Key Works in Radical Constructivism
BOLD VISIONS IN EDUCATIONAL RESEARCH

Keyworks

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Key Works in Radical Constructivism
Ernst von Glasersfeld

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Marie Larochelle

Comments by:
Edith Ackermann
Gérard Fourez
Jacques Désautels
Leslie P. Steffe

Postscript by:
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Only the first of the twenty-two chapters in this volume deals explicitly with education. The others discuss language, theory of knowledge, and the formation of concepts. These are three topics in which traditional educational research has rarely shown an interest. I am suggesting that this disregard can to a large extent be blamed for the dismal state of education today. By and large the notion that language, if used properly, is an efficient conveyer of knowledge was unquestioningly accepted. Any detailed analysis of a lesson and its effect on the students shows that this is an illusion. A closer look at some aspects of the actual workings of language may therefore not be a waste of time.

Western philosophers have with few exceptions perpetuated the belief that true knowledge exists apart from the knower. Although they have been unable to show how it might enter the human head, they have staunchly condemned as ‘genetic fallacy’ any investigation of how the knower might generate it. The essays in the section on the theory of knowledge are intended to provide teachers, parents, and perhaps students themselves with a more fruitful philosophical background.

The concepts analyzed in the third section are but a sample that needs to be greatly expanded. But they constitute a scaffolding for the construction of others. They provide the teacher with a model of how it could be done—and how concepts can be built up is surely something teachers should be able to show their students.

The list of the sources of the collected papers shows that they were written for diverse occasions and diverse audiences. Although I have always tried to take this into account, the essential elements of my position could not be changed. I regret that the reader of the present book, therefore, has to bear with a number of repetitions.

I thank Ken Tobin for his Postscript and Edith Ackermann, Jacques Désautels, Gérard Fourez, and Leslie P. Steffe for commentaries, which greatly enhance the value of this book.

To Marie Larochelle I am immensely grateful for many suggestions and the painstaking editorial work she has done; and I appreciate the willingness of Sense Publications to publish a book that may cause some ripples.

E.v.G.
November 2006
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Leslie P. Steffe is a Distinguished Research Professor of Mathematics Education at the University of Georgia. He collaborated with Ernst von Glasersfeld on the project ‘Interdisciplinary Research on Number’ (IRON) at the University of Georgia. The influence of radical constructivism originated in that work in IRON and has spread through the work of many investigators.
ACKNOWLEDGEMENT OF SOURCES


Ch. 3 Facts and the self from a constructivist point of view. Poetics, 18(4-5), 435-448, 1989.

Ch. 4 Signs, communication, and language. Journal of Human Evolution, 3, 465-474, 1974.

Ch. 5 How do we mean? A constructivist sketch of semantics. Cybernetics and Human Knowing, 6(1), 9-16, 1999.


Ch. 10 The simplicity complex. Original of ‘Il complesso di semplicità’. In G. L. Bocchi & M. Ceruti (Eds.), La sfida della complessità (pp. 103-111). Milan, Italy: Feltrinelli, 1985.

Ch. 11 The logic of scientific fallibility. Expanded version of a paper presented at the 8th Biennial Conference, Mental Research Institute, San Francisco, 1987.

Ch. 12 The incommensurability of scientific and poetic knowledge. Expanded version of a talk given at the International Congress on Science, Mysticism, Poetry, and Consciousness, Instituto Piaget, Lisbon, April 1994.


Ch. 22 Author’s translation of ‘Universalien als Konstruktion’. In P. M. Hejl (Ed.), Universalien und Konstruktivismus (pp. 68-75). Frankfurt: Suhrkamp, 2001.
INTRODUCTION

ERNST VON GLASERSFELD’S WAY
OF WORLDMAKING

In three short stories said to have inaugurated the genre of detective fiction, Edgar Allan Poe brought to life a character, Auguste C. Dupin, whose comments in many ways bring to mind the comments and thoughts of that other well-known character Ernst von Glasersfeld, the pioneering thinker of radical constructivism whose principal texts in education are the subject of this book.¹ Auguste C. Dupin and Ernst von Glasersfeld are alike in holding the view that we are always arriving too late on the scene to be able to behold a pure, as-yet un-interpreted world. Rather, the world that we are seeing and experiencing is one that has been configured according to both the notions that we entertain about it and the distinctions with which we have laden it; further, such notions and distinctions constitute practical means of our own invention, devised to co-ordinate and manage our experience of the world (Glasersfeld, 1993). Ultimately, whenever we claim to describe the world-in-itself (or the ‘ontologically preexisting world’), we in fact are describing the product of the mapping process that has enabled us to make our way in this world and to actualize our projects within it (inclusive even of the ‘dud’ roadmaps—that is, the cognitive itineraries that have proved non-viable or indeed fatal to our assumptions and views). In short, we are describing what can be done in the world and not, to paraphrase Geertz (1988), seeing the world as it really is when only God is looking! Thus, from this perspective, knowledge is viewed as “a search for fitting ways of behaving and thinking” (Glasersfeld, 1988, p. 41), and thus said to be operative, as it allows us to operate, act and anticipate, just as it can, obviously, lead us into dead-ends, as is shown in one of the cases narrated by detective Dupin.

