Mathematical tasks have long been recognized as crucial mediators between mathematical content and the mathematics learner. For many people, the mathematics classroom is defined by the type of tasks one finds there - and this is appropriate. Mathematical tasks are the embodiment of the curricular pretext that brings each particular set of individuals together in every mathematics classroom. In other contexts, individuals come together to engage in musical performances or dramatic performances. The performances of the mathematics classroom are largely the performance of mathematical tasks and if we are to understand and facilitate the learning that is the purpose of such settings then we must understand the nature of the performances that we find there.

The classroom performance of a task is ultimately a unique synthesis of task, teacher, students and situation. Of particular interest are differences in the function of mathematically similar tasks when employed by different teachers, in different classrooms, for different instructional purposes, with different students. By making comparison possible between the classroom use of mathematical tasks in different classrooms around the world, the analyses reported in this book reveal the profound differences in how each teacher utilises mathematical tasks, in partnership with their students, to create a distinctive form of mathematical activity.

The Learner’s Perspective Study aims to juxtapose the observable practices of the classroom and the meanings attributed to those practices by classroom participants. The LPS research design documents sequences of at least ten lessons, using three video cameras, supplemented by the reconstructive accounts of classroom participants obtained in post-lesson video-stimulated interviews, and by test and questionnaire data, and copies of student written material. In each participating country, data generation focuses on the classrooms of three teachers, identified by the local mathematics education community as competent, and situated in demographically different school communities within the one major city. The large body of complex data supports both the characterisation of practice in the classrooms of competent teachers and the development of theory.
Mathematical Tasks in Classrooms around the World

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DEDICATION

With the greatest respect and affection this book is dedicated to

GODFREY ISHMAEL SETHOLE
25 SEPTEMBER 1967 TO NOVEMBER 16, 2007

Godfrey Sethole was a respected mathematics educator in South Africa. He was an
Associate Professor and Head of the Mathematics, Science and Technology
Department at the Tshwane University of Technology and played an important role
in the development of the Association for Mathematics Education of South Africa
(AMESA) in the North West Province. Godfrey was much loved and respected by
all those of us who had the privilege (and the fun) of working with him on the LPS
project. He will be missed by all who knew him: for his quick intelligence, for his
warmth and for his sense of humour. The LPS community is the richer for
Godfrey’s contribution and we miss him deeply.
SERIES PREFACE

The Learner’s Perspective Study provides a vehicle for the work of an international community of classroom researchers. The work of this community will be reported in a series of books of which this is the third. The documentation of the practices of classrooms in other countries causes us to question and revise our assumptions about our own practice and the theories on which that practice is based. International comparative and cross-cultural research has the capacity to inform practice, shape policy and develop theory at a level commensurate with regional, national or global priorities. International comparative research offers us more than insights into the novel, interesting and adaptable practices employed in other school systems. It also offers us insights into the strange, invisible, and unquestioned routines and rituals of our own school system and our own classrooms. In addition, a cross-cultural perspective on classrooms can help us identify common values and shared assumptions, encouraging the adaptation of practices from one classroom for use in a different cultural setting. As these findings become more widely available, they will be increasingly utilised in the professional development of teachers and in the development of new theory.

David Clarke
Series Editor
# TABLE OF CONTENTS

Acknowledgements........................................................................................................... xi

1. The Role of Mathematical Tasks in Different Cultures...........................................1
   Yoshinori Shimizu, Berinderjeet Kaur, Rongjin Huang and David Clarke

2. A Study of Mathematical Tasks from Three Classrooms in Singapore.............15
   Berinderjeet Kaur

3. Mathematical Tasks as Catalysts for Student Talk: Analysing Discourse
   in a Norwegian Mathematics Classroom....................................................................35
   Ole Kristian Bergem and Kirsti Klette

4. Engaging Students with Mathematical Tasks in a Large Class ....................63
   Florenda Gallos Cronberg

5. A Task-Specific Analysis of Explicit Linking in the Lesson Sequences
   in Three Japanese Mathematics Classrooms......................................................87
   Yoshinori Shimizu

   of the Czech Republic..........................................................................................103
   Jarmila Novotná and Alena Hošpesová

7. Comparison of Learning Task Lesson Events between Australian
   and Shanghai Lessons..........................................................................................119
   Ida Ah Chee Mok

8. Implementing Mathematical Tasks in US and Chinese Classrooms...............145
   Rongjin Huang and Jinfa Cai

9. Student-Created Tasks Inform Conceptual Task Design...............................165
   Gaye Williams

10. A Functional Analysis of Mathematical Tasks in China, Japan, Sweden,
    Australia and the USA: Voice and Agency......................................................185
    Carmel Mesiti and David Clarke

Appendix A: The LPS Research Design.................................................................217
    David Clarke

Author Index.............................................................................................................233

Subject Index...........................................................................................................239
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THE CENTRALITY OF TASKS IN MATHEMATICS CLASSROOM INSTRUCTION

Mathematics classroom instruction is generally organised around and delivered through students’ activities on mathematical tasks (Doyle, 1988). International comparative studies on mathematics classroom instruction tend to report analyses of such students’ activities engaged in tasks that occupy various amounts of lesson time in classrooms. Notably, in all of the seven countries that participated in the TIMSS 1999 Video Study, eighth-grade mathematics was most commonly taught by spending at least 80% of lesson time in mathematics classrooms working on mathematical tasks (Hiebert et al., 2003).

Classroom activities are coherent actions shaped by the instructional context, in general, and, in particular, by what is taught through the use of tasks (Stodolsky, 1988). Individual teachers arrange instruction very differently, depending on what they are teaching, and students respond to instruction very differently, depending on the structure and demands shaped by tasks enacted in the classroom. The tasks that teachers assign can determine how students come to understand what is taught. In other words, tasks serve as a context for students’ thinking, during and after instruction. Doyle argues the point that

tasks influence learners by directing their attention to particular aspects of content and by specifying ways of processing information. (Doyle, 1983, p.161)

Mathematics tasks are important vehicles for classroom instruction that aims to enhance students’ learning. To achieve quality mathematics instruction, then, the role of mathematical tasks to stimulate students’ cognitive processes is crucial (Hiebert & Wearne, 1993).

In summary, the centrality of tasks in mathematics classroom is evident from theoretical perspectives as well as in empirical results from international comparative studies. The role of mathematical tasks provides a key to any attempt to understand teaching and learning in research on classroom practices in mathematics.
MATHEMATICAL TASKS AND THE LEARNER’S PERSPECTIVE STUDY

This book is the third in a series arising from the international collaborative project called The Learner’s Perspective Study (LPS). The LPS documented sequences of at least ten lessons, using three video cameras, supplemented by the reconstructive accounts of classroom participants obtained in post-lesson video-stimulated interviews, and by test and questionnaire data, and copies of student written material (Clarke, 1998, 2001, 2003). In each classroom, formal data generation was preceded by a one-week familiarisation period in which the research team undertook preliminary classroom videotaping and post-lesson interviewing until such time as the teacher and students were accustomed to the classroom presence of the researchers and familiar with the research process. In each participating country, the focus of data generation was the classrooms of three teachers, identified by the local mathematics education community as competent, and situated in demographically different school communities within the one major city. For each school system (country), this design generated a data set of at least 30 ‘well-taught’ lessons (three sequences of at least ten lessons), involving 120 video records, 60 student interviews, 12 teacher interviews, plus researcher field notes, test and questionnaire data, and scanned student written material. Well-taught, in the context of this study, meant that the teachers in each country were recruited according to local criteria for competence: visibility as presenters at conferences for other teachers, leadership roles in professional organisations, and, acclamation by colleagues and students. It is not surprising, therefore, that the classroom of a competent teacher in Uppsala might look a little different from the classroom of a competent teacher in Shanghai or San Diego. The local construction and enactment of competence was one of the most appealing aspects of this study. Greater detail on data generation procedures is provided in the appendix to this book. Signature elements of the LPS Research Design are (i) the commitment to studying ‘competent’ teachers as these are locally defined; (ii) the recording of a sequence of at least ten lessons constituting a mathematics topic; and (iii) the use of classroom videos in video-stimulated reconstructive interviews with teacher and students as soon as possible after the recorded lesson. The teacher and student interviews offer insight into both the teacher’s and the students’ participation in (and reconstruction of) particular lesson events and the significance and meaning that the students associated with their actions and those of the teacher and their classmates.

