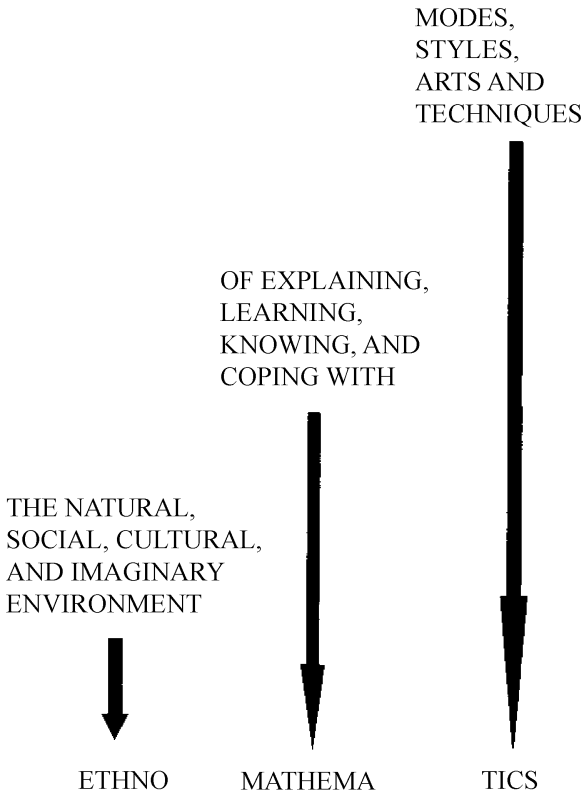
Ethnomathematics

The adventure of the human species is identified with the acquisition of styles of behaviors and of knowledge to survive and transcend in the distinct environments it occupies, that is, in the acquisition of



ETHNOMATHEMATICS

Ethnomathematics
Link between Traditions and Modernity

By

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PREFACE

Ubiratan D'Ambrosio is without a doubt the most well-known mathematics educator in Brazil, and the most well-known Brazilian mathematics educator abroad. In fact, the same could be said substituting mathematics historian, or holistic philosopher, for mathematics educator. But the key word associated with this man is ethnomathematics. Over the last thirty years, D'Ambrosio has developed this idea into a concept and a program which has inspired many researchers and educators throughout the world to emphasize listening in the classroom, and to respect social and cultural diversity. Many have come to believe, as he does, that mathematics is not solely academic mathematics, and that the "s" at the end of mathematic***s*** implies plural, meaning that mathematics can therefore have different expressions. In this book, he illustrates, in his story-telling manner, how different forms of mathematics have developed as a result of the interaction of humans with the environment. He and his followers have argued with strong support from the all the different fields they draw from – anthropology, education, history, psychology, biology, philosophy - that academic mathematics is nothing more than one expression of an incredible variety of (ethno)mathematics developed throughout different moments in history in different parts of the world.

Ubi, as many know him, has lectured in many different countries, and contributed to mathematics education in different parts of the world. He has played key roles in numerous meetings of the International Congress on Mathematics Education (ICME), which take place every four years. This conference is organized by ICMI (International Commission of Mathematics Instruction), a commission in which he has also played a key role. He has helped to broaden the notion of mathematics education by bringing the concerns and needs of developing countries to international forums. He has also played an important role in the initiation of the scientific phase of mathematics education by helping to establish graduate programs in Brazil. For example, he helped establish the first

Masters program in mathematics education at UNESP, Rio Claro, São Paulo, twenty-two years ago, as well as the Doctoral program in the same institution nine years later.

D'Ambrosio was recently presented with the second Felix Klein Medal of the International Commission of Mathematics Instruction. This very important award of the mathematics education community recognizes the importance that D'Ambrosio has had for mathematics education throughout the world. In 2001, D'Ambrosio was also awarded the Kenneth O. May Medal of the International Commission of History of Mathematics/ICHM. Both, ICMI and the ICHM, are specialized commissions of the International Mathematical Union.

Although D'Ambrosio has numerous publications in various languages, this is his first book published in the English language. It is our hope that it will serve to fill a gap for mathematics educators throughout the world who have wanted to read more of his writings, but do not read Portuguese. This book was first published in Brazil in 2001 as the first in a collection of books, which now has 12 volumes, called "Trends in Mathematics Education" of which I am the coordinator. Since the book was published, it has generated, like all of D'Ambrosio's writings, considerable debate, including criticism, in the very dialogical fashion he praises so much.

With this new concept of book developed by Sense Publishers, access to his ideas will be incommensurable. Readers will be able to understand that when he mentions "Program Ethnomathematics", it is not a problem in the translation. He seeks to emphasize that a program is something more stable, developed by a collective of researchers in the same broad sense that he understands investigation in mathematics education. The study of ethnomathematics has become established; in contrast, the notion of ethnomathematics presented in this book is dynamic and does not fit in a "definition; it is in constant construction and reconstruction. That is why he explores different dimensions of ethnomathematics throughout the whole book. He presents the reader with many examples, and may entice some readers into learning Portuguese, as many of the references used for this purpose are available only in that language.

I believe the reader will find great inspiration in this book for reflection and new research. I have been Ubi's student since 1984, although I stopped studying with him formally in 1987 when he, as a member of my Masters committee, helped me in the defense of the first thesis using the notion of ethnomathematics. I continue to learn

with his new ideas, but mostly with his young spirit! I am sure that the reader will share with me this experience of being his eternal student.