In ‘The Purloined Letter’, published circa 1845, Dupin comments on the failure of the Paris Prefect of Police to locate a letter of paramount importance, tying this inability to the police chief’s habits of comprehending the world and figuring the capacities of others—in this instance, the thief, Minister D—I, who also happened to be a poet. As Dupin informs us, in the Prefect’s view a poet is by definition a fool and a scatterbrain—therefore, the kind of person who would think to hide the letter nowhere else than in some unlikely spot or other. On the basis of this assumption, the Prefect and his men painstakingly searched the thief’s apartment, ripping up the inlaid pieces of the parquet floor, scrutinizing the bindings of his entire book collection beneath a microscope, peering inside the hollows of the chair legs and sinking long needles into the chair cushions—all to no avail. Further,
throughout their searches, they remained completely oblivious to the letter that had been placed prominently on display atop a fireplace mantle!

All of which goes to show—and on this point Dupin the detective and Glaserfeld the epistemologist again think alike—the importance of developing a reflexive understanding of the world; in other words: an understanding that is conscious of its assumptions and that, as a result, is conscious of being one manner of understanding or one ‘take’ among other possible manners of understanding or ‘takes’. By the same token, this does not mean that all takes or intellectual constructions are equal or interchangeable. Indeed, the Prefect’s failed efforts at finding the letter offers a telling illustration of how this is not so. On the other hand, if he and his men had previously developed the habit of thinking reflexively, they might have been able to vary their investigative approaches somewhat. In addition, they might well have been able to work up not one but several composite drawings of the thief and, as a result, would have multiplied their potentialities for action, as Glaserfeld would say.

* * *

For more than 40 years now, Ernst von Glaserfeld has invited us to take up detective work and reflection on the question of ‘how we know what we know’. This he has done not only at numerous seminars and workshops, but also through hundreds of written contributions in a broad range of fields (such as psychology, philosophy, linguistics, cybernetics and, of course, education) across an equally wide spectrum of heavyweight themes (such as the notions of truth, objectivity and the transparency of language), and with a style of writing that knowingly and knowledgeably grapples with the epistemological genre framing the debate at hand. Regardless of the subject contemplated (such as poetic wisdom, the conceptual construction of time, or scientific fallibility) or the field involved, ongoing inquiry into radical constructivism has been his main, steadfast concern, which has been shaped by his acute sense that the things that are said are said from within a perspective that cannot claim to be all-knowing and all-encompassing.

In short, with Glaserfeld’s work, it is a line of thought we are contemplating—or rather, an oeuvre. All the more so, to borrow from Foucault (1975), Glaserfeld is more than simply the author of texts, for he has indeed made possible the formation of other texts and other discourses. He has, by the same token, inaugurated a way of viewing. In other words, as is testified to by the numerous publications and curriculum designs drawing on constructivism, Ernst von Glaserfeld has opened up both intellectual and institutional space for revisiting the issues of cognition and learning (and in the process, moreover, has managed to chip away at certain privileges).2

Accordingly, the idea for this book—to which Kenneth Tobin and Joe Kincheloe, editors of the “Bold Visions in Educational Research” collection gave the initial impetus and to which Glaserfeld personally contributed thereafter—grew out of a desire to make his ideas on all these different issues more accessible and thus to highlight not only the originality, fecundity and consistency of his line
ERNST VON GLASERSFELD’S WAY OF WORLDMAKING

of thought but also the breaks with ‘realist metaphysics’ that it necessarily entails. Initially, we wished to confine ourselves to texts on education (according to the generally accepted meaning of this term). Before long, however, we discarded this plan, which would have had the effect of only partially illustrating the reversal of perspective implied in radical constructivism, and of ignoring the principles and larger implications of this reversal. We thus decided to take the approach outlined below, which brings together a number of interpretations and conceptual analyses through which Glasersfeld takes up the different issues in question, while also expanding the purview of inquiry beyond the ‘school form’ of these issues, nonetheless.

The first three parts of this book contain texts by Glasersfeld that are intended as a window onto the constructivist way of configuring the issues of cognition and learning and on the attendant—and frequently drastic—reconceptualization of closely related themes. Thus, Part I, which opens with a text dedicated to the set of issues surrounding teaching and learning, is concerned with reconceptualizing the concepts of knowledge and fact (chapters 2 and 3), as well as the concepts of communication, interpretation and meaning (chapters 4, 5 and 6). It is also concerned with reinterpreting Piagetian epistemology and bringing out its family ties to radical constructivism (chapter 7).

Part II outlines various theoretical frameworks that have shaped constructivism (such as cybernetics, chapter 15) and presents the major thinkers and researchers who have contributed to the development of this line of thought, such as Vico, Berkeley, Piaget (chapter 8), Ceccato (chapter 15) and Foerster (chapter 13). This part also proposes the outline of a pragmatic conception of science-making (chapter 14), as well as a discussion of the staple notions underpinning realist discourse, such as objectivity and truth (chapters 9, 11 and 13). Finally, it is also a question of the ways and means (rational or other) used to bring order to one’s experience (chapters 10 and 12).

Part III is a collection of conceptual analyses bearing, among other things, on processes of change, abstraction, deduction and anticipation (chapters 16, 17, 18 and 21), on the construction of time and ‘universals’ (chapters 20 and 22), and on the experiential—as opposed to the ontological—nature of the foundations of mathematics (chapter 19).