The classroom use of mathematical tasks has been addressed in previous publications from the Learner’s Perspective Study (LPS). For example, the first book in the LPS series (Clarke, Keitel, & Shimizu, 2006) included a chapter on “Setting a Task” (Keitel, 2006) and another on “The Role of the Textbook and Homework” (Kaur, Low, & Seah, 2006), and the second LPS book (Clarke, Emanuelsson, Jablonka, & Mok, 2006) included a chapter on “Learning Tasks” (Mok & Kaur, 2006). It is difficult to imagine any substantial investigation of the mathematics classroom that did not address the tasks that characterise such
settings. This book, the third in the LPS series, is devoted entirely to research into the role of mathematical tasks in the classrooms of different countries.

THE ROLE OF MATHEMATICAL TASKS IN DIFFERENT CULTURES

THE NATURE OF MATHEMATICAL TASKS IN CLASSROOMS

A mathematical task has been defined as a set of problems or a single complex problem that focuses students’ attention on a particular mathematical idea (Stein, Grover, & Henningsen, 1996). The construct ‘task’ also includes the intellectual and physical products that are expected of students, such as the operations that students are to use to obtain the desired results, and the resources that are available for students to produce the products (Doyle, 1983). In his elaboration of the construct ‘task,’ Doyle (1988) included the following components: The product(s), such as numbers in blanks on a worksheet; the operation(s) required to produce the product; the resources drawn upon in completing the task, such as notes from textbook information; and, the significance or ‘weight’ of a task in the accountability system of a class.

In their critique of ‘minimal guidance’ instruction, Kirschner, Sweller and Clark (2006) make the insightful observation that

it may be an error to assume that the pedagogic content of the learning experience is [should be] identical to the methods and processes (i.e., the epistemology) of the discipline being studied. (p. 84)

In particular, their assertion that “The practice of a profession is not the same as learning to practice the profession” (p. 83) highlights a critical issue in the design of instruction in mathematics. How is classroom mathematical activity related to the activity of the mathematician? While we may classify the tasks of the mathematics classroom in a variety of ways, we should not confuse those tasks with the tasks of the mathematician: they are fundamentally different in purpose.

Mathematical tasks employed in educational settings have been variously categorised under designations such as ‘authentic,’ ‘rich’ and ‘complex.’ The classification ‘authentic’ has particularly emotive overtones – suggesting that some mathematical tasks might be classified as ‘inauthentic.’ The most common usage of the term ‘authentic’ in this regard seems to refer to an assumed correspondence between the nature of the task and other mathematical activities that might be undertaken outside the classroom for purposes other than the learning of mathematics. The value attached to ‘authentic mathematical tasks’ seems to appeal to a theory of learning that measures mathematical understanding by the capacity to employ mathematical knowledge obtained in the classroom in non-classroom (‘real-world’) settings and which constructs the process of mathematical learning as ‘legitimate peripheral participation’ (Lave & Wenger, 1991) in the mathematical activities of a community larger than a mathematics class.

To illustrate another classificatory scheme: Williams and Clarke (1997) developed a framework that identified the following forms of task complexity: linguistic, contextual, numerical, conceptual, intellectual, and representational. In her subsequent research, Williams (2000, 2002) identified a phenomenon she
called ‘discovered complexity.’ This research identified situations in which students discovered complexities in the course of attempting a mathematical task posed by the teacher. These discovered complexities in combination with other instructional and learning conditions would stimulate the creation by students of their own mathematical tasks. These student-created tasks provide the focus of Chapter 9.

An academic task can be examined through a wide variety of attributes. When a teacher selects a task for her/his teaching, she/he may think of such task attributes as: context, complexity, degree of openness, form and representation. In the mathematics education research tradition, considerable research has been devoted to task attributes which affect students’ mathematical problem solving (Goldin & McClintock, 1984). Each of the attributes just listed has its own structure and variations – as has just been illustrated in the case of complexity.

As these earlier studies pointed out, a systematic analysis of task attributes has direct implications for teaching and learning in mathematics classroom, and particularly for teaching via problem solving. As discussed below, it is especially important that attention be given to the analysis of the cognitive demands enacted by tasks presented in the classroom and to the situated nature of the task as it is enacted by teacher and students in the classroom.

Task attributes need to be considered in relation both to the teacher and the learners in a mathematics classroom as well as in relation to broader contextual influences such as the curriculum, social expectations and so on. Also, considerations of the ternary relations such as Teacher-Task-Learner or Learner-Task-Mathematics are needed to explore fundamental aspects in mathematics teaching and learning (Christiansen & Walther, 1986). Study of the nature of such relations will reflect the choice of theoretical framework.

Although attention to the nature of mathematical tasks is important, attention to the classroom processes associated with mathematical tasks is equally needed. Such student activities as making conjectures, abstracting mathematical properties, explaining their reasoning, validating their assertions, and discussing and questioning their own thinking and the thinking of others do not fit well with the tasks and task use employed in many ‘traditional classrooms’ (Lampert, 1990). Yet such activities are the explicit goal of many curricular initiatives (for example, NCTM, 1989) and it is mathematical tasks that provide the pretext and the catalyst for such activities.

Any consideration of the nature of mathematical tasks in classrooms must attend to elements such as: task complexity, social participants and the nature of participation, socio-cultural context, and, most importantly, the purpose for which the task has been introduced into the mathematics classroom. Some of the chapters of this book explore such purposes: mathematical tasks as catalysts for student talk (Chapter 3 by Bergem and Klette); the function of mathematical tasks in connecting mathematical ideas (Chapter 5 by Shimizu and Chapter 6 by Novotná and Hošpesová); and the use of mathematical tasks “to scaffold student entry to idiosyncratic exploration” (Chapter 9 by Williams). In Chapter 2, Kaur distinguishes tasks according to their use for learning, review, practice or
THE ROLE OF MATHEMATICAL TASKS IN DIFFERENT CULTURES

assessment, and then employs an analytical framework synthesised from other studies to carry out a more fine-grained analysis of task use in three Singapore classrooms.

It is the intention of this book to contribute to the extensive and diverse literature on mathematical tasks in two ways: by examining the classroom use of mathematical tasks from a variety of perspectives; and, by making comparison possible between forms of classroom use of mathematical tasks in different classrooms around the world. Neither approach is intended to be comprehensive, but it is hoped that the chapters that follow, both individually and in combination, will address issues of interest to the mathematics education community internationally. The remainder of this introductory chapter sets out some of the theoretical and contextual considerations that should be taken into account in any reading of this book.

THEORETICAL ALTERNATIVES IN CONSIDERING THE CLASSROOM USE OF MATHEMATICAL TASKS

There are many different theories currently being employed in mathematics education. Activity Theory, for example, is an obvious contender in considering how the classroom use of mathematical tasks might be situated theoretically. Recent developments in the conceptualisation of Activity Theory (eg Engeström, 2001) have increased the breadth of phenomena and contexts able to be addressed using Activity Theory. In particular, mathematical tasks can be situated naturally within the tools available for use in pedagogic activity systems. Our purpose here is not to catalogue all the different theories that might be employed in researching the classroom use of mathematical tasks, but to examine briefly a selection of relevant theories that informed the analytical work of the LPS community represented in the chapters of this book. In this context, Activity Theory serves as a useful example of an eminently eligible theory, used in other LPS publications for consideration of classroom discourse, but which was not explicitly employed in any of the chapters that follow. This example provides an opportunity to re-emphasise the commitment of the LPS community to an inclusive selectivity, by which the theory guiding each analysis is chosen pragmatically for its consistency with the purposes of that analysis.

Gellert (2008) usefully contrasts ‘interactionist’ and ‘structuralist’ perspectives on mathematics classroom practice. In the consideration of the classroom use of mathematical tasks, the interactionist perspective offers insight into the negotiative processes that interact with individuals’ use in classroom settings for the socially-mediated constitution of learning. Chapters 4 (Gallos Cronberg) and 7 (Mok) report interactionist analyses that emphasise student engagement and mediated learning respectively.

In Chapter 3, Bergem and Klette suggest that greater emphasis be given to mathematical communication in research and theory. They argue that cognitive psychology, social constructivism, distributed cognition, semiotics and socio-cultural theory all draw our attention to the essential role that
reflective discourse and discursive practices have for fostering mathematical understanding (p. 36).

