty$. In the second case, a classic argument to explain the behavior of the product when $x \to -\infty$ is that the exponential function tends to zero “faster” than the quadratic function goes to $+\infty$, and since the exponential function is “stronger” than the polynomial function, the product of the two functions tends to zero.

Going back to the excerpt presented above, we can note that the approach of the course, which was developed using chat, focused not only on mathematical discussions, but also on educational issues related to this field of knowledge, characterizing a reflection on pedagogical practices based on the experiences of the teacher-students.

In another Trends course, the theme discussed was also functions, based on experimental activities. The problem being discussed had originated from a face-to-face class in an applied mathematics course for biology undergraduates at UNESP, Rio Claro. Using graphing calculators, students had been asked to vary the parameters $a$, $b$, and $c$ of the function $f(x) = ax^2 + bx + c$, $a \neq 0$ and discuss the behavior of the graph. As they carried out the activity, a student named Renata had presented the following conjecture: whenever $b$ is positive, the parabola will cross the $y$-axis with the increasing part of the parabola. Whenever it is negative, it will cross it with the decreasing portion of the parabola. Thus, we proposed to the teacher-students in the distance course that they investigate the parameters $a$, $b$, and $c$ of the second degree function to explore Renata’s statement and justify their responses.

One of the teacher-students, Carlos, began the debate by telling that he had proposed to his students that they also investigate the parameters $a$, $b$, and $c$ with the help of Winplot, and that during the class, one of them had presented the following conjecture: “when $a$ is negative, or $b$ is positive, the parabola moves more to the right, but when $a$ is negative and $b$ is also negative, the parabola moves to the left”. Since the student’s hypothesis was intimately related to the activities we had proposed, we asked the teacher-students to discuss Carlos’ student’s conjecture, as well, and to emphasis the algebraic questions.

(19:53:15) MarceloBorba: The solution that Carlos’ student gave
regarding a and b; does anyone have an algebraic explanation for it?

(19:54:53) Taís: It has to do with the coordinate x of the vertex of the parabola.

(19:55:30) Carlos: After trying various times (they made various graphs varying the value of the a, b, and c coefficients), they concluded that the proposal in question 2 [presented by Renata, the biology student] was really valid.

The arguments presented differed. While Carlos affirmed that his students had concluded, through experimentation, that Renata’s statement was correct, Tais presented evidence of a conclusion based on the formula of the vertex of the second degree function, i.e., she began to give an algebraic explanation for the problem. As we can observe in the discussion that followed, the arguments overlap:

(19:57:07) Taís: Xv=-b/2a...If a and b have different signs, Xv is positive.

(19:59:16) Norma: I made many graphs and verified it was true; afterwards, I went to the coordinates of the vertex of the parabola Xv= -b / 2a, and I analyzed the sign of b as a function of positive or negative a; then I checked the sign of the vertex crossing the concavity opening up or down and checking to see if it was on the increasing or decreasing part. I don’t know if I made myself clear.

Norma and Tais presented similar ideas to confirm the biology student’s conjecture. Sandra and Marcelo elaborated yet another justification for the statement based on the derivative of the second degree function, i.e. f’(x) = 2.a.x + b.

(20:00:43) Sandra: I understand, but I also understand that the increasing part of the parabola is when the first derivative is positive, i.e. for x>-b/2a.

(20:01:18) MarceloBorba: Why, Sandra? Say more about how you saw that!

(20:03:55) Sandra: If we have the equation ax^2+bx+c= y, then we have that dy/dx = 2ax+b, so if we want to know when the parabola is increasing, we have to calculate when dy/dx >0.

(20:05:04) Sandra: So, x > - b/2a.

(20:07:03) MarceloBorba: Sandra, except I saw it a bit differently. I saw calculating y'(0)=b, so when b is positive, the parabola will increase, and analogously . . .

Since few of the participants understood the explanation, Marcelo and Badin explained the solution in greater detail.

(20:10:59) MarceloBorba: Sandra, upon calculating the value of y’, I
have that if \( y' > 0 \), then the function is increasing, and therefore I take \( y'(0) \), which is equivalent to the point where \( y \) crosses the \( y \)-axis and \( \text{the function } y'(0) = b \), and so \( b \) decides it!!! Understand?

(20:13:43) Badin: The point where the graph “crosses” the \( y \)-axis is \( f(0) \), therefore, to be in the increasing part of the parabola, we should consider two cases:

1. \( a > 0 \): we should have the \( x \) of the vertex “before” the abscissa of the point where the graph crosses the \( y \)-axis, i.e. \( (-b/2a) < 0 \). Since \( a > 0 \), this occurs for \( b > 0 \).