Part IV, differing somewhat in form the preceding portions, consists of commentaries penned by colleagues and friends of Glasersfeld. Their common objective is to point out the avenues of reflection that he has been instrumental in opening up, and to indicate which among these paths, today, warrant undertaking new efforts to delineate them better or to reassess their import more fully. In other cases, the goal should be to suggest linking routes worth exploring between the main well-travelled avenues.

In the first commentary (chapter 23), Edith Ackermann proposes a set of reflections on our interactions with artifacts, which she views as offering a valuable opportunity for offsetting constructivism’s tendency to focus on the subject’s assimilative activity. In particular, she highlights the process of accommodation that is a necessary part of the effort to co-ordinate objects that cannot be bent to
one’s will. So doing, she grounds the subject in a world requiring him or her to
deal with objects that come laden with affordances (that is, guidelines and indeed
injunctions concerning the way such objects are to be used) and invites us to
examine how the subject will go about appropriating it.

Gérard Fourez, who signs the second commentary (chapter 24), advocates re-
examining the concept of representation and, at the same time, urges rehabilitating
the latter—not in its iconic form, obviously, but instead as a model, a map or chart
of our possibilities for action and, hence, as a tool with which to develop our world
and to engage in deliberations about it. He suggests taking the same approach with
the concept of fact, noting that the standardized, convention-ruled character of facts
argues powerfully for developing a kind of constructivism that fully accounts for
the sociopolitical dimension of the construction and legitimization of knowledge.

In the third commentary (chapter 25), Jacques Désautels also pleads for a
variety of radical constructivism that opens onto the social and the political. He
shows that through the concepts of intersubjectivity and recursive processes alone,
constructivism already provides a basis for calling into question the classic division
between individual and society. What is more, however, by conceiving of society
and individual from the perspective of interdependence, it is possible, he argues, to
take this challenge one step further. Indeed, by drawing on work in science studies,
it is possible to seriously re-examine the other ‘Great divide’ that frequently
dominates the landscape of scientific literacy in schools—namely, the separation of
questions and issues pertaining to science from those pertaining to society.

The last commentary (chapter 26), by Leslie P. Steffe, urges us to pay greater
attention to the context of reception for radical constructivism, and particularly in
the field of mathematics education. In this setting, constructivism often appears to
take form according to a logic reminiscent of that which characterized the Paris
Prefect of Police and that prompts us to underestimate the competencies of
others—and of students in particular. In other words, it would seem as if we
continued to labour under the assumptions of the cognitive deficit model that we
first adopted to reflect on learning/teaching relationships; it would seem as if we
were shying away from entering the world of ‘co-inspiration’ and the
‘interweaving (in French, *enroulement*) of perspectives’ that is promoted by
constructivism.

Finally, in his postscript to this book, Kenneth Tobin recalls, from his privileged
vantage point, several of the episodes and events that signalled the emergence of
constructivism in the field of science education. His account serves to bring out the
revolutionary quality of this perspective at a time when the field was still largely
under the sway of a realist ontology. Furthermore, it relates how his own field
work and research led him to frame constructivism within the terms of culture and,
in the process, contribute to the development of this line of thinking by adopting a
sociocultural approach to issues of cognition and education. Kenneth Tobin has
thus crafted the conclusion to a book which, in an approach recalling of the notion
of *équilibration majorante* so dear to Piaget, bears witness to the unceasing
reflexive elaboration to which constructivism is submitted—not only in the work of
Glasersfeld but also in the texts of all those who have chosen to work from within this perspective.

* * *

It goes without saying that this edition of Glasersfeld’s texts, which was brought out in part through funding from the Social Sciences and Humanities Research Council of Canada, has benefited from the close collaboration and valued contributions of a number of individuals. I would like to thank my colleagues Suzanne Vincent and Serge Desgagné for their suggestions, Bernard Jobin for his assistance in assembling the bibliography, and Sylvie Côté for her work creating the tables. I would also like to express my gratitude to my Faculty’s administration for its financial contribution toward translation costs. Finally, I wish to thank Donald Kellough who, in connection with several contributions, has once again tackled the challenge of translating (i.e., re-mapping) into English a ‘Francophone way of worldmaking’.

Marie Larochelle, Editor
Université Laval, Québec

NOTES

1 The first paragraphs of this introduction are based on excerpts from the address I gave on June 18, 2006, on the occasion of the Université Laval (Québec, Quebec) graduation ceremony at which Ernst von Glasersfeld was awarded the degree of doctor of education honoris causa. A slightly modified version of these passages can also be found elsewhere (Larochelle & Désautels, 2007). It was only shortly after having written this address, while re-reading Ernst von Glasersfeld’s book entitled The Construction of Knowledge (published in 1987 by Intersystems Publications) that I realized that Heinz von Foerster, in his preface to this publication, also noted a similarity between the work of Glasersfeld and that of Dupin, particularly on the question of language. It had been some 20 years since I last read this preface, with the result that in June 2006, I genuinely imagined that I was working from an association that had heretofore gone unnoticed!