For Bergem and Klette, therefore, mathematical conversations and the discourses such conversations might embody warrant close research scrutiny. The situatedness of any such conversations is seen as critical to their realisation in consequent student learning.

The structuralist perspective potentially offers very different insights into the deployment and function of mathematical tasks in classrooms. Focusing attention on differentiated participation, a structuralist analysis aspires to explain such differentiation in terms of hierarchies and power relationships. In the case of mathematical tasks, these hierarchies reflect the enactment of an entrenched social order and the privileging of particular forms of knowledge. Within the structuralist perspective, particular pedagogies can be seen as embodying systems of social and academic privilege (Bernstein, 1996) and in the mathematics classroom it is primarily through the performance of mathematical tasks that these pedagogies are enacted. The chapter by Kaur (Chapter 2) reports an analysis that could be considered structuralist. The analysis differentiates usefully between forms of knowledge and the key resources that structure instruction in the three Singapore classrooms studied.

The chapter by Mesiti and Clarke (Chapter 10) argues that mathematical tasks should only be considered ‘as performed,’ since the same mathematics problem can provide a vehicle for the realisation of very different social and educational purposes. This performative emphasis is echoed in Chapter 5 by Shimizu, where it is asserted that the functions of mathematical tasks posed in classroom settings need to be considered within the contexts in which they are undertaken.

The Theory of Didactical Situations in Mathematics (TDSM) (Brousseau, 1997) is a coherent, well-elaborated instructional theory, capable of supporting the explicit advocacy of particular practices. Within TDSM, Brousseau carefully distinguishes the work of the mathematician, the work of the student and the work of the teacher. Particular attention is given to ‘The notion of problem.’

A student isn’t really doing mathematics unless she is asking herself questions and solving problems. (Brousseau, 1997, p. 79)

Similar care is given within TDSM to critical considerations such as ‘epistemological obstacles’ and ‘didactic problems.’ Novotná and Hošpesová make use of TDSM to theorise about the teacher’s scaffolding of linkages between different mathematical concepts and procedures, using specific examples from two mathematics classrooms in the Czech Republic to illustrate the key points.

By way of comparison, Variation Theory is a similarly coherent theory that provides clear criteria for instructional advocacy. Variation Theory privileges constructs such as ‘the object of learning’ and ‘dimensions of variation’ (Marton & Tsui, 2004). In Chapter 7, Mok draws attention to one Shanghai teacher’s deliberate partial variation of the content and constraints between problems and questions.
The choice of the theoretical lens focuses analytical attention on some aspects of the role of mathematical tasks and ignores others. This is inevitable. It is a strength of the combination of analyses reported in this book that they are not restricted to the application of a single theory. Instead, the tasks of the mathematics classroom are examined from a variety of theoretical perspectives. In several cases, the same tasks occur in different chapters, to be re-examined from the perspective of different theories. The reader is encouraged to compare the treatment of the same task in the different analyses and to reflect on which analysis connects most usefully with the reader’s concerns, interests and purposes.

Another entry point for consideration of the theories relevant to the classroom use of mathematics tasks are the three related issues of Abstraction, Context and Transfer. In some discussions, abstract mathematics seems to be treated as simply decontextualised mathematics. Clarke and Helme have argued that there is no such thing as decontextualised mathematics (Clarke & Helme, 1998), since all mathematical activity is undertaken in a context of some sort. If abstraction in mathematics is to have any legitimacy or relevance, then it must reside in some form of generalisability of the mathematical matter under consideration, in the sense that the principle, concept or procedure can be thought of as transcending any particular context or instance. But, to argue that an exercise in Euclidean geometry or in pure number is an abstract task is to deny the social situatedness that has become accepted even from the most cognitivist of perspectives (Lave & Wenger, 1991).

In relation to mathematical tasks, Clarke and Helme distinguished the social context in which the task is undertaken from any ‘figurative context’ that might be an element of the way the task is posed. In this sense, the task:

Siu Ming’s family intends to travel to Beijing by train during the national holiday, so they have booked three adult tickets and one student ticket, totalling $560. After hearing this, Siu Ming’s classmate Siu Wong would like to go to Beijing with them. As a result they buy three adult tickets and two student tickets for a total of $640. Can you calculate the cost of each adult and student ticket? (Shanghai School 3, Lesson 7, Train task)

has a figurative context that integrates elements such as the family’s need to travel by train and the familiar difference in cost between an adult and a student ticket. The social context, however, could take a wide variety of forms, including: an exploratory instructional activity undertaken in small collaborative groups; the focus of a whole class discussion, orchestrated by the teacher to draw out existing student understandings; or, an assessment task to be undertaken individually. In each case, the manner in which the task will be performed is likely to be quite different, even though we can conceive of the same student as participant in each setting. The significance of the context in which mathematical tasks are undertaken has been persuasively demonstrated by Nunes, Schliemann, Carraher and their co-workers through the well-known series of studies contrasting the performance of
mathematically equivalent tasks in school and everyday settings (Nunes, Schliemann, & Carraher, 1993).

If we take ‘transfer’ not as a description of a particular cognitive process, but as a metaphor for a skill developed in one context being used in a different context, then it is reasonable to ask, “Under what conditions (and through the instructional use of what tasks) will the likelihood of transfer be maximised?” A cognitivist might direct attention to the selective variation of task attributes with the intention of successively focusing student attention on salient aspects of the mathematical concept or procedure to be learned. Variation Theory (Marton & Tsui, 2004) identifies learning with an increasing capacity to discern relevant attributes in the object of learning. From such a perspective, particular tasks and particular sequences of tasks can be critiqued as more or less conducive to directing student attention appropriately and thereby to the optimal promotion of the discernment that is identified with learning.

Distributed Cognition (Hutchins, 1995) and other theories with a material semiotic character accord significance to artefacts as participating in cognition. Once representational forms are included in the broad class of artefacts, then mathematical tasks cease to be either the objects to which we apply our cognitive tools nor merely the social catalysts for their deployment. Rather, mathematical tasks become the embodiment of performed cognition, integrating, as they do, representational forms, socio-cultural imperatives and mathematical entities. In Chapter 10, Mesiti and Clarke attempt to portray mathematical tasks performatively in order to examine the role each task plays in affording or constraining agency and voice in the social settings in which the tasks are communally performed.

COGNITIVE DEMANDS OF DIFFERENT TASKS FOR DIFFERENT LEARNERS

Selecting and setting appropriate tasks is key to the success of teaching mathematics (Doyle, 1988; Hiebert & Wearne, 1993; Stein & Lane, 1996; Martin, 2007). Tasks can vary not only with respect to mathematics content but also with respect to the cognitive processes involved in working on them. Worthwhile tasks are those that offer students the opportunity to extend what they know and stimulate their learning.

Doyle (1988) argues that tasks with different cognitive demands are likely to induce different kinds of learning. Tasks that require students to solve complex problems can be considered to be cognitively demanding tasks. In contrast, cognitively undemanding tasks are those that give less opportunity for the students to engage in high-level cognitive processes. Thus, the nature and the role of tasks that entail different types of cognitive demand are key to students’ opportunities for learning. Chapter 8 by Huang and Cai compares the cognitive demands of the mathematical tasks used in the US and Chinese classrooms. In this chapter, reference is made to ‘declining cognitive demand’ and the important point is introduced that the cognitive demand of a task depends not just on the
mathematical form of the initial task, but on the manner in which the task is enacted in the classroom.

The cognitive demands of mathematical tasks can change as tasks are introduced to students and/or as tasks are enacted during instruction (Stein, Grover, & Henningsen, 1996). The progression of mathematical tasks can be modeled by identifying the phases of enactment from their original form as they appear in the pages of textbooks or other curriculum materials to the tasks that teachers actually provide to students and then to the tasks as enacted by the teacher and students in classroom lessons (see Figure 1).

**Figure 1. Progression of mathematical tasks (adapted from Stein, Grover, & Henningsen, 1996)**

The first two arrows in the diagram identify critical phases in the instructional life of tasks at which cognitive demands are susceptible to being altered. The tasks, especially as enacted, have consequences for student learning of mathematics, as is shown by the third arrow in the figure. The features of an instructional task, especially its cognitive demand, often change as a task passes through these phases (Stein, Grover, & Henningsen, 1996).