Marcelo C. Borba
Rio Claro, São Paulo, Brazil
April, 2006

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Although Anne Kepple is responsible for the translation, we would like to thank Arthur Powell, of Rutgers University, USA, for his helpful comments on parts of the translation, as well as the author, Ubiratan D'Ambrosio, for reviewing the entire English translation and contributing in a valuable way to the challenging task of translating his ideas into English.

INTRODUCTION

In this book, I seek to provide a general overview of ethnomathematics, focusing on the theoretical aspects.

Ethnomathematics is today considered a sub-field of the History of Mathematics and Mathematics Education, with a very natural relation to Anthropology and Cognitive Sciences. The political dimension of ethnomathematics is evident.

Ethnomathematics is the mathematics practiced by cultural groups, such as urban and rural communities, groups of workers, professional classes, children in a given age group, indigenous societies, and so many other groups that are identified by the objectives and traditions common to these groups.

In addition to this anthropological character, ethnomathematics has an undeniable political focus. Ethnomathematics is imbedded in ethics, focused on the recovery of the cultural dignity of the human being.

The dignity of the individual is violated by social exclusion, which often occurs as a result of failing to pass the discriminatory barriers established by the dominant society, including, and principally, in the school system; but also by making costumes of the traditional garb of marginalized peoples; folklore of their myths and religions; crimes of their medical practices; and for making of their traditional practices and their mathematics, mere curiosity, when not the target of derision.

In subordinating the disciplines, and scientific knowledge itself, to the greater objective of prioritizing the human being and his dignity as a cultural entity, ethnomathematics, and ethnosciences in general, and multicultural education, have been the object of criticism; by some, due to a lack of understanding, and by others, as a perverse protectionism. For these, the great goal is to maintain the status quo, covering up with the deceptive discourse of more-of-the-same [sameness] with quality.

In a way, this book continues to build on the ideas expressed in my book *Etnomatemática. Arte ou técnica de explicar e conhecer*, Editora Ática, São Paulo, 1990. Many of my more recent works in

the field can be found on my Web site http://sites.uol.com.br/vello/ubi.htm.

Studies in ethnomathematics have been intensifying for nearly fifteen years, when the *International Study Group of Ethnomathematics/ISGEm* was founded. With broad international participation, the ISGEm has encouraged, recognized, and publicized research in ethnomathematics. The *ISGEm Newsletter* has been published bi-annually, in English and Spanish, since August, 1995, under the responsibility of Patrick J. Scott and, since 2000, by Daniel Ness and Daniel Orey. The collection of the first thirteen years of publication, 26 issues, reunited as a compendium, constitutes a broader vision of what is available to us with respect to this new field of research. There we find reviews of studies and books, research reports, news of events, methodological suggestions, ultimately, everything necessary to be integrated into the field. The newsletters, in English and Spanish, and other information are available at http://www.rpi.edu/~eglash/isgem.htm.

Various Ethnomathematics events have taken place. In addition to the regular sessions held at the *International Congress of History of Science*, which meets every four years, the *International Congress of Mathematics Education*, which also meets every fours years, and the annual meetings of the *National Council of Teachers of Mathematics*, in the United States, the following conferences have also been held: the First International Ethnomathematics Congress in Granada, Spain, in 1998, and the second in Ouro Preto, Brazil, in 2002; the First Bolivian Congress in Ethnomathematics in Santa Cruz, Bolivia, in 1999; and the First Brazilian Congress in Ethnomathematics in São Paulo, in 2000. The proceedings of these three congresses have already been published.

Numerous theses and dissertations have been defended in universities in various countries, including Brazil, with ethnomathematics as the central theme. Also, the prestigious journal, *The Chronicle of Higher Education*, opened a debate on ethnomathematics on its site: http://chronicle.com/colloquy/2000/ethnomath/ethnomath.htm

All this justifies considering ethnomathematics as a new field of research on the international academic scene. We are not dealing with a passing fad.

I will not be presenting a “state of the art” of ethnomathematics. In addition to the *ISGEm Newsletter* mentioned above, I recommend a collection of writings that shows the most relevant work being

done in ethnomathematics throughout the world: *Ethnomathematics. Challenging Eurocentrism in Mathematics Education*, eds. Arthur B. Powell and Marilyn Frankenstein, SUNY Press, Albany, 1997. In this book, Paulus Gerdes wrote a *Survey of Current Work in Ethnomathematics*, which presents a state of the art up to 1997.

Equally relevant, with a focus on history, is the book *Mathematics Across Cultures. The History of Non-Western Mathematics*, ed. Helaine Selin, Kluwer Academic Publishers, Dordrecht, 2000.

There are various publications in Portuguese, including theses and dissertations, about ethnomathematics. Some are listed in the Appendix, and some are referred to in the footnotes to the chapters.

CHAPTER 1

WHY ETHNOMATHEMATICS?