2 By holding that there is not one but many ‘ways of worldmaking’, to borrow Goodman’s (1992) felicitous expression, radical constructivism undertakes more than the promotion of a pragmatic conception of knowledge. He de-centers what Bourdieu (1980) called the ‘racism of intelligence’, which consists in ascribing the ability to produce valid bodies of learning and knowledge to certain groups only. In other words, students, for example, may not know what we would like them to know, but, following radical constructivism, they are not lacking in knowledge. Through their learning paths and interactions, they too have elaborated ‘authentic knowledge’, to use Bensaude-Vincent’s (2000) phrase—they have, in other words, engaged with their experiences of the world and circumstances.

3 I borrow the concept of ‘co-inspiration’ from Maturana (1988) and that of the ‘interweaving (enroulement) of perspectives’ from Rocher (2005).

4 That is, as Glasersfeld has pointed out (1981), “the incremental equilibration that proceeds in spiral form, incorporating more and more items and events in the developing organism’s experience” (p. 93).
PART I

LEARNING, LANGUAGE AND THE RADICAL THEORY
CHAPTER 1

LEARNING AS CONSTRUCTIVE ACTIVITY

The general topic I was given for this chapter is ‘Research in Mathematics Education from an Epistemological Perspective’. That sounds no more dangerous than so many other academic topics. But don’t let the prosaic surface deceive you. To introduce epistemological considerations into a discussion of education has always been dynamite. Socrates did it, and he was promptly given hemlock. Giambattista Vico did it in the 18th century, and the philosophical establishment could not bury him fast enough. In our own time there was Jean Piaget. He really wanted to stay out of education but allowed himself to be drawn in—and we know what has happened to his epistemology at the hands of interpreters and translators. It seems that to discuss education from an epistemological point of view was a sure way of committing intellectual suicide. Recently, however, the world of education may have begun to change. At least the particular discipline that is represented in this meeting, the discipline that is concerned with numbers, with arithmetic, and ultimately with mathematics, is manifesting symptoms that indicate the will to change.

The rapid shifts in the methods of mathematics education that have taken place in the last few decades—from simplistic associationism to ‘New Math’ and ‘Back to Basics’—did not work the miracles that were expected of them. Their failure has created a mood that no longer fosters enthusiasm for new gimmicks. Today, I think it is fair to say, there is a more or less general disillusionment. This disillusionment is healthy and propitious because it pushes us closer to the point where we might be ready to review some of the fundamental presuppositions of the traditional theories of education. Among these presuppositions are our conception of teaching and learning and, most fundamental of all, the conception of what it is ‘to know’.

Ten or 15 years ago, it would have been all but inconceivable to subject educators or educational researchers to a talk that purported to deal with a theory of knowledge. Educators were concerned with getting knowledge into the heads of their students, and educational researchers were concerned with finding better ways of doing it. There was, then, little if any uncertainty as to what the knowledge was that students should acquire, and there was no doubt at all that, in one way or another, knowledge could be transferred from a teacher to a student. The only question was, which might be the best way to implement that transfer—and educational researchers, with their criterion-referenced tests and their sophisticated statistical methods, were going to provide the definitive answer.

Something, apparently, went wrong. Things did not work out as expected. Now there is disappointment, and this disappointment—I want to emphasize this—is not restricted to mathematics education but has come to involve teaching and the didactic methods in virtually all disciplines. To my knowledge, there is only one

Glasersfeld, E. von, Key Works in Radical Constructivism (Edited by M. Larochelle), 3–19. © 2007 Sense Publishers. All rights reserved.
exception that forms a remarkable contrast: the teaching of physical and, especially, athletic skills. There is no cause for disappointment in that area. In those same 10 or 15 years in which the teaching of intellectual matters has somehow foundered, the teaching of skills such as tennis and skiing, pole jumping and javelin throwing, has advanced quite literally by leaps and bounds. The contrast is not only spectacular but it is also revealing. I shall return to this phenomenon at a later point when, I hope, we will be able to consider an analogy which, at this moment, might seem utterly absurd.

If educational efforts are, indeed, failing, the presuppositions on which, implicitly or explicitly, these efforts have been founded must be questioned and it seems eminently reasonable to suggest, as did those who formulated the topic for this discussion, that we begin by inspecting the commodity that education claims to deal in, and that is ‘knowledge’.

This chapter is an attempt to do three things. First, I shall go back to what I consider the origin of the troubles we have had with the traditional conception of knowledge. This historical review will not only be sketchy, but it will also be quite biased, because I have rather strong views on the subject. However, considering the mess in which the theory of knowledge has been during the last 50 years in the ‘hard’ sciences, my attempt will, I hope, not be deemed unjustified.

Second, I shall propose a conceptualization of ‘knowledge’ that does not run into the same problem and that, moreover, provides another benefit in that it throws helpful light on the process of communication. As teachers, I said a moment ago, we are intent upon generating knowledge in students. That, after all, is what we are being paid for, and since the guided acquisition of knowledge, no matter how we look at it, seems predicated on a process of communication, we should take some interest in how that process might work. In my experience, this is an aspect that has not been given much thought. Educators have spent and are rightly spending much time and effort on curriculum. That is, they do their best to work out what to teach and the sequence in which it should be taught. The underlying process of linguistic communication, however, the process on which their teaching relies, is usually simply taken for granted. There has been a naive confidence in language and its efficacy. Although it does not take a good teacher very long to discover that saying things is not enough to ‘get them across’, there is little if any theoretical insight into why linguistic communication does not do all it is supposed to do. The theory of knowledge which I am proposing, though it certainly does not solve all problems, makes this particular problem very clear.