In order to track changes in cognitive demand, tasks can be classified at different levels of cognitive demand. Implementing cognitively challenging tasks in ways that maintain students’ opportunities to engage in high-level cognitive processes is not a trivial endeavour (Henningsen & Stein, 1997). In the TIMSS 1999 video study, the ability to maintain the high-level demands of cognitively challenging tasks during instruction was the central feature that distinguished classroom teaching in different countries (Hiebert et al., 2003). One of the questions addressed in several of the chapters in this book is whether it is useful or even feasible to classify a task as cognitively challenging, independent of the manner in which it is performed in the classroom. Instead, the progression in task form and demand as depicted in Figure 1 resembles the distinction made by Variation Theory between the intended object of learning, the enacted object of learning, and the experienced object of learning. The TIMSS 1999 Video Study sensibly distinguished between the potential demands of a task and the extent to which those demands were realised in the actual practice of the classroom. As can be seen, there is a strong convergence of view towards seeing mathematical tasks as performatively defined rather than as static objects enshrined in text.
DISTINGUISHING BETWEEN THE USE OF MATHEMATICAL TASKS FOR INSTRUCTION AND ASSESSMENT

If the use of a mathematical task is intended to promote the student’s learning of mathematics, that use can reasonably be designated *instructional*. If the use of a mathematical task is intended to generate information about student learning or, relatedly, about the effectiveness of instruction, then we can characterise that use as being for the purposes of assessment. Of course, the same task can serve both purposes, even at the same time, as is evident from the extensive literature on formative assessment (Wiliam & Black, 1996). It does not necessarily follow, however, that a given task (or task type) is equally effective when used for instructional purposes or assessment purposes. This distinction must be seen as indicative rather than definitive. It can be argued that all tasks constitute assessment in as much as all tasks reveal information regarding any students’ attempts at the task (particularly given the contemporary prioritisation of formative assessment). Equally, even test items can be instructional, serving to convey messages about what is and what is not correct and valued mathematical performance.

The chapter by Kaur (Chapter 2) classifies tasks as learning, review, practice and assessment tasks. The distinction between the instructional and assessment uses of mathematical tasks in the classrooms studied draws attention to the lack of alignment of the espoused curricular goals with either instructional or assessment practices in the three Singapore classrooms.

Earlier, reference was made to the designation ‘rich mathematical tasks.’ One context in which this characterisation of tasks has been used is assessment. In that context, ‘rich’ is taken to signify a task of sufficient complexity as to admit a variety of approaches and therefore to have the capacity to reveal differences in student conceptions of relevant mathematical concepts and procedures.

Assessment tasks should be sufficiently open to invite a range of responses from students and so allow students to disclose their level of competence and understanding at a number of levels. Such rich assessment tasks:

- connect naturally with what has been taught
- address a range of outcomes in the one task
- allow all students to undertake the activity
- can be successfully undertaken using a range of methods or approaches
- encourage students to disclose their own understanding of what they have learned
- allow students to show connections they are able to make between the concepts they have learned
- are themselves worthwhile activities for children’s learning
- draw the attention of teachers to important aspects of mathematical activity
• help teachers to decide whether it is appropriate to move on for most students and what specific help others may require. (Clarke, 1996, pp. 361-362)

There is no question that many of the tasks discussed in this book are ‘rich tasks’ by the criteria just listed. Whether a particular task should be categorised as an instructional task or an assessment task or as both depends on the way in which the task was utilised by the particular teacher. Certainly, it is clear in several chapters that teachers made use of the information provided by many tasks to guide their instruction.

The premise of Kaur’s chapter is an important one: however we might classify tasks (whether as serving instructional or assessment purposes), a crucial consideration is the coherence with which valued mathematical performance is nurtured, promoted, revealed, evaluated and acknowledged in the classroom and it is through our choice and use of mathematical tasks that this coherence is displayed and communicated.

THE FOCUS AND THE STRUCTURE OF THIS BOOK

The following chapters in this book explore and discuss the nature and roles of mathematical tasks in LPS classrooms with a focus on their impact on students’ learning. It is an essential theme of the Learner’s Perspective Study that international comparative research offers unique opportunities to interrogate established practice, existing theories and entrenched assumptions (Clarke, Emanuelsen, Jablonka, & Mok, 2006; Clarke, Keitel, & Shimizu, 2006). In this book, focusing on the nature, role and implementation of mathematical tasks, we offer the reader a variety of images of classrooms from the countries participating in the Learner’s Perspective Study.

It was of interest in the development of this book whether the enactment of mathematical tasks observed in the classrooms of one country showed consistency of form and purpose, sufficiently different from other classrooms, such as to suggest a culturally-specific character. Because of the highly selective nature of the classrooms studied in each country, no claims can be made about national typification of practice, however any regularities of practices sustained across thirty lessons demand some consideration as to the possible causes of such consistency. Whether or not such identifiable characteristics in the treatment of tasks exist as cultural traits, the Learner’s Perspective Study was predicated on a belief that international comparative studies are likely to reveal patterns of practice less evident in studies limited to a single country or community.

‘Teaching’ is not only about teaching what is conventionally called content, but also teaching what a lesson is and how to participate in it. Students learn not only what the answer is to a task but also how to approach academic tasks. The performative nature of tasks and the nature of student participation in the classroom activities catalysed by mathematics tasks are explored in many different ways in the chapters that follow.
This book includes chapters with very different approaches, such as: comparisons of the features of tasks within a lesson sequence in different cultures, the cognitive demands of different tasks for different learners, mathematical tasks as experienced from the learner’s perspective, the role of tasks as vehicles for interaction among classroom participants, or the analysis of particular emphases within tasks: representations, real-world contexts, proof, problem solving, or types of reasoning and argumentation. The chapters have been organised as a progression from the consideration of mathematical tasks in the classrooms of single countries to comparative analyses across classrooms situated in several countries. While each individual chapter rewards careful reading, it is our hope that in combination the different perspectives on the classroom use of mathematical tasks will provide insights through differences in the settings in which the tasks are performed, differences in ways in which classroom participants are positioned in relation to the tasks, and profound differences in how each teacher utilises mathematical tasks, in partnership with their students, to create a distinctive form of mathematical activity.

REFERENCES


THE ROLE OF MATHEMATICAL TASKS IN DIFFERENT CULTURES


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CHAPTER TWO

A Study of Mathematical Tasks from Three Classrooms in Singapore

INTRODUCTION

The basic aim of a mathematics lesson is for learners to learn something about a particular topic that the teacher has planned. To do this the teacher engages them in mathematical tasks during the lesson. Mathematical tasks in this context refer to what the students are asked to do, be it computations to be performed, symbols to be manipulated, diagrammatic representations to be made or translations of word problems into mathematical statements or models (Mason & Johnston-Wilder, 2006). Experiences gained by students (through the mathematical tasks that teachers engage them in to actualise the intended curriculum) form the bedrock of their knowledge and perception of mathematics. Therefore the nature and source of the tasks are important contributors to the pedagogical goals of the lessons in which they are enacted. In this chapter we examine the source and nature of tasks used by three competent mathematics teachers in grade eight classrooms from Singapore schools.

This study has emerged from two main considerations, namely the source and nature of mathematical tasks, which specifically are learning, review, practice and assessment tasks used by the three teachers to implement the intended curriculum and the link between the tasks and the primary goal of the school mathematics curriculum, that is, mathematical problem solving (Ministry of Education, 2006). A learning task (Mok, 2004) is an example the teacher uses to teach the students a new concept or skill. A review task is a task used by the teacher to review previously learnt concepts and/or skills so as to facilitate the learning of new concepts and skills. Practice tasks are tasks used during the lesson to either illuminate the concept or demonstrate the skill further and the teacher asks students to work through tasks during the lesson either in groups or individually or during out of class time. Assessment tasks are tasks used to assess the performance of the students. Based on these considerations, we have attempted to examine the tasks used by the teachers, in particular the source of the tasks and aspects of the demands the tasks make on the learners.
RESEARCH FRAMEWORK

Stein and Smith (1998) in their framework of mathematical tasks clarified the multiple roles of tasks that may be available in curricular instructional materials such as textbooks. These tasks may be set up by teachers during their instruction as learning tasks to introduce new concepts and skills, make connections between old and new concepts or skills, or illustrate the application of concepts in problem solving. Such tasks may also be set up by teachers as review tasks to review past knowledge. Lastly, tasks from such materials may also be used as practice and assessment tasks. Practice and assessment tasks are completed or implemented by students with no interjections from the teacher. The purpose of assessment tasks is evaluation, be it formative or summative. In all of the above instances, the teacher’s goal for setting the tasks is student learning.