ANTECEDENTS

The voyages of the great explorers synthesized non-academic knowledge in 15th Century Europe. Although it is recognized that Portuguese universities participated in the enterprise of discovery, these discoveries surprised the Renaissance thinkers in universities and academies of other European countries to a certain extent. Mathematical knowledge of the time, which was fundamental to the discoveries, cannot be identified as a body of knowledge. It is found in various directions, in societal groups with distinct objectives.[1]

Although the first great voyages and the feat of circumnavigating the globe pertained to Spain and Portugal (Christopher Columbus, 1492; Vasco da Gama, 1498; Pedro Álvares Cabral, 1500; and Fernando de Magalhães, 1520), soon other European nations recognized the economic and political possibilities of expansion, and a new vision of the world was incorporated in the academic environment in Europe, contributing decisively to modern science.

There was surprise and curiosity throughout Europe regarding the new lands and new peoples. The discoveries, above all, of the American continent, the New World, captured the imagination of the Europeans. The Old World, Eurasia, and Africa, were familiar, as the cultural and economic exchanges recognized by historians of Antiquity date back thousands of years. Thus, these peoples and lands provoked less controversy. The new was to be found in the New World.

Spanish and Portuguese chroniclers are responsible for important literature describing the nature, phenomena, and peoples found there. Reports of other ways of thinking encountered in the lands visited are vast, always emphasizing the exotic, the curious. Particularly interesting is how the other, the new man, is viewed in the literature. One example is Shakespeare's *The Tempest*.[2]

However, recognition of other ways of thinking as systems of knowledge is belated in Europe. At the peak of colonialism, there is a great interest on the part of the European nations to come to know the peoples and lands of the planet. Great scientific expeditions emerge. In the 18th and 19th Centuries, controversy unfolds

regarding the "inferiority" of the inhabitants, the fauna and flora, and the geology itself, of the New World.[3]

Of the great scientific expeditions, the one that had the largest impact may have been that of Alexander Humboldt (1768−1859), who, already at an advanced age, synthesized his vision of a harmonious universe in his book *Cosmos*. Humboldt is explicit in his adherence to Eurocentric Rationalism:

> "It is to the inhabitants of a small section of the temperate zone that the rest of mankind owe the earliest revelation of an intimate and rational acquaintance with the forces governing the physical world. Moreover, it is from the same zone (which is apparently more favorable to *the progress of reason, the softening of manners, and the security of public liberty*) that the germs of civilization have been carried to the regions of the tropics (italics mine)." [4]

The phrase emphasized above reveals the acceptance of the "incivility" found in the New World as being intrinsic to the New World, thus justifying a civilizing mission of the immigrant. We shall not forget that *Cosmos* was a best-seller, widely translated throughout Europe. The immigrant arriving with a mission to civilize could rarely recognize the local culture, a mixture of the cultures of the first colonizers with the cultures of the indigenous people and the Africans who were brought as slaves. It is enough to observe that the most widely spoken language in Brazil when the Royal family arrived was a variation of Tupi. The opinion that there lacked capacity to organize a political system has much to do with the political framework that was implanted in the Americas following independence. The essential difference of the independence of the United States compared to other countries of the New World is a fundamental question pointed out by the historian Herbert Aptheker when he said that the American revolution was, in fact, an English revolution that took place on the other side of the Atlantic. The characteristics of the formation of American nations following their independence varied greatly.

Returning to Humboldt, he does not fail to recognize that there is something fundamental that differentiates the knowledge and behavior of the rest of the other peoples of the planet from those that have their origin in the Mediterranean civilizations. In *Cosmos* one reads:

> "We find even among the most savage nations (as my own travels enable me to attest) a certain vague, terror-stricken sense of the all-powerful unity of natural forces, and of the existence of an invisible, spiritual essence manifested in these forces, whether in unfolding the flower and maturing the fruit of the nutrient tree, in upheaving the soil of the forest, or in rending the clouds with the might of the storms. We may here trace the revelation of a bond of union, linking together the visible world and that higher spiritual world which escapes the grasp of the senses. The two become unconsciously blended together, developing in the mind of man, as a simple product of ideal conception, and independently of the aid of observation, the first germ of a Philosophy of Nature." [5]

Soon after the end of World War I, a German philosopher, Oswald Spengler (1880-1936) proposed a philosophy of history that sought to understand the West through a new lens, viewing culture as an organic whole. The book *The Decadence of the West. Form and Reality,* published in 1918, was soon followed by a second volume, *The Decadence of the West. Perspectives of the Universal History*, published in 1922. The books were taken out of circulation in 1933. This book, which has an encyclopedic character, opened up new possibilities for understanding the nature of mathematical thought. Spengler wrote:

> From this follows a decisive circumstance which, until now, has escaped mathematicians themselves. If mathematics were a mere science, like astronomy or mineralogy, it would be possible to define its object. There is, however, no single mathematics; there are many mathematics. What we call "the" history of mathematics − a supposed progressive approximation to a single, immutable ideal − becomes, in reality, once the deceiving image of the surface of history is dismissed, a plurality of independent processes, complete in themselves; a sequence of births of worlds of forms, new and distinct, that are incorporated, transformed, abolished; a purely organic florescence, of fixed duration, followed by phases of maturity, of decay, of death.[6]

Spengler seeks to understand mathematics as a living cultural manifestation, going so far as to say that the Gothic cathedrals and Doric temples are petrified mathematics. Spengler declares himself to be an admirer of the thinking of Goethe, who was criticized by Humboldt, and sees mathematics as totally integrated with the other manifestations of a culture.[7]

Although he refers exclusively to the West, Spengler's ideas serve to encourage the examination of the mathematics of other cultures.