Lastly, having provided what I would like to call a model of ‘knowing’ that incorporates a specific view of the process of imparting knowledge, I shall briefly explore a way to apply that model to the one thing all of us here are interested in: how to introduce children to the art, the mystery, and the marvelous satisfaction of numerical operations.
LEARNING AS A CONSTRUCTIVE ACTIVITY

THE INSTRUMENTALIST ANSWER TO THE SCEPTICS’ ATTACK

The nature of knowledge was a hotly debated problem as far back as the 6th century B.C. The debate has been more or less continuous, and while in many ways it has been colorful, it has been remarkably monotonous in one respect. The central problem has remained unsolved throughout, and the arguments that created the major difficulty at the beginning are the very same that today still preclude any settlement of the question.

The story begins with the first documents on epistemology that have come down to us, the so-called ‘fragments’ of the pre-Socratics. The ideas these men struggled with and tried to clarify must have arisen some time before them, but since we have no earlier written records, that background is extremely hazy. The pre-Socratics, at any rate, exhibit a degree of sophistication that is unlikely to have been acquired in one or two generations. Fragmentary though they are, their pronouncements leave no doubt that, towards the close of the 5th century B.C., the process of knowing had been conceptually framed in a relatively stable general scenario. By and large, the thinkers who concerned themselves with the cognizing activity tacitly accepted the scenario in which the knower and the things of which, or about which, he or she comes to know are, from the outset, separate and independent entities.

I want to stress that this dichotomy does not coincide with the split between the knowing subject and the subject’s knowledge. That second dichotomy appears whenever an actor becomes aware of his or her own activity or when a thinker begins to think about his or her own thinking. That second problem of self-consciousness is not identical with the problem of cognition. Though the two are related in that they interact (e.g., in an analysis of reflective thought, which will enter our discussion later), I here want to deal only with the first. The pre-Socratics, in any case, took for granted the human ability to be aware of knowing. What they began to wonder about was how it was possible that one could come to know the world. It is in this quest that the cognitive scenario they accepted and that has been perpetuated by almost all epistemologists after them, is of decisive importance. Once it was chosen as the basis for the construction of a theory of knowledge, that construction was saddled with a paradox. The paradox is inescapable and it has haunted philosophers incessantly in the 2,500 years since then.

The reason why that particular cognitive scenario was adopted is very simple. It reflects the situation as it initially appears to any experiencer. The question, how it comes about that we know anything, is not likely to be asked at the beginning of a prospective knower’s development. A six-year-old who bicycles home from school would be a very peculiar six-year-old if she suddenly asked herself, ‘How on earth do I manage to find my way home?’ or ‘What exactly happened when I learned to ride my bicycle?’

I am not suggesting that these are questions a six-year-old or, indeed, anyone should ask. I am merely saying that if we ever do ask them, it will be at a somewhat later age. The same goes for the question, ‘How is it that I can know
what I do know?’ Those who have felt such epistemological curiosity probably formulated their first relevant question in their middle teens or later. That is to say, they began to question their knowledge at a point in their cognitive career when they had already acquired an enormous amount of know-how and learning. Inevitably, nearly all they knew at that point was tacitly assumed to be knowledge about the environment, about the world in which they found themselves living. It is not surprising that this should be the case. Once one has learned to manage things, there is no reason to suspect that they might not be what they seem.

If a person whose knowledge has been growing and expanding over the years then raises questions about how one comes to have all that knowledge, it seems reasonable to postulate at the beginning an inexperienced and totally ignorant knower, who comes into the world, much as an explorer might come into a terra incognita, with both the need and the will to discover what that world is like. The first if not the only tools that seem to be available for such a task are obviously the senses. Therefore, the senses are at once categorized as organs, or channels, through which the experiencer receives messages from the environment. On the basis of these messages, the experiencer then must, and apparently can, build up a ‘picture’ of the world. In our contemporary jargon, this is often expressed by saying that the senses convey information which enables the experiencing subject to form a representation of the world. Usually this seems to work quite well. Occasionally, of course, the senses turn out to be somewhat deceptive, but by and large they work well enough for us to build up a modus vivendi. Provided we remain patient and flexible, we will continue to make adjustments, and as long as things work moderately well, there will be no need to question the over-all validity of whatever picture of the world we have built up.

The pre-Socratics started out in this thoroughly normal way, but because there were some highly original thinkers among them, they came up with mutually incompatible pictures of the world.1 Obviously, that was felt to be a problem and it led to two closely connected questions: One, how could anyone compose a picture of the world out of sensory messages and, two, how could one be certain that a particular picture of the world was ‘true’? Attempts to answer these questions soon ran into troubles, some of them so serious that they have not yet been overcome.

Here I want to focus on the second problem because it is inherent and unavoidable in the discoverer’s scenario. If experience is the only contact a knower can have with the world, there is no way of comparing the products of experience with the reality from which whatever messages we receive are supposed to emanate. The question, how veridical the acquired knowledge might be, can therefore not be answered. To answer it, one would have to compare what one knows with what exists in the ‘real’ world—and to do that, one would have to know what ‘exists’. The paradox, then, is this: to assess the truth of your knowledge you would have to know what you come to know before you come to know it.