Mathematical tasks can be examined from a variety of perspectives including the demands of the tasks and the presentation of the tasks. However it is not always possible to subject all the tasks to the same type of analysis. As learning tasks, taken from textbooks or other sources, are set up for specific goals of instruction during the instructional cycle, these tasks cannot be treated in the same vein as review, practice and assessment tasks because the corresponding classroom discourse has a lot to do with how the pupils engage with it. Drawing on the framework for the analysis of learning task lesson events proposed by Mok and Kaur (2006), three levels of the first aspect, differentiation of the learning process, namely

- **Level 1**: introducing new concepts and skills
- **Level 2**: making connections between new and old concepts or skills
- **Level 3**: introducing knowledge or information beyond the scope of the curriculum requirement or textbook

were found relevant to the present study.

Koh and Lee (2004) as part of a core project of the Centre for Research in Pedagogy and Practice (CRPP) at the National Institute of Education (NIE) in Singapore created and validated a set of standards for scoring teacher assignments (practice tasks) and assessment tasks in languages (English, Chinese, Malay and Tamil), mathematics, science, and social studies. Six standards were selected to analyse the practice and assessment tasks used by the teachers in the study. The first standard, depth of knowledge, is about the type of knowledge the task requires. The second standard, knowledge criticism, is about what students are required to do with the knowledge. The third standard, knowledge manipulation, is about the nature of thinking skills the task requires students to engage in. The next three standards: supportive task framing, clarity and organisation, and explicit performance standard or marking criteria are about the form of the tasks. Details of the standards are shown in Table 1.
### Table 1. Relevant standards and dimensions (Koh & Lee, 2004)

#### Standard 1 - Depth of knowledge
- **Dimension 1 – factual knowledge**
  Possible indicators are tasks that require students to recognise mathematical terms; state concepts, facts or principles; identify objects, patterns, or list properties; recall rules, formulae, algorithms, conventions of number, or symbolic representations; describe simple mathematical facts and computational procedures and perform routine arithmetic operations.
- **Dimension 2 – procedural knowledge**
  Possible indicators are tasks that require students to know how to carry out a set of steps; use a variety of computational procedures and tools; perform strategic or non-routine arithmetic operations and manipulate the written symbols of arithmetic.
- **Dimension 3 – advanced knowledge**
  Possible indicators are tasks that require students to expand definitions; relate facts and concepts; make connections to other mathematical concepts and procedures; explain one or more mathematical relations; understand how one major math topic relates to another and understand how a mathematical topic relates to other disciplines or real world situations.

#### Standard 2 – Knowledge criticism
- **Dimension 1 – presentation of knowledge as truth or given**
  Possible indicators are tasks that require students to accept or present ideas and information as truth or affixed body of truths; perform a well-developed algorithm; follow a set of preordained procedures and perform a clearly defined series of steps.
- **Dimension 2 – comparing and contrasting information or knowledge**
  Possible indicators are tasks that require students to identify the similarities and differences in observations, data and theories; classify. Organise, and compare data, and develop heuristics to identify, organise, classify, compare and contrast data, observations or information.
- **Dimension 3 – critiquing information or knowledge**
  Possible indicators are tasks that require students to comment on different mathematical solutions, theories, and procedures; discuss and evaluate approaches to mathematics-related problems; make mathematical arguments, and pose and formulate mathematical problems.

#### Standard 3 – Knowledge manipulation
- **Dimension 1 – reproduction**
  Possible indicators are tasks that require students to reproduce facts or procedures; recognise equivalents; recall familiar mathematical objects and properties; perform a set of preordained algorithms; manipulate expressions containing symbols and formulae in standard form; carry out computations; apply routine mathematical procedures and technical skills, and apply mathematical concepts and procedures to solve simple and routine problems.
- **Dimension 2 – organisation, interpretation, analysis, synthesis or evaluation**
  Possible indicators are tasks that require students to interpret given mathematical models (equations, diagrams, etc); organize, analyse, interpret, present or generate data or information; interpret tables, graphs and charts; predict...
mathematical outcomes from the trends in the data; interpret the assumptions and relations involving mathematical concepts and consider alternative solutions or strategies.

- **Dimension 3 – application or problem solving**
Possible indicators are tasks that require students to apply mathematical concepts and processes to solve non-routine problems; apply the signs, symbols and terms used to represent concepts and use problem-solving heuristics for non-routine problems.

- **Dimension 4 – generation or construction of knowledge new to students**
Possible indicators are tasks that require students to come up with new proofs or solutions to a mathematical problem; generalize mathematical procedures, strategies and solutions to new problem situations and apply modelling to new contexts.

**Standard 4 – Supportive task framing**
The task provides students with appropriate framing or scaffolding (written or graphic guidance in view of the students’ skill levels and prior knowledge) in order to support them to complete the task given

- **Dimension 1 – content scaffolding**
- **Dimension 2 – procedural scaffolding**
- **Dimension 3 – strategy scaffolding**

**Standard 5 – Clarity and Organisation**
The task is framed logically and has instructions that are easy to understand.

**Standard 6 – Explicit performance standard or marking criteria**
The task is provided with the teacher’s clear expectations for students’ performance and the marking criteria are explicitly clear to the students.

Aspects of Stein and Smith’s (1998) task analysis guide were also drawn on to establish the cognitive demands of the tasks used by the teachers in their classrooms. A brief outline of Stein and Smith’s guide with adaptations made by the author for the analysis of data presented in this chapter is shown in Table 2.
Table 2. Levels of cognitive demand

<table>
<thead>
<tr>
<th>Levels of cognitive demand</th>
<th>Characteristics of tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 0 – [Very Low]</td>
<td>Reproduction of facts, rules, formulae</td>
</tr>
<tr>
<td>Memorisation tasks</td>
<td>No explanations required</td>
</tr>
<tr>
<td>Level 1 - [Low]</td>
<td>Algorithmic in nature</td>
</tr>
<tr>
<td>Procedural tasks without connections</td>
<td>Focused on producing correct answers</td>
</tr>
<tr>
<td></td>
<td>Typical textbook word - problems</td>
</tr>
<tr>
<td></td>
<td>No explanations required</td>
</tr>
<tr>
<td>Level 2 [High]</td>
<td>Algorithmic in nature</td>
</tr>
<tr>
<td>Procedural tasks with connections</td>
<td>Has a meaningful / ‘real-world’ context</td>
</tr>
<tr>
<td></td>
<td>Explanations required</td>
</tr>
<tr>
<td>Level 3 – [Very High]</td>
<td>Non-algorithmic in nature</td>
</tr>
<tr>
<td>Problem Solving / Doing Mathematics</td>
<td>Requires understanding of mathematical concepts and application of</td>
</tr>
<tr>
<td></td>
<td>Has a ‘real-world’ context / a mathematical structure</td>
</tr>
<tr>
<td></td>
<td>Explanations required</td>
</tr>
</tbody>
</table>

Hence, appropriate aspects of the works of Stein and Smith (1998), Mok and Kaur (2006) and Koh and Lee (2004) contributed towards the analytical framework used for the analysis of learning tasks, practice tasks, and assessment tasks. However, the framework was not able to provide for the analysis of the review tasks, which were a part of the data analysed and presented in this chapter.