The 20th Century sees the emergence of anthropology and much attention given to understanding the ways of thinking of other cultures. However, perhaps the first explicit recognition of other systems of rationalization and their pedagogical implications is owed to the noted Japanese algebrist, Yasuo Akizuki, in 1960:

> I can, therefore, imagine that other modes of thinking can also exist, even in mathematics. Thus, I think that we should not limit ourselves to directly applying the methods that are currently considered as the best in Europe and America, but should study the mathematical instruction appropriate for Asia.[8]

The recognition, albeit belated, of other ways of thinking, including mathematical, encourages broader reflections about the nature of mathematical thought, from the cognitive, historical, social and pedagogical points of view. This is the objective of the Ethnomathematics Program.

THE PROGRAM ETHNOMATHEMATICS

The great motivator for the research program known as Ethnomathematics is to seek to understand mathematical knowing/doing throughout the history of humanity, in the contexts of different interest groups, communities, peoples and nations. This denomination will be justified throughout this book.

Why do I talk about Ethnomathematics as a research program and, at the same time, often use the term Program Ethnomathematics?

The principle reason results from my concern regarding attempts to propose an epistemology, and as such, an explanation for Ethnomathematics. Upon insisting on the name Program Ethnomathematics, I seek to make evident that the intention is not to propose another epistemology, but rather to understand the adventure

of the human species in the search for knowledge and the adoption of behaviors.

The critics of the epistemological proposals that polarized the philosophy of science in the 1970's around Popper and Kuhn, and that placed Lakatos and Feyerabend in oddly opposed camps, had an influence on my interest in ethnomathematics. I see the denomination Program Ethnomathematics as being at the same time more in line with the posture of permanent seeking, proposed by transdisciplinarity, and more immune to the attacks from both sides that are sparring in the so-called "science wars".[9]

Research in ethnomathematics should be done with much rigor, but the subordination of this rigor to a language and a standard methodology, even with an interdisciplinary character, can be deleterious to the Program Ethnomathematics.[10] In recognizing that it is not possible to arrive at a final theory of the ways of knowing/doing mathematics of a culture, I want to emphasize the dynamic character of this research program. I highlight the fact that we need to always be open to new foci, new methodologies, and new views of what science is and how it evolves, which results in a dynamic historiography.[11]

Every living person develops knowledge and has behavior that reflects this knowledge, which in turn becomes modified as a function of the results of the behavior. For every individual, her behavior and knowledge are in permanent transformation, and relate to each other in a truly symbiotic way, in total inter-dependence.

THE NOTION OF CULTURE

The drive for survival, of the individual and of the species, which characterizes life, manifests itself when the individual turns to nature for survival and seeks and finds the other, of the same species, albeit biologically different (male/female) to continue the species.

The human species also obeys this instinct. Individuals seek and find others, exchange knowledge and behaviors, and common interests, which are communicated between them, keeping them in association and in societies, organized at various levels: groups of common interests, families, tribes, communities, nations.

The everyday life of groups, families, tribes, communities, associations, professions, and nations takes place in different regions of the planet, in different ways and at different paces, as the result of

certain priorities, among many factors, due to environmental conditions, models of urbanization and production, systems of communication, and power structures.

Upon recognizing that the individuals of a nation, community, or group share their knowledge, such as language, systems of explanation, myths and spiritual gatherings, customs and culinary habits, and that their behaviors are made compatible with and subordinated to value systems agreed to by the group, we say that these individuals pertain to a culture. In sharing knowledge and making behavior compatible, the characteristics of a culture are synthesized. Thus we speak of the culture of the family, the tribe, the community, the association, the profession, the nation.

A dynamic of interaction that is always present in the encounter of individuals makes it impossible to speak with precision about cultures, final or extant. Cultures are in incessant transformation, obeying what we could call a cultural dynamic.[12]

The distinct ways of doing (practice) and knowing (theory) that characterize a culture are part of the shared knowledge and the behavior that has become compatible. Like behavior and knowledge, the ways of knowing and doing are in permanent interaction. The dichotomies between knowing and doing are false, as are those between theory and practice.

NOURISHMENT, SPACE, AND TIME

The need to eat, in competition with other species, is the great stimulus for the development of instruments to help obtain food. Thus, there is evidence of instruments made of carved stone that, close to two million years ago, were used to carve meat, thus improving the quality and quantity of food available. Of course the stone carved with this objective must have the appropriate dimensions to accomplish this end. The evaluation of the appropriate dimensions for carved stone may be the first mathematical manifestation of the species. Fire, which became widely used some 500,000 years ago, even lends characteristics of social organization to food provisioning.[13]

From the use of dead animal carcasses, humans began killing live prey. The invention of the lance brought greater security to man for killing prey, which were generally larger and stronger than he was. Wooden lances, of approximately 2.5 meters, appeared around

250,000 years ago. Man's use of the lance, his muscular coordination, perception of target, and recognition of vulnerable parts of the prey, demonstrate the development of a great capacity for observation and analysis.