The argument that the likeness or trustworthiness of a picture can be assessed only by looking at both the picture and what it is supposed to depict, was brought forth already at the time of the pre-Socratics and it has been the mainstay of all
scepticism ever since. The history of Western epistemology is the history of more or less imaginative attempts to circumvent it. None of these attempts was satisfactory. Plato’s poetic genius almost succeeded in eliminating the dilemma by undercutting the role of experience. He placed the real reality into the world of ideas and turned sensory experience into a secondary affair, murky, unreliable, and ultimately irrelevant to the quest for truth. Since the world of ideas was accessible only to the thinking mind, this arrangement bred the notion of solipsism, the notion that there is no real world beyond the world the mind creates for itself.

Alternatively one could, as Descartes suggested, place one’s faith in God and trust the divine maker not to have been so malicious as to have provided his creatures with deceptive senses.

Neither of the two alternatives provides a durable solution. Solipsism turns into absurdity whenever an idea we have conceived is shattered by experience. In fact, that is not a rare occurrence. We are constantly reminded that the world we live in is not quite the world we would like and that there is, indeed, a hard and unforgiving ‘reality’ with which we have to cope. On the other hand, the Cartesian notion boils down to a simple injunction to believe, and that does not satisfy the philosopher’s need. If epistemology must be founded on the blind faith that our senses convey a true picture, it cannot accomplish what it sets out to do, namely, provide a rational basis for the generation and assessment of knowledge. Actually, Descartes’ injunction to trust our God-given senses merely shifts the problem. If the senses were thought to be trustworthy, the fact that we so often draw the wrong conclusion from their messages should show that there is a serious difficulty of interpretation; and if we cannot be sure how to interpret what the senses tell us, we again have to admit that we have no certain knowledge of the world and that the picture we come to have of it remains questionable.

The problem, as I suggested at the beginning, is intrinsic to the traditional scenario. It arises from the ‘iconic’ conception of knowledge, a conception that requires a match or correspondence between the cognitive structures and what these structures are supposed to represent. Truth, in that conception, inevitably becomes the perfect match, the flawless representation. The moment we accept that scenario, we begin to feel the need to assess just how well our cognitive structures match what they are intended to represent. But that ‘reality’ lies forever on the other side of our experiential interface. To make any such assessment of truth we should have to be able, as Hilary Putnam (1981) recently put it, to adopt a ‘God’s eye view’. Since we are not, and logically cannot be, in a position to have such a view of the ‘real’ world and its presumed representation, there is no way out of the dilemma. What we need is a different scenario, a different conception of what it is ‘to know’, a conception in which the goodness of knowledge is not predicated on likeness or representation.

The first explicit proposal of a different approach originated in those quarters that were most concerned with faith and its preservation. When, for the first time, the revolutionary notion that the Earth might not be the center of the universe seriously threatened the picture of the world which the Church held to be unquestionable and sacred, it was the defenders of the faith who proposed an
alternative scenario for the pursuit of scientific knowledge. In his preface to
Copernicus’ treatise *De revolutionibus*, Osiander (1627) suggested:

> There is no need for these hypotheses to be true or even to be at all like the
> truth; rather, one thing is sufficient for them—that they yield calculations
> which agree with the observations. (in Popper, 1968, p. 98)

This introduces the notion of a second kind of knowledge, apart from faith and
dogma, a knowledge that fits observations. It is knowledge that human reason
derives from experience. It does not represent a picture of the ‘real’ world but
provides structure and organization to experience. As such it has an all-important
function: it enables us to solve experiential problems.

In Descartes’ time, this instrumentalist theory of knowledge was formulated and
developed by Mersenne and Gassendi. It was then extended by Berkeley and
Vico, given strong but unintended support by Hume and Kant; and at the end of the
last century, it was applied to physics and science in general by Ernst Mach and to
philosophy by Georg Simmel. It was not and still is not a theory popular with
traditional philosophers. The idea that knowledge is good knowledge if and when it
solves our problems is not acceptable as criterion to those who continue to hope
that knowledge, ultimately, will at least approximate a true picture of the ‘real’
world.

Karl Popper (1968, chapter 3), who has given a lucid account of the beginnings
of instrumentalism, has struggled hard to convince us that, though reasonable, it is
an unsatisfactory theory. As he reiterates in his latest work:

> What we are seeking in sciences are true theories—true statements, true
descriptions of certain structural properties of the world we live in. These
theories or systems of statements may have their instrumental use; yet what
we are seeking in science is not so much usefulness as truth; approximations
to truth; explanatory power, and the power of solving problems: and thus,
understanding. (1982, p. 42)

This suggests that ‘descriptions’, ‘explanations’, and ‘understanding’ can indeed
capture aspects of ‘the world we live in’. Whether we can or cannot agree with this
statement will depend on how we define ‘the world we live in’. There is no doubt
that Popper intended an objective world, that is, a ready-made world into which we
are born and which, as explorers, we are supposed to get to know. This is the
traditional realist view, and Popper does his best to defend it, in spite of all
arguments one can hold against it. The realists and the sceptics are once more in
the familiar deadlock.