2. \( a < 0 \): we should have the \( x \) of the vertex “after” the abscissa of the point where the graph crosses the \( y \)-axis, i.e. \( (-b/2a) > 0 \). Since \( a < 0 \), this occurs for \( b > 0 \).

By this time, the teacher-students had been debating the problem and its solutions – vertex and derivative – for approximately 40 minutes. The time interval gaps in the transcriptions indicate where some parts were omitted to make it easier to follow the discussion. Other actors were involved in the debate and in refining the solutions to this problem, but we have opted to present only some of them for the sake of clarity.

We can highlight that the justifications for the conjectures presented via chat were written “naturally”, using “natural language”, given that the written word is the means of communication in a chat session. In addition, participants tried to make themselves understood through their “written words”. In the face-to-face class, when Renata presented her conjecture, orality was the main actor in communication, and the students did not write their conclusions and justifications, but only spoke them. The professor of the Applied Mathematics class relied on the chalkboard to formalize Renata’s conjecture, presenting the vertex solution to the class.

Example from Spatial Geometry

Some aspects of writing in chat are qualitatively different from writing with paper and pencil, and as we discuss mathematical questions in this environment, “doing” mathematics becomes transformed online. Writing, like multilogue, shapes the production of mathematical knowledge in different ways in online environments.

While this seemed reasonable when the content being addressed was functions, we decided to investigate what would happen with the mathematical discussion in the virtual world when the content was spatial geometry. Our research group was curious to know how the tension would play out between the model of the notion of space that most of us have, spatial geometry, and the Internet, which appears to destroy the notion of space and/or time that we had up until the end of the 20th Century, and constitutes a new notion. Readers interested in this theme can learn more by reading a detailed discussion in Santos (2006).
In 2005, the Trends course was the stage for this investigation about mathematical production in the chat room. Various trends in mathematics education were discussed in this edition of the course, led by the first author of this book, and Silvana Santos led discussions on mathematical activities. Her work (SANTOS, 2006) revealed various particularities of the nature of “doing” mathematics in the context of online distance education using diverse resources, such as manipulative materials, the free software Wingeom\(^1\), as well as books on the subject. One of her conclusions emphasizes that, despite some limitations, the chat tool enabled discussion and made it possible for “mathematical production to become consolidated in a very specific way” (p.12). For her, these media conditioned the way the participants discussed the conjectures formulated during the geometrical constructions, thus transforming mathematical production. In her analysis, she highlighted the ways chat, the software, the coordinated use of different ICTs, investigation, and visualization are present and act in the production of mathematics in a distance course. Another aspect she emphasized is related to mathematical demonstration in a virtual environment, and to illustrate this, we present an example. It should be noted that the activities proposed by Santos were of an investigative nature, and that the teacher-students were expected to use the Wingeom software to develop them.

In one of the problems, participants were asked to construct a rectangular parallelepiped and then sketch some segments and observe what would happen to the planes formed by points belonging to these segments and others belonging to their vertices. The statement of the problem\(^12\) was as follows:

1. Insert a parallelepiped of length \(\#\), width 2 and height 6;
2. Using the menu Anim/Variation of \(\#\) type, in the window that opens, 0 and then click fix L. In the same way, type 3 and click fix R;
3. Sketch segments AC, CH, and AH;
4. Click on See/Thickness of segment, select a color for the segments you sketched, and then click on add;
5. Sketch segments BE, EG, and BG;
6. As before, add a color for these segments;
7. Animate your construction and observe what happens;
8. What can you say about the planes defined by points ACH and BEG? Justify your answer.
9. What happens when the value of \(\#\) is zero?

CHAPTER II
The Wingeom software has animation functions based on specific commands. The proposed activity requested fixed width and height for the parallelepiped, but the length could vary from 0 to 3. Thus, the solid could be “animated” based on the variation of its length. FIG 2 illustrates the construction of this activity in Wingeom.

The chat discussion began when one of the participants, Maria, from Argentina, presented her conclusion: “In this activity, the conclusion I got was that the planes (GEB and HAC) are parallel (if #=0 are coincident), but I saw Marie’s demonstration” (20:17:15). Marie, another teacher-student, had posted her demonstration earlier in the portfolio of TelEduc and referred to it again in the chat:

(20:21:08) Marie: Look how I justified it: Planes ACH and BGE are parallel. We can justify it remembering that a straight line is parallel to a plane when it is parallel to a line on that plane. Considering that segment BG is parallel to segment AH (both diagonals of opposite and parallel faces of the parallelepiped) which belongs to plane ACH, we can state that segment BG is parallel to plane ACH. If we apply the same reasoning to segments EG and EB, we conclude that the planes are parallel.

(20:22:22) Marie: Look at my figure in the portfolio. You can understand it better when you can visualize the figure.