The circumstantial and occasional slaughter of prey obviously had an irregular character in the social organization. However, upon creating the possibility of slaughtering herds, the organization of hunting groups became necessary, with a hierarchical structure and leadership, distribution of functions, and organization of space. The social life thus became much more complex. The learning of habits and behaviors by the species, not only by individuals, shows the development of the capacity to classify objects (individuals) according to specific qualities.

This was a decisive step, recognized as occurring around 40,000 years ago, in the evolution of the human species, giving origin to the organization of the first societies. The cooperation between groups of relatively numerous individuals, centered on myths and symbolic representations, was probably responsible for the emergence of song [**time**] and dance [**space**], which led groups of individuals to form distinct families to be together, their symbolic universe situated in time and space. According to William H. McNeill, song and dance were the first great innovation distinguishing the evolutionary course of humans from their closest relatives, the chimpanzees.[14] Dance and song are intimately associated with mathematical representations of time and space. From the meeting of these larger groups, it is probable that language evolved, such as articulated speech and grammar.

All these inventions presaged agriculture, which developed around 10,000 years ago, and was the most important conceptual transition in the history of humanity. Agriculture made possible patterns of subsistence that could not possibly be achieved by groups of hunters and gatherers. The human species encountered, thanks to agriculture, its best manner of food provisioning.[15]

The emergence of agriculture represented, especially among civilizations around the Mediterranean, a conceptual transition from a matriarchal to a patriarchal vision of the world. Up until the invention of agriculture, the great divinities were feminine. It was with the emergence of agriculture that a god identified as masculine manifested itself.[16]

The populations grew and the need emerged for intellectual instruments for planning planting, harvest, and storage, and

consequently, organization of land occupation, organized production, and work, providing a basis for structures of power and economy that are still prevalent today. Myths and rituals linked to seasonal phenomena affecting agriculture emerged. It became important to know where [**space**] and when [**time**] to plant, harvest, and store.

Geometry [*geo*=land, *metry*=measure] is the result of the pharaohs' practice that made it possible to feed the population in years of low productivity, distribute productive lands along the banks of the Nile River, and measure them following floods to collect the part destined for storage, a form of taxation.[17]

Calendars synthesize the knowledge and behaviors necessary for the success of the stages of planting, harvest, and storage, and are obviously associated with the myths and rituals directed at the entities responsible for this success, which guaranteed the survival of the community. Therefore, calendars were local.

Although the internationally recognized calendar was the one proclaimed by Pope Gregorio XIII, in effect since October 15, 1582, there are around forty calendars currently in use today. The construction of calendars, i.e., the counting and recording of time, is an excellent example of ethnomathematics.[18]

Many may find it strange that I give so much emphasis to understanding food provisioning and issues of agriculture. Without a doubt, food provisioning, nourishment for survival, was always the main necessity before all others for all living things. With the emergence of agriculture, the first organized societies begin to be identified. Geo-metry and calendars are examples of an ethnomathematics associated with a system of production, in response to the principle need of organized societies to feed their people.

Knowledge and behaviors are shared and made compatible, making the continuation of these societies possible. This knowledge and these behaviors are recorded, orally or graphically, and disseminated and passed from generation to generation. Thus is born the history of groups, families, tribes, communities, and nations.

This has great importance in education. A mathematics project centered on the construction of home gardens, developed by José Carlos Borsato, is among the first ethnomathematics projects oriented toward pedagogical practice. At that time, the term ethnomathematics was not in use.[19]

More recently, the work of Gelsa Knijnik[20] and Alexandrina Monteiro,[21] among many others, focuses on ethnomathematics

developed and practiced by families involved in organized actions to occupy unused farmland. These actions are, in Brazil, organized as the *Movimento dos Sem Terra/MST*.

MATHEMATICS IN EVERYDAY LIFE

Among the different ways of doing and knowing, some privilege comparing, classifying, quantifying, measuring, explaining, generalizing, inferring, and, in some way, evaluating. We are then talking of a knowing/doing mathematics that seeks explanations and ways of dealing with the immediate and remote environment. Obviously, this knowing/doing mathematics is contextualized and responds to natural and social factors.

Everyday life is impregnated in the knowledge and practices of a culture. At all times, individuals are comparing, classifying, quantifying, measuring, explaining, generalizing, inferring, and, in some way, evaluating, using material and intellectual instruments that belong to their culture.

There are innumerable studies about the ethnomathematics present in everyday life. It is an ethnomathematics that is not learned in school, but rather in the family environment, the environment of toys and work, received from friends and colleagues. How does this learning take place? Maria Luisa Oliveras, working with artisans in Granada, Spain, identified what she called ethno-didactics.[22]

We recognize the practical mathematics of market vendors. The pioneering research of Terezinha Nunes, David Carraher, and Ana Lúcia Schliemann recognized how children helping their parents in open-air markets in Recife, Brazil, acquire very sophisticated arithmetic practices to deal with money, make change, and offer discounts while still making a profit.[23]

The use of day-to-day shopping to teach mathematics reveals practices learned outside the school environment, a true ethnomathematics of commerce. An important component of ethnomathematics is making a critical view of reality possible using instruments of a mathematical nature. Comparative analysis of prices, bills, budgets, provides excellent pedagogical material. The work of Marilyn Frankenstein in the U.S.A. is pioneering in proposing critical mathematics in the schools.[24] A similar proposal,

using products found in supermarkets as a reference, was developed in Italy by Cinzia Bonotto.[25]