Yet, there is another possibility. ‘The world we live in’ can be understood also
as the world of our experience, the world as we see, hear, and feel it. This world
does not consist of ‘objective facts’ or ‘things-in-themselves’ but of such invariants
and constancies as we are able to compute on the basis of our individual
experience. To adopt this reading, however, is tantamount to adopting a radically
different scenario for the activity of knowing. From an explorer who is condemned
to seek ‘structural properties’ of an inaccessible reality, the experiencing organism
now turns into a builder of cognitive structures intended to solve such problems as
the organism perceives or conceives. Fifty years ago, Piaget (1937) characterized
this scenario as neatly as one could wish: “Intelligence organizes the world by
organizing itself” (p. 311). What determines the value of the conceptual structures
is their experiential adequacy, their goodness of fit with experience, their viability
as means for the solving of problems, among which is, of course, the never-ending
problem of consistent organization that we call understanding.

The world we live in, from the vantage point of this new perspective, is always
and necessarily the world as we conceptualize it. ‘Facts’, as Vico saw long ago, are
made by us and our way of experiencing, rather than given by an independently
existing objective world. But that does not mean that we can make them as we like.
They are viable facts as long they do not clash with experience, as long as they
remain tenable in the sense that they continue to do what we expect them to do.

This view of knowledge, clearly, has serious consequences for our
conceptualization of teaching and learning. Above all, it will shift the emphasis
from the student’s ‘correct’ replication of what the teacher does, to the student’s
successful organization of his or her own experience. But before I expand on that I
want to examine the widespread notion that knowledge is a commodity that can be
communicated.

COMMUNICATION AND THE SUBJECTIVITY OF MEANING

The way we usually think of ‘meaning’ is conditioned by centuries of written
language. We are inclined to think of the meaning of words in a text rather than of
the meaning a speaker intends when he or she is uttering linguistic sounds. Written
language and printed texts have a physical persistence. They lie on our desks or can
be taken from shelves, they can be handled and read. When we understand what we
read, we gain the impression that we have ‘grasped’ the meaning of the printed
words, and we come to believe that this meaning was in the words and that we
extracted it like kernels out of their shells. We may even say that a particular
meaning is the ‘content’ of a word or of a text. This notion of words as containers
in which the writer or speaker ‘conveys’ meaning to readers or listeners is
extraordinarily strong and seems so natural that we are reluctant to question it. Yet,
it is a misguided notion. To see this, we have to retrace our own steps and review
how the meaning of words was acquired at the beginning of our linguistic career.

In order to attach any meaning to a word, a child must, first of all, learn to
isolate that particular word as a recurrent sound pattern among the totality of
available sensory signals. Next, she must isolate something else in her experiential
field, something that recurs more or less regularly in conjunction with that sound
pattern. Take an ordinary and relatively unproblematic word such as ‘apple’. Let us
assume that a child has come to recognize it as a recurrent item in her auditory
experience. Let us further assume that the child already has a hunch that ‘apple’ is
the kind of sound pattern that should be associated with some other experiential
item. Adults interested in the child’s linguistic progress can, of course, help in that
process of association by specific actions and reactions, and they will consider their
‘teaching’ successful when the child has come to isolate in her experiential field something that enables her to respond in a way which they consider appropriate. When this has been achieved, when the appropriate association has been formed, there is yet another step the child must make before she can be said to have acquired the meaning of the word ‘apple’. The child must learn to re-present to herself the designated compound of experiences whenever the word is uttered, even when none of the elements of that compound are actually present in her experiential field. That is to say, the child must acquire the ability to imagine or visualize, for instance, what she has associated with the word ‘apple’ whenever she hears the sound pattern of that word.4

This analysis, detailed though it may seem, is still nothing but a gross summary of certain indispensable steps in a long procedure of interactions. In the present context, however, it should suffice to justify the conclusion that the compound of experiential elements that constitutes the concept an individual has associated with a word cannot be anything but a compound of abstractions from that individual’s own experience. For each one of us, then, the meaning of the word ‘apple’ is an abstraction which he or she has made individually from whatever apple-experiences he or she has had in the past. That is to say, it is subjective in origin and resides in a subject’s head, not in the word which, because of an association, has the power to call up, in each of us, our own subjective representation.

If you grant this inherent subjectivity of concepts and, therefore, of meaning, you are immediately up against a serious problem. If the meanings of words are, indeed, our own subjective construction, how can we possibly communicate? How could anyone be confident that the representations called up in the mind of the listener are at all like the representations the speaker had in mind when he or she uttered the particular words? This question goes to the very heart of the problem of communication. Unfortunately, the general conception of communication was derived from and shaped by the notion of words as containers of meaning. If that notion is inadequate, so must be the general conception of communication.

The trouble stems from the mistaken assumption that, in order to communicate, the representations associated with the words that are used must be the same for all communicators. For communication to be considered satisfactory and to lead to what we call ‘understanding’, it is quite sufficient that the communicators’ representations be compatible in the sense that they do not manifestly clash with the situational context or the speaker’s expectations.