METHOD

Source of Data

Three competent mathematics teachers participated in the study and their competence was locally defined by the community in which they worked. Data was collected in accordance with the protocol set out in the Learner’s Perspective Study (LPS) (Clarke, 2006). Teacher 1 (T1) from school 1 (SG1) was a female with 21 years of teaching experience and taught a class of 37 students. Teacher 2 (T2) from school 2 (SG2) was also a female with 27 years of teaching experience and taught a class of 40 students and Teacher 3 (T3) was a male with 15 years of teaching experience and taught a class of 40 students. For each teacher, 10 consecutive grade eight mathematics lessons, L01–L10, were observed and video-taped by the researchers. For the purpose of this chapter, a topic of a particular textbook chapter that the teachers taught during the 10-lesson period was selected for study. The topic was defined by a sequence of lessons that captured the curricular coherence between lessons. For all the three teachers, the topic selected was made up of learning tasks, review tasks and practice tasks. As assessment tasks were only administered during class tests we have included these ‘test items’ in our analysis.
to represent assessment tasks. Studying the tasks used in a topic would help us to document how students’ learning is developed through the use of different tasks from the introductory level to the application level. In addition, this would allow us to see the kind of opportunity that existed for students to engage in problem solving by applying the skills they have acquired. From the corpus of data of schools SG1, SG2 and SG3, the source of data was primarily the lesson tables. A lesson table is a chronological narrative account of activities that take place during the lesson. This table also details all the tasks (learning, review, practice and assessment) that the teacher used during the lesson, and their source.

Content and Lessons

Table 3. Content of lessons

<table>
<thead>
<tr>
<th>SG1-T1</th>
<th>SG2-T2</th>
<th>SG3-T3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Topic:</strong> Power of Ten and Standard Form</td>
<td><strong>Topic:</strong> Algebraic Manipulation and Formulae</td>
<td><strong>Topic:</strong> Pythagoras’ Theorem</td>
</tr>
<tr>
<td></td>
<td>Part IV [L05] – Addition and subtraction of algebraic fractions</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Part V [L06] – Changing the subject of a formula</td>
<td></td>
</tr>
</tbody>
</table>

Each topic comprised several lessons and during a lesson a part of the topic was covered. Every part contained some of the following tasks: learning tasks, review tasks and practice tasks. The lesson during which the class test was administered or discussed contained all the assessment tasks. Table 3 shows the overview of the main content of a specific topic covered by each teacher and the class test topic/s being tested during the class test lesson in each school. For SG1, the topic on power of ten and standard form was chosen and T1 took three lessons to complete
this topic. For SG2, we selected the topic on algebraic manipulation and formulae and this topic was covered by T2 over five lessons. For T3 the topic on Pythagoras’ theorem was selected and it took three lessons to complete. In general, during the lessons, learning tasks were used to introduce new knowledge, review tasks to recall prior knowledge and practice tasks were given to the students to practice during the lesson either in groups or individually or at home after each part of the topic was covered.

The tests were the only source of assessment tasks. For all three schools during the ten-lesson period of observation and videotaping, only one class test was administered or reviewed. For SG1, a test was conducted during L06. For SG2, the corrections of a test administered during the assessment week were reviewed during L07. For SG3, a test was conducted during L03. For both SG1 and SG2, the tests contained the respective topics selected for the analysis in this chapter, but for SG3 this was not the case, as the test was on a topic taught just before the start of the observation and videotaping period of SG3.

Data Analysis

All the tasks from the selected lessons were compiled and their sources traced, that is, where they were taken from. Next the tasks were analysed using appropriate frameworks elaborated in the first part of the chapter. The match of frameworks and task types was as follows:

– framework proposed by Mok and Kaur (2006) was used to ascertain the role of the learning tasks in the learning process differentiated by levels
– framework of Koh and Lee (2004) was used to examine against selected standards and their corresponding dimensions the nature of practice and assessment tasks
– framework of Stein and Smith (1998) was used to establish the cognitive demands of the practice and assessment tasks.

As none of the frameworks were suitable to characterise the review tasks an exploration of these tasks was carried out to develop a possible framework.

Using the appropriate frameworks, the author and a research assistant analysed all the learning, practice and assessment tasks independently. The overall rate of agreement was 80%. Next, they jointly examined the ‘disputed tasks’ and after extensive discussion reached consensus on them. The review tasks were analysed differently. The purpose of each review task was examined by referring to the lesson during which it was enacted as well as the lesson prior to it.
FINDINGS

Number and Sources of Tasks

Table 4. Number and sources of review, learning, practice and assessment tasks

<table>
<thead>
<tr>
<th>School-Lesson</th>
<th>Review tasks</th>
<th>Learning tasks</th>
<th>Practice tasks</th>
<th>Assessment tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Text book</td>
<td>Other</td>
<td>Text book</td>
<td>Other</td>
</tr>
<tr>
<td>SG1-L01</td>
<td>-</td>
<td>-</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>SG1-L02</td>
<td>-</td>
<td>-</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>SG1-L03</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>SG1-L06*</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Total</td>
<td>-</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>SG2-L02</td>
<td>-</td>
<td>-</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>SG2-L03</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>SG2-L04</td>
<td>1</td>
<td>-</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>SG2-L05</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>SG2-L06</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>SG2-L07**</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Total</td>
<td>1</td>
<td>7</td>
<td>1</td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>School-Lesson</th>
<th>Review tasks</th>
<th>Learning tasks</th>
<th>Practice tasks</th>
<th>Assessment tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Text book</td>
<td>Other</td>
<td>Text book</td>
<td>Other</td>
</tr>
<tr>
<td>SG3-L02</td>
<td>-</td>
<td>-</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>SG3-L03*</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>SG3-L04</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>SG3-L05</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>Total</td>
<td>-</td>
<td>1</td>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>

* class was having a test
** teacher went through the test corrections

Table 4 shows the number and sources of review, learning, practice and assessment tasks. From the table, it is apparent that T3 used less tasks in total compared to both T1 and T2. It must be noted that all the lessons of both T1 and T2 that contribute towards the data of this chapter were an hour in duration, while that of T3 were only 35 minutes in duration. The purpose of this table is not to provide any comparison between numbers of tasks used by the teachers but rather to explore the sources of the tasks. Only T1 and T2 used review tasks in their lessons. Review tasks used by T1 were non-textbook materials, real-life objects such as the Russian dolls and cutlery, which were collections of similar solids and self-made simple mathematical tasks, such as, “How would you write the number 392.5 in standard form?” and common factors, for example, \(5x + 10y = 5(x + 2y)\); \(4ab + bc = b(4a + c)\). The only review task T2 used was taken from the textbook that both the students and teacher of SG2 used for their mathematics lessons. It was “Find the Highest Common Factor (HCF) and Lowest Common Multiple (LCM) of \(3a^2b\) and \(4ab^2\),” which led to a review of the three methods: prime factorisation, division and inspection that may be used to find the HCF and LCM of two or more algebraic polynomials.
As for learning tasks, T1 used both the textbook and herself as a source, while T2 relied mainly on the textbook for tasks, while T3 used mainly non-textbook sources such as the internet and self. For all three teachers, the textbook was their main source for practice tasks. The assessment tasks were sourced from past school and national examination papers. The textbook appears to be a significant source of learning and practice tasks (Kaur, Low, & Seah, 2006), while past school and national examination papers appear to be a significant source of the assessment tasks. This alignment suggests that to a large extent the textbook tasks drive the implementation of the school curriculum, while the “collection of past examination tasks” assists teachers in assessing the performance of their students benchmarked against “examination standards.”

### Learning Tasks

Table 5 shows the purpose of the learning tasks used by the three teachers, as defined by Mok and Kaur (2006). It appears that all the learning tasks used by T1 and T2 were of level 1 type, that is, used to introduce new concepts and skills as stipulated by the curriculum guides. However, this was not the case for T3, as two of his six learning tasks were of level 3 type, that is, used to introduce knowledge or information beyond the scope of the textbook requirement. As an introductory task, T3-L02-L1 (Teacher 3, Lesson 2, Learning Task 1) for the topic Pythagoras theorem, T3 used the internet and showed the students the portrait of Pythagoras and also his contribution in the field of mathematics. The other level 3 type of learning task used by T3 (T3-L02-L5) introduced students to two algebraic methods of finding the Pythagorean triplets. The first was \( \{2m, m^2 - 1, m^2 + 1\} \) for integer values of \( m \geq 1 \) and the second \( \{2pq, p^2 - q^2, p^2 + q^2\} \) for integer values of \( p \) and \( q \) where \( p > q > 1 \). Using algebraic methods to list Pythagorean triplets is beyond the scope of the grade eight mathematics curriculum, but T3 has certainly shown the students that the Pythagorean triplets are not random and can be
generalised. Two of the learning tasks used by T1 were of both levels 1 and 2 as they were used to bridge the past knowledge of students, for example, multiplication of $10^3$ and $10^4$ using ordinary notation [T1-L01-L1], division of $10^6$ by $10^4$ using ordinary notation and also the introduction of the laws of indices: $a^m \times a^n = a^{m+n}$ and $a^m \div a^n = a^{m-n}$. None of the learning tasks used by T2 and T3 were of level 2 and this is perhaps due to the nature of the topics T2 and T3 were teaching, as students are introduced to algebraic fractions and transformation of formulae and also Pythagoras theorem for the first time in grade eight.