Seeking to perceive the influence that the parents' profession has on their children's achievement in school, Adriana M. Marafon identified mathematical practices of mechanics who repair and change tires.[26]

Groups of professionals practice their own ethnomathematics. Observing numerous surgical operations, Tod L. Shockey identified, in his doctoral dissertation, mathematical practices of heart surgeons, focusing on criteria for making decisions about time and risk, and topological notions in the manipulation of the suture knots.[27] Maria de Carmo Villa researched the manner in which fruit juice vendors decide, using a probability model, the quantity of each fruit juice that they should have available in their juice stand to satisfactorily meet the demands of their customers.[28] N.M. Acioly and Sergio R. Nobre identified the mathematics practiced by *bicheiros*, illegal betting ticket vendors, to practice an attractive and compensatory scheme of bets.[29] The mathematics of the *jogo de bicho*, a widely-practiced illegal betting game, had already attracted the interest of Malba Tahan.[30] Marcelo de Carvalho Borba analyzed the way children living in a poor neighborhood organized to build a soccer field, obeying the official dimensions in scale.[31]

The recognition of mathematical practices in everyday life in Africa has been the object of important studies.[32] A very interesting example is the use of percussion instruments, an integral part of traditions originating in Africa. The rhythm that accompanies the percussion instruments can be studied to aid the understanding of ratios.[33]

The work of Claudia Zaslavsky deserves special mention. Her pioneering book, published in 1973, recognized that many of the mathematical practices found in Africa have characteristics of their own, a true ethnomathematics, although the term was not yet in use at that time.[34]

The interest in the ethnomathematics of African cultures has grown tremendously. The work of Paulus Gerdes and collaborators in Mozambique deserves to be highlighted, with publications in Portuguese and English, studying largely traditional basket weaving, textiles, and games of southern Africa.[35]

In the Americas, ethnomathematics appears strongly in the remaining native cultures. There is a great interest in historical study of the mathematics existing during the arrival of the conquistadors

and practiced during the colonial period[36]; however, descendents of native cultures continue to practice their ethnomathematics.

Conciliating the need to teach the dominant mathematics and, at the same time, give recognition to the ethnomathematics of their traditions, is the great challenge for education for indigenous peoples. This theme was discussed by Samuel López Bello, working together with teachers from the Quechua tradition in Bolivia.[37]

Economic relations and systems of production are important factors in the development and transformation of ethnomathematics as a body of knowledge, as Chateaubriand Nunes Amâncio showed.[38]

The vast bibliography available today makes it impossible, in one short book, to attempt a type of "State of the Art" of research in ethnomathematics. In fact, this is not the intention of this publication. However it is justifiable to give some guidance to those who wish to delve deeper into ethnomathematics, from the point of view of research as well as pedagogy.

A special volume of *Teaching Children Mathematics*, a periodical of the National Council of Mathematics Teachers, focused on *"Mathematics and culture"*. It is a collection of various studies, all focused on the school.

As I sought to demonstrate in this chapter, ethnomathematics is part of everyday life, which is the universe in which the expectations and anxieties of children and adults are situated.

CHAPTER 2

THE VARIOUS DIMENSIONS OF ETHNOMATHEMATICS

THE CONCEPTUAL DIMENSION

Ethnomathematics is a research program about the history and philosophy of mathematics, with obvious implications for teaching.

I shall begin with a reflection on the origin of mathematical ideas. How did mathematics come to be?

Mathematics, like knowledge in general, is a response to the drive for survival and transcendence, which synthesize the existential question of the human species. The species creates theories and practices that resolve the existential question. These theories and practices are based on the elaboration of knowledge, and decisions regarding behavior, based on representations of reality. The representations respond to the perception of space and time. The virtuality of these representations, which is manifested in the elaboration of models, distinguishes the human species from other animal species.

For all living species, the question of survival is resolved by behaviors of immediate response, here and now, elaborated on the real, and falling back on the previous experiences [knowledge] of the individual and the species [incorporated into the genetic code]. Behavior is based on knowledge and, at the same time, produces new knowledge. This symbiosis of behavior and knowledge is what we refer to as instinct, which resolves the question of survival of the individual and the species.

In the human species, the question of survival is accompanied by that of transcendence: the "here and now" is broadened to include the "where" and "when". The human species transcends space and time to go beyond the immediate and the perceivable. The present is prolonged by the past and the future, and the perceivable is broadened to include the remote. The human being acts as a function of his sensorial capacity, which responds to the material [artifacts], and of his imagination, often called creativity, which responds to the abstract [mentifacts].

The material reality is the accumulation of facts and phenomena accumulated from the beginning. What is the beginning, in space and time? That is the greatest question of all religious, philosophical, and scientific systems.

The reality perceived by each individual of the human species is the natural reality, together with the total of artifacts and mentifacts [experiences and ways of thinking] accumulated by him and by the species [culture]. This reality, through genetic and sensorial mechanisms and memory [knowledge] inform each individual. Each individual processes this information, which defines their action, resulting in their behavior and the generation of more knowledge. The accumulation of knowledge shared by the individuals of a group has the consequence of making the behavior of these individuals compatible, and, accumulated, this shared knowledge and "compatibilized" behavior constitute the culture of the group.