A simple example may help to make this clear. Let us assume that, for the first time, Jimmy hears the word ‘mermaid’. He asks what it means and is told that a mermaid is a creature with a woman’s head and torso and the tail of a fish. Jimmy need not have met such a creature in actual experience to imagine her. He can construct a representation out of familiar elements, provided he is somewhat familiar with and has established associations to ‘woman’, ‘fish’, and the other words used in the explanation. However, if Jimmy is not told that in mermaids the fish’s tail replaces the woman’s legs, he may construct a composite that is a fish-tailed biped and, therefore, rather unlike the intended creature of the seas.
Jimmy might then read stories about mermaids and take part in conversations about them for quite some time without having to adjust his image. In fact, his deviant notion of a mermaid’s physique could be corrected only if he got into a situation where the image of a creature with legs as well as a fish’s tail comes into explicit conflict with a picture or with what speakers of the language say about mermaids. That is, Jimmy would modify the concept that is subjective meaning of the word only if some context forced him to do so.

How, you may ask, can a context force one to modify one’s concepts? The question must be answered not only in a theory of communication but also in a theory of knowledge. The answer I am proposing is essentially the same in both.

The basic assumption is one that is familiar to you. Organisms live in a world of constraints. In order to survive, they must be ‘adapted’ or, as I prefer to say, ‘viable’. This means that they must be able to manage their living within the constraints of the world in which they live. This is a commonplace in the context of biology and evolution. In my view, the principle applies also to cognition—with one important difference. On the biological level, we are concerned with species, that is, with collections of organisms which, individually, cannot modify their biological make-up. But since they are not all the same, the species ‘adapts’ simply because all those individuals that are not viable are eliminated and do not reproduce. On the cognitive level, we are concerned with individuals and specifically with their ‘knowledge’ which, fortunately, is not immutable and only rarely fatal. The cognitive organism tries to make sense of experience in order better to avoid clashing with the world’s constraints. It can actively modify ways and means to achieve greater viability.

‘To make sense’ is the same activity and involves the same presuppositions whether the stuff we want to make sense of is experience in general or the particular kind of experience we call communication. The procedure is the same but the motivation, the reason why we want to make sense, may be different.

Let me begin with ordinary experience. No matter how one characterizes cognizing organisms, one of their salient features is that they are capable of learning. Basically, to have ‘learned’ means to have drawn conclusions from experience and to act accordingly. To act accordingly, of course, implies that there are certain experiences which one would like to repeat rather than others which one would like to avoid. The expectation that any such control over experience can be gained must be founded on the assumptions that: (1) some regularities can be detected in the experiential sequence; and (2) future experience will, at least to some extent, conform to these regularities. These assumptions, as David Hume showed, are prerequisites for the inductive process and the knowledge that results from it.

In order to find regularities, we must segment our experience into separate pieces so that, after certain operations of recall and comparison, we may say of some of them that they recur. The segmenting and recalling, the assessing of similarities, and the decisions as to what is to be considered different, are all our doing. Yet, whenever some particular result of these activities turns out to be useful (in generating desirable or avoiding undesirable experiences), we quickly forget
that we could have segmented, considered, and assessed otherwise. When a scheme has worked several times, we come to believe, as Piaget has remarked, that it could not be otherwise and that we have discovered something about the real world. Actually we have merely found one viable way of organizing our experience. ‘To make sense’ of a given collection of experiences, then, means to have organized them in a way that permits us to make more or less reliable predictions. In fact, it is almost universally the case that we interpret experience either in view of expectations or with a view to making predictions about experiences that are to come.

In contrast, ‘to make sense’ of a piece of language does not usually involve the prediction of future non-linguistic experience. However, it does involve the forming of expectations concerning the remainder of the piece that we have not yet heard or read. These expectations concern words and concepts, not actions or other experiential events. The piece of language may, of course, be intended to express a prediction, for example, ‘tomorrow it will rain’, but the way in which that prediction is derived from the piece of language differs from the way in which it might be derived from, say, the observation of particular clouds in the sky. The difference comes out clearly when it is pointed out that, in order to make sense of the utterance ‘tomorrow it will rain’ it is quite irrelevant whether or not there is any belief in the likelihood of rain. To ‘understand’ the utterance it is sufficient that we come up with a conceptual structure which, given our past experience with words and the way they are combined, fits the piece of language in hand. The fact that, when tomorrow comes, it doesn’t rain, in no way invalidates the interpretation of the utterance. On the other hand, if the prediction made from an observation of the sky is not confirmed by actual rain, we have to conclude that there was something wrong with our interpretation of the clouds.

In spite of this difference between the interpretation of experience and the interpretation of language, the two have one important feature in common. Both rely on the use of conceptual material which the interpreter must already have. ‘Making sense’, in both cases, means finding a way of fitting available conceptual elements into a pattern that is circumscribed by specific constraints. In the one case, the constraints are inherent in the way in which we have come to segment and organize experience; in the other, the constraints are inherent in the way in which we have learned to use language. In neither case is it a matter of matching an original. If our interpretation of experience allows us to achieve our purpose, we are quite satisfied that we ‘know’; and if our interpretation of a communication is not countermanded by anything the communicator says or does, we are quite satisfied that we have ‘understood’.

The process of understanding in the context of communication is analogous to the process of coming to know in the context of experience. In both cases, it is a matter of building up, out of available elements, conceptual structures that fit into such space as is left unencumbered by constraints. Though this is, of course, a spatial metaphor, it illuminates the essential character of the notion of viability and it brings out another aspect that differentiates that notion from the traditional one of ‘truth’; having constructed a viable path of action, a viable solution to an