**Review Tasks**

<table>
<thead>
<tr>
<th>Task</th>
<th>Purpose (Teacher’s goal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1-L02-R1†</td>
<td>Recall prior knowledge</td>
</tr>
<tr>
<td>T1-L02-R2</td>
<td>Recall prior knowledge Make connection between newly acquired and past knowledge</td>
</tr>
<tr>
<td></td>
<td>Provide procedural scaffolding for subsequent tasks</td>
</tr>
<tr>
<td>T1-L06-R1</td>
<td>Relate newly acquired mathematical concepts to real life examples</td>
</tr>
<tr>
<td>T2-L04-R1</td>
<td>Recall prior knowledge</td>
</tr>
</tbody>
</table>

† Teacher 1, Lesson 2, Review Task 1

Table 6 shows the review tasks and the corresponding teacher’s goals. It is apparent that T1 used review tasks to help her students recall prior knowledge, provide procedural scaffolding for later work during a lesson, connect newly acquired and past knowledge, and relate newly acquired mathematical concepts to real life examples. T2 used only one review task and it was to help her students recall prior knowledge while T3 used none. Based on this small sample of review tasks, which T1 and T2 used during the teaching of a topic each, a preliminary framework for the analyses of review tasks may be proposed as follows:

- **Type 1**: recall of prior knowledge
- **Type 2**: provide scaffolding for subsequent tasks
- **Type 3**: connect newly acquired knowledge with past knowledge
- **Type 4**: relate newly acquired knowledge to real life examples
### Practice Tasks

#### Table 7. Standards and dimensions of practice tasks

<table>
<thead>
<tr>
<th>Task</th>
<th>Standards and dimensions</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>S5</th>
<th>S6</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1-L01-P1-2</td>
<td></td>
<td>✓</td>
<td>✓</td>
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<td>✓</td>
</tr>
<tr>
<td>T1-L01-P3</td>
<td></td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>T1-L01-P4</td>
<td></td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
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<td>✓</td>
</tr>
<tr>
<td>T1-L01-P5-6</td>
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<td>✓</td>
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<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>T1-L02-P1-6</td>
<td></td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>T1-L03-P1-4</td>
<td></td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
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</tr>
<tr>
<td>T1-L03-P5-7</td>
<td></td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
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</tr>
<tr>
<td>T2-L02-P1-7</td>
<td></td>
<td>✓</td>
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<td>✓</td>
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</tr>
<tr>
<td>T2-L03-P1-2</td>
<td></td>
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<td>✓</td>
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</tr>
<tr>
<td>T2-L04-P1-5</td>
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</tbody>
</table>

**Legend**

- **S1**: Depth of knowledge
  - 1- factual knowledge
  - 2- procedural knowledge
  - 3- advanced knowledge

- **S2**: Knowledge criticism
  - 1- presentation of knowledge as truth or given
  - 2- comparing and contrasting information or knowledge
  - 3- critiquing information or knowledge

- **S3**: Knowledge manipulation
  - 1- reproduction
  - 2- organisation, interpretation, analysis, synthesis or evaluation
  - 3- application or problem solving
  - 4- generation or construction of knowledge new to students

- **S4**: Supportive task framing
  - 1- content scaffolding
  - 2- procedural scaffolding
  - 3- strategy scaffolding

- **S5**: Clarity and organisation

- **S6**: Explicit performance standard or marking criteria

The analysis of the practice tasks using Koh and Lee’s (2004) framework is presented in Table 7, which shows the standards and dimensions of the tasks used by the three teachers during their course of teaching a topic each. Only three of the tasks used by all the three teachers dealt with factual knowledge, while the rest dealt with procedural knowledge. None of the tasks were about advanced
knowledge. All except one of the tasks were about the presentation of knowledge as truth.

Complete the table:

<table>
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<tr>
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<th>Result</th>
</tr>
</thead>
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<tr>
<td>1000</td>
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<td>$10^{-4}$</td>
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<tr>
<td>0.00001</td>
<td>$10^{-5}$</td>
</tr>
</tbody>
</table>

Figure 1. Task T1-L01-P3

PQRS is a rectangle in which PQ = 9 cm and PS = 6 cm. T is a point on PQ such that PT = 7 cm and RV is the perpendicular from R to ST. Calculate ST and RV.

Figure 2. Task T3-L05-P1

Make "a" the subject by completing the following:

$$e = \sqrt{5a - 8}$$

$$\sqrt{5a - 8} = e$$

$$(5a - 8)^{\frac{1}{2}} = e$$

.................

.................

Figure 3. Task T2-L06-P1

Generate Pythagorean triplets $(a, b, c)$ such that $a^2 + b^2 = c^2$

Figure 4. Task T3-L02-P1

The exception was a task given by T1 to her students to compare and contrast factual knowledge. The task, T1-L01-P3 (Teacher 1, Lesson 1, Practice Task 3), shown in Figure 1 required students to analyse the facts presented and predict the
MATHEMATICAL TASKS FROM SINGAPORE CLASSROOMS

missing bits of information. It was framed to provide content scaffolding and was clearly presented in an organised manner. All except three of the tasks demanded the lowest level of knowledge manipulation, that is, reproduction. Of the three non-reproduction type of tasks, two required students to analyse, predict and evaluate knowledge while one (T3-L05-P1 shown in Figure 2) required students to apply their conceptual knowledge to solve a mathematical problem. The only task that provided students with procedural scaffolding was T2-L06-P1 shown in Figure 3. All the practice tasks given by the three teachers to their students were clearly presented, but students were not given any explicit performance or marking criteria along with the tasks.

Table 8. Levels of cognitive demand of practice tasks

<table>
<thead>
<tr>
<th>Task</th>
<th>Levels of cognitive demand</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Level 0</td>
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<td>T1-L01-P1-2</td>
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</tr>
<tr>
<td>T1-L01-P3</td>
<td>✓</td>
</tr>
<tr>
<td>T1-L01-P4-6</td>
<td>✓</td>
</tr>
<tr>
<td>T1-L02-P1-5</td>
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</tr>
<tr>
<td>T1-L02-P6</td>
<td>✓</td>
</tr>
<tr>
<td>T1-L03-P1-4</td>
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<tr>
<td>T1-L03-P5-7</td>
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<td>T2-L02-P1-7</td>
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<tr>
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</tr>
<tr>
<td>T3-L05-P1</td>
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</tbody>
</table>

Legend
Level 0 - Memorisation tasks
Level 1 - Procedural tasks without connections
Level 2 - Procedural tasks with connection
Level 3 - Problem solving / Doing mathematics

Table 8 shows the cognitive demands of the practice tasks according to Stein and Smith’s (1998) task analysis guide. All except five of the tasks were of memorisation type [level 0], that is, they required of students to reproduce facts, rules and formulae without any explanations. Of the five non-memorisation type of tasks, three were of the procedure without connections type [level 1], that is, they required students to use algorithms to do typical textbook type of exercises, which contextualise concepts and skills taught. Figure 2, shows an example of one such task, which is classified as an application of knowledge type of task according to the standards and dimensions analysis using Koh and Lee (2004) framework. The two tasks that were of the procedures with connections type are shown in Figures 1
and 4. Both these tasks required the students to do mathematics in meaningful and mathematically rich contexts.

Assessment Tasks

The analysis of the assessment tasks using Koh and Lee’s (2004) framework is presented in Table 9 which shows the standards and dimensions of the tasks that were given to the students as part of their mathematics assessment. All the tasks tested procedural knowledge. All the tasks also required students to present knowledge as truth rather than as comparing and contrasting knowledge or critiquing knowledge. All except five of the tasks required students to reproduce facts or procedures; manipulate algebraic expressions; carry out computations and apply mathematical concepts and procedures to solve simple and routine problems. Of the five non-reproduction type of tasks, four required students to analyse information given and use it to carry out a computation or interpret the data given,
make a graphical representation and find the solution. Three such tasks, which also had procedural scaffolding, are T3-L04-A4 (Teacher 3, Lesson 4, Assessment Task 4) shown in Figure 5, T2-L07-A6 shown in Figure 7 and T2-L07-A7 shown in Figure 8. The only task, T1-L06-A7, that required students to engage in problem solving is shown in Figure 6. This was a ‘non-routine’ task for the students. All the tasks were clearly presented in an organised way showing the marks allocated to each or parts of it.