THE HISTORICAL DIMENSION

We live at the moment of apogee of modern science, which is a system of knowledge that originated in the Mediterranean basin nearly 3,000 years ago, and which imposed itself on the whole planet. This rapid evolution is a short period in the entire history of humanity, and there is no indication that it will be permanent. What will come afterwards? Without a doubt, as has always happened with other systems of knowledge, modern science itself will develop the intellectual instruments to criticize itself and to incorporate elements of other systems of knowledge.

These intellectual instruments depend strongly on a historical interpretation of the knowledge of the Egyptians, Babylonians, Jews, Greeks, and Romans, from which modern knowledge originated.

Throughout nearly three millennia, transitions between the qualitative and the quantitative can be noted in the analysis of facts and phenomena. What could be called the quantitative reasoning of the Babylonians gave way to the qualitative reasoning characteristic of the Greeks, which prevailed throughout the entire Middle Ages.

Modernity came about with the incorporation of quantitative reasoning, made possible thanks to arithmetic [*tica*=art + *aritmos*=numbers], done with Indo-Arabic algorithms, and later, with the extensions of Simon Stevin [decimals] and John Neper

[logarithms], culminating with computers. In this evolution, quantitative reasoning, which can be considered the essence of modernity, was privileged. More recently, we see an intense search for qualitative reasoning, particularly through artificial intelligence. This trend is in step with the intensification of interest in ethnomathematics, whose qualitative character is strongly predominant.

Another aspect of the evolution of Western thought that should be noted is the subordination of global thought, such as predominated in the southern Mediterranean cultures, by sequential thought, which came to characterize Greek philosophy. This culminated in the thought of René Descartes, resulting in the organization of disciplines, which prevailed over the holistic proposals of Jan Comenius.

We are now living in a time which resembles the intellectual effervescence of the Middle Ages. It is thus justifiable to speak of a new renaissance. Ethnomathematics is one of the manifestations of this new renaissance.

It is important to note that the acceptance and incorporation of other ways of analyzing and explaining facts and phenomena, as in the case of ethnomathematics, always occurs in parallel with other manifestations of culture. This is evident in the two attempts to introduce the Indo-Arabic system in Europe. The first attempt, by Gerbert de Aurillac, who was ordained as Pope Silvester II in 999, was unsuccessful.[1] The second attempt, almost three centuries later, was promoted by the merchant Leonardo Fibonacci, of Pisa, with the publication *Líber Abaci*, in 1202. The new system taught by Sylvester II contributed little for the economic model and technology that prevailed in the 11th Century. However, for the merchant trade that began to develop in the 13th Century, as well as for the advances in experimental science in the late Middle Ages, the arithmetic learned from the Arabs was essential.

This parallel between mathematical ideas and economic models was recognized by Frei Vicente de Salvador when he commented on the arithmetic of the indigenous Brazilians. The historian explained that they counted with their fingers and, if necessary their toes. With this, all of their day-to-day needs [of survival] and for their systems of explanation [of transcendence] were perfectly satisfied. They knew no other systems because their was no reason to.[2] Today, native Brazilians want calculators because they are essential for their commercial relations.

CHAPTER 2

It would not be possible to understand the behavior of today's youth and, thus, evaluate the state of education, without turning to an analysis of the cultural moment in which youth are living. This leads us to an examination of what is happening with the central discipline of the curricula, which is mathematics; not only in the discipline itself, which necessarily leads us to intercultural reflections on the history and philosophy of mathematics, but just as importantly, on how mathematics is situated today in the individual and collective experience of each individual.

THE COGNITIVE DIMENSION

Mathematical ideas, in particular, comparing, classifying, quantifying, measuring, explaining, generalizing, inferring, and to an extent, evaluating, are forms of thinking present in the entire human species. The attention of scientists who study cognition has been increasingly directed to this characteristic of the species.

The emergence of mathematical thinking in individuals, and in the human species as a whole, has been the subject of intense research. Much is already known about the brain, and we know much about cranial matter. It has even been attempted to privilege brain lobes with specific actions! But where is the capacity to prefer one color over another, and the reason why a smell awakens emotions? There is a fundamental difference between the mind and the brain. The new science of cognition has received great contributions from neurologists. [3]

The attention of researchers is focused on studies of the mind, or of consciousness. Many call this field of science the frontier of science. What is thinking? What is consciousness? Studies of the mind or of consciousness, common among neurologists, including neurosurgeons, have attracted the growing attention of mathematicians and theoretical physicists. [4]

In order to understand humans, it is, of course, important to know about those living beings that have some similarity with humans, in particular, the primates. It is enough to note that 98% of the genome of the *rhesus*, is identical to humans. Primates have been the subject of considerable research. The emergence of thinking of a mathematical nature, privileging the quantitative, has been noted in primates. [5]

It is equally important to create automated devices and models that, at least partially, carry out functions similar to those performed by humans. Without a doubt, calculators and computers have proven efficient for tasks involving quantification. But the greatest challenge is qualitative thinking, which includes emotions. This lies within the sphere of robotics and artificial intelligence. One fascinating theme is the study of autonomous "mental" development in robots, as a result of experiences with the natural environment. [6]

But let us return to our species, where the ideas of comparing, classifying, quantifying, measuring, explaining, generalizing, inferring, and to an extent, evaluating, appear as characteristics.