(20:22:28) Silvana: Marie, I don’t understand . . . We can justify it remembering that a straight line is parallel to a plane when it is parallel to a line on that plane??? Sorry . . .

(20:23:12) Marie: Yes, that’s what descriptive geometry says.
Maria: What Marie said is true: if a straight line is parallel to another on a plane, it is parallel to the plane (I have the Theorem here).

Silvana: Ah, OK!!!

In this excerpt, we notice that Marie does not state that she demonstrated, but rather that she justified her conclusion, and she suggests that it is easier to understand if one looks at the figure she constructed and posted in the portfolio. Another point we would like to emphasize is related to the various resources used by the teacher-students. Silvana Santos, who was co-teaching the course, did not understand Marie’s justification, and Maria, who had a book by her side when she said “I don’t know all the theorems . . . I have the reference book next to me here!!! Ha!!!” (20:34:07), confirmed what Marie had said based on a theorem.

More arguments, doubts, and demonstrations were presented by the teacher-students as the debate continued:

Dias: Marie, another doubt is noticeable that the planes GEF and…

Claudia: Sil, we used the following, but I’m not sure

Def.: Two planes are parallel, if and only if they have no points in common or are equal.

Sufficient condition: EG and BG to the plane BEG and are concurrent, having AC and AH on plane ACH and EG // AC and BG // AH, plane EGB is parallel to ACH.

Dias: ACH are “sub-planes” of parallel planes, but can we conclude from that that they are parallel?

Maria: What is ACH “sub-planes”?

Carlos: (a small correction) EG and BG belong to plane BEG and are concurrent, having AC and AH belonging to plane ACH and EG // AC and BG // AH, so planes EGB is parallel to ACH.

Marie: Let’s think in practical terms. You are probably in a room, right? Think of the walls as planes. Think of a door, and consider one of the door posts as a straight line. Isn’t this “straight line” parallel to a straight line on the wall opposite the wall where the door is? Think of a straight line as where the walls meet. So then you can say that the door is parallel to the wall because it is parallel to the intersection of the two walls, which is considered a straight line. Is that clear, or did it confuse things more?

Maria: Dias, planes are infinite...

In this excerpt, we can see that demonstration, in a chat, as Santos emphasized, is done “in pieces”, in other words,

One part is presented and then another message appears from someone else, about the same subject or not, and meanwhile, the one who was
presenting the demonstration is typing and presents another part, and the process repeats itself until the demonstration is concluded. (SANTOS, 2006, p.98)

In addition, Marie used a visual image to illustrate her conjectures and tried to get the other participants to “see” the figure. In response, others presented more statements, continuing the justifications posted earlier.

(20:29:50) Dias: from the same idea as subset. Taking the same example as Marie’s: Take the wall that represents a plane and a chalkboard attached to it; the chalkboard is a plane subset of the wall.

(20:30:42) Silvana: Marie, I like your strategy of making things more “visible”…But let me think better about this door…haha.

(20:31:39) mborba: I thought Marie’s demonstration was terrific [uma beleza], as Paulo Freire would say… simpler than Silvana’s… And Claudia’s ….. I’m not sure I understood. What do you guys think? Claudia, can you explain to me better…

(20:31:41) Maria: Yes I understand ACH, but I saw it as a plane and not as a subset of the plane.

(20:31:45) Marie: Maria, we make a great pair. You know all the theorems and I know things from practice, from having worked and tried hard to make geometry “concrete” for my students.

(20:34:33) Carlos: We tried to show that there are two concurrent straight lines in a plane that are parallel to the other plane.

(20:39:59) mborba: Claudia, I understood your demonstration, with Carlos’ explanation, and I think it is the simplest! Congratulations! Nice work!

We notice in this excerpt that terms presented in the course of the debate were clarified with the help of other participants. In addition, Marie highlighted distinct approaches to dealing with geometrical problems: mathematical formalization and “practical” work, which we interpret as being exploratory activities with software, manipulative material, etc. In this way, we can identify that the two approaches are complementary, from the perspective of the teacher-students as well as the professors, since it becomes explicit that this activity, in particular, addresses both aspects. We also note that various mathematical argumentations were presented during the debate, and that some were easier to understand than others for some of the participants. Perhaps chat itself, due do its nature, allows various demonstrations and justifications to be presented simultaneously, thus characterizing a transformation in the mathematics produced using chat.

In this chapter, we presented some examples of mathematical problems discussed in chat, and we pointed out that transformations occur in mathematics education when activities that are specific to this field of knowledge are debated in chat sessions. The nature of mathematical
discussions in chat is qualitatively different from those that take place via videoconference, and to exemplify this, we present some examples in the following chapter of GPIMEM’s experiences with teacher education courses in which the mathematic production is conditioned by another actor, the videoconference, which has unique aspects such as orality and sharing of images.