Table 10 shows the cognitive demands of the assessment tasks according to Stein and Smith’s (1998) task analysis guide. All except eight of the tasks were of memorisation type [level 0], that is, they required students to reproduce facts, rules and formulae without any explanations. The eight non-memorisation types of tasks were of the procedures without connections type [level 1]. These tasks were algorithmic in nature and required students to apply facts, rules and formulae to standard textbook type of problems. Figures 5, 7 and 8 show three examples of such tasks.

Table 10. Levels of cognitive demand of assessment tasks

<table>
<thead>
<tr>
<th>Task</th>
<th>Levels of cognitive demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1-L06-A1-4</td>
<td>Level 0</td>
</tr>
<tr>
<td>T1-L06-A5-7</td>
<td>Level 0</td>
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<tr>
<td>T2-L07-A1-5</td>
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<td>T2-L07-A8-9</td>
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<td>T3-L03-A1</td>
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<tr>
<td>T3-L03-A2-4</td>
<td>Level 0, Level 1</td>
</tr>
</tbody>
</table>

Legend:
Level 0 - Memorisation tasks
Level 1 - Procedural tasks without connections
Level 2 - Procedural tasks with connections
Level 3 - Problem solving / Doing mathematics

The whole of this question must be answered on a single sheet of graph paper.

On a Saturday, Ahmad left home on his bicycle at 8 a.m. and travelled 4 km at a speed of 8 km/h directly to his school. After staying in school for CCA for 2 hours, he returned home by the same route in 20 minutes.

(a) Use a scale of 2 cm to represent 20 minutes for the time taken from 08 00 to 11 00 on the horizontal axis and 2 cm to represent 1 km for the distance on the vertical axis, and draw the travel graph. [3 marks]

(b) From the graph, find
(i) the time at which Ahmad arrived home [1 mark]
(ii) the speed at which he travelled on the return journey. [1 mark]
12 chickens took 5 days to eat 3 bags of corn. How many days will 10 chickens take to finish 4 bags of corn? [3 marks]

Figure 6. Task T1-L06-A7

If $10^x = 3$, find the value of $10^{3x}$ [2 marks]

Figure 7. Task T2-L07-A6

Given that $(x + y)^2 = 36$ and $xy = 4$, find the value of
(a) $x^2 + y^2$ [1 mark]
(b) $(3x - 3y)^2$ [2 marks]

Figure 8. Task T2-L07-A7

DISCUSSION

Figure 9 shows the framework of Singapore’s school mathematics curriculum, the primary goal of which is mathematical problem solving (Ministry of Education, 2006). As shown in the figure the development of mathematical problem solving ability is stated to be dependent on five inter-related components, namely, Concepts, Skills, Processes, Attitudes and Metacognition. The findings from the study of the tasks used by the three teachers who participated in the study will be examined in relation to three components, namely concepts, skills and processes of the framework.

Figure 9. Framework of school mathematics curriculum
Learning tasks used by the teachers either introduced new concepts and skills, made connections between new and old concepts or skills, or introduced students to knowledge or information that might excite them (example T3 showing them the history of Pythagoras via the internet) or explained some of their observations (example T3 working through the generalised representations of Pythagorean triplets). These tasks were either taken from the textbook or sourced by teachers from their personal resources. A closer examination of the learning tasks showed that the tasks were used for dissemination of knowledge via teacher talk. In other words the only activity the tasks led to were teacher–whole class discourse that culminated with teacher telling and showing. None of the tasks were exploratory, that is, allowing students to engage in some activity before conjecturing and finally making knowledge one’s own.

Also the significantly large number of practice tasks that preceded a learning task showed that there was great emphasis on ‘practice makes perfect’. Most of the practice tasks were taken from the textbook and were procedural in nature. The textbooks used in the three classrooms adopted the exposition-examples-exercises model (Love and Pimm, 1996) and therefore the exercises of the textbook for the relevant topic formed the bulk of the practice tasks. These tasks were mainly procedural and algorithmic in nature. The few tasks that were word problems contextualised in some ‘real-world’ context also failed to provide students with the experience of applying their knowledge to solve novel problems, as the deictic relation of mathematics textbook questions to the real world is a highly problematic one (Love and Pimm, 1996). Only two tasks, T1-L01-P3 (shown in Figure 1) and T3-L02-P1 (shown in Figure 4), provided students with opportunities to engage in thinking skills such as comparing, inductive reasoning and systematic listing.

It may be said that in all the three classrooms, concepts and skills were developed during the mathematics lessons but the range of processes that students were given opportunities to engage in were limited. The scope of the term ‘processes’ in the framework shown in Figure 9 is skills involved in acquiring and applying mathematical knowledge. This includes reasoning, communications and connections, thinking skills and heuristics, and application and modelling. (Ministry of Education, 2006, p. 4)

The assessment tasks were taken from past examination papers. These tasks mainly tested the reproduction of facts or procedures, manipulation of algebraic expressions, computations and application of mathematical concepts and procedures to solve simple and routine problems. Bearing in mind the limitations of pencil-paper tests, these items appeared to largely test for concepts and skills. The task T1-L06-A7 is noteworthy as it attempted to engage grade eight students, who have not worked with complex variation tasks, to attempt it using problem solving heuristics. The assessment tasks were to a large extent aligned to the practice tasks and it may be claimed that the nature of assessment tasks push textbook writers to mould their exercises accordingly and teachers to provide their students with purposeful practice!
The data analysed and presented in this chapter were part of the LPS data from Singapore. Three grade eight mathematics teachers who were deemed competent by their local community participated in the LPS from Singapore. The goal of this chapter was to study the source and nature of mathematical tasks, specifically the learning, review, practice and assessment tasks used by the three teachers to implement the intended curriculum and ascertain the extent of the link between the tasks and the primary goal of the school mathematics curriculum, in other words, mathematical problem solving (Ministry of Education, 2006). The questions that may be answered by the data and analysis in this chapter are:

“What drives the classroom instruction?” and

“Is mathematical problem solving the end goal of both the instructional and assessment practice in the three classrooms?”

The source of the learning tasks shows that teachers draw on both the textbook and personal resources where appropriate, to assist students with the learning of concepts and skills. This is followed by practice, which is mainly driven by the textbook that is used by both the teacher and students. With the exception of the two practice tasks that engaged students in more than algorithmic and procedural activity, it may be said that most of the practice tasks were textbook exercises to hone skills and application of concepts in simple routine tasks. The assessment tasks were closely aligned with the practice tasks, and it appears that the assessment tasks influence the textbook exercises. The marks for each part of the assessment tasks were clearly indicated, but no other explicit marking criteria, for example the tension between product and process were given. Hence it may be claimed that in these three classrooms the instruction was driven mainly by the textbook and somewhat by the teachers’ personal resources. Although textbooks used in Singapore schools must have the approval from the Ministry of Education and satisfy the curriculum content requirements, they have their limitations too. The data presented in this chapter shows that although problem solving is the official end goal of both instruction and assessment as stipulated by the framework for the school mathematics curriculum, the instructional practice and assessment practice of the three classrooms appear to be poorly aligned with this goal.

This chapter has also made a contribution towards a framework for the analysis of review tasks. An analysis of the purpose of the review tasks used by the three teachers has resulted in a preliminary framework for the analysis of such tasks. This framework, which comprises the following types:

- **Type 1**: recall of prior knowledge
- **Type 2**: provide scaffolding for subsequent tasks
- **Type 3**: connect newly acquired knowledge with past knowledge
- **Type 4**: relate newly acquired knowledge to real life examples

will be used to analyse more review tasks from the corpus of LPS data in Singapore to test for its validity and expansion where feasible.
NOTES

‘Teacher assignments – this term is coined and means written assignments given by teachers to their students for either in class or out of class follow-up work subsequent to a lesson.

ACKNOWLEDGEMENTS

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REFERENCES


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