The species *Homo sapiens sapiens* is new, having been identified approximately 40 thousand years ago. The species that preceded it, the Australopithecus, appeared nearly 5 million years ago, close to what is known today as Tanzania, and spread over the planet. In this expansion, the species underwent transformation, influenced by the climate, food habits, and various other factors, and developed techniques and abilities that allowed their survival in the new regions they encountered. When confronted with new situations, we gather experiences from previous situations, adapting them to new circumstances, thus incorporating to the memory new ways of knowing and doing. Thanks to an elaborate system of communication, the ways and modes of dealing with situations are shared, transmitted, and disseminated.

Although knowledge is generated individually, from information received from reality, perhaps one of the characteristics that most distinguishes the human species from other species is the phenomenon of communication, which takes place in the encounter with the other. Through communication, the information received by one individual is enriched by the information received by another. The knowledge generated by the individual, which is a result of processing the totality of information available, is, also through communication, shared, at least partially, with the other. This is extended, obviously, to others and to the group. Thus, the knowledge shared by the group is developed.

The behavior of each individual, associated with her knowledge, is modified by the presence of the other, largely by the knowledge of the consequences for the other. This is reciprocal, and thus, the behavior of one individual is made compatible with the behavior of the other. Obviously, this extends to the others and to the group. Thus, the *compatibilized* behavior of the group is developed.

Culture is the set of shared knowledge and *compatiblized* behavior.

As I already mentioned in the introduction, we have evidence of a species, a type of Australopithecus, that lived about 2.5 million years ago and used instruments made of chiseled stone to clean animal carcasses. It is easy to understand that, when eating a slaughtered animal, the existence of an instrument such as a chiseled stone would make it possible to scrape the bone, and thus, not only make use of every piece of meat, but also extract nutrients from the bone that would not otherwise be available using the teeth alone. The species began to have more food of higher nutritional value. This appears to have been a decisive factor in the refinement of the brain of the species that dominated this technology.

What does this have to do with ethnomathematics?

At the moment this Australopithecus selected and chiseled a piece of stone with the objective of de-boning a piece of meat, his mathematical mind revealed itself. To select the stone, one must evaluate its dimensions; and in order to chisel it as needed and sufficiently to fulfill the objectives for which it is intended, it is necessary to evaluate and compare dimensions. Evaluating and comparing dimensions is one of the most elementary manifestations of mathematical thinking. Thus, a first example of ethomathematics was that practiced by this Australopithecus.

As they evolved and spread throughout the various regions of the planet in small groups, the species that preceded us continued to refine the material and intellectual instruments needed to deal with their environment, and to develop new instruments.

In various regions of the planet, the different languages began to take form, making it possible to organize systems of knowledge. Records began to be made. Especially rich are the records of Eurasia, and a pre-history of the mathematical ideas from which academic mathematics originated. In pre-history and in history, ethnomathematics is identified as a knowledge system.[7]

Man seeks explanations for everything, and very naturally associates these explanations with what he sees but does not understand: the climate, day and night, the stars in the sky. What is occurring, what is perceived and felt at every moment, may be indicators of what is going to occur. This is the mystery. Seeking explanations for the mystery that relates causes and effects is an important step in the evolution of the *homo* species.

Systems of explanation for the first causes are organized [myths of creation]. Death, which is so evident, may not be the end, but rather an encounter with the first causes. What happens after death? An even more practical question occurs: what will happen next, in the following moment? What are the consequences of what I am seeing now? Of what I am doing now? Only whoever is responsible for the first causes [a diety] could know the mystery of what will come to pass.

How to ask the deity what will come to pass? Through the techniques of "consulting" the deity. These techniques are called divination arts. How to influence the deity so that desirable, necessary, and pleasant things happen? Through worship, sacrifice, and magic.

Religions are systems of knowledge that allow us to delve into the past - explaining the first causes, developing a sense of history and organizing traditions - and influence the future. The knowledge of these traditions is shared by the group. Continuing to belong to the group, even after death, depends on behaving, in life, in response to his shared knowledge. This knowledge, which is compatible and shared by the group, is subordinated to parameters that we call values.

THE CHALLENGES OF EVERYDAY LIFE

One of the most important things in our relationship with the environment is obtaining nutrition and protection from the elements. Knowing the environment, we can act in such a way that our ability to nourish and protect ourselves depends less on factors such as the weather.

Once they had mastered techniques of agriculture, raising livestock, and construction, humans were able to remain in the same location, to be born and die in the same place. They perceived the time needed for germination and gestation, the time that passed between planting and harvest. At a certain moment, a configuration of the sky coincided with the sprouting of the little plants. It is a divine message. It was learned how to interpret these messages, which are generally translated into periods characteristic of what are known as seasons of the year.

Insemination was more difficult to perceive, but the time from gestation to birth is more easily recognized. The regularity of the

menstrual cycle, and the relationship between its interruption and gestation, have long been recognized. Recognizing and recording the menstrual cycle, associated with the phases of the moon, appear to have been among the first forms of ethnomathematics.

Agriculture had a large influence on the history of ideas of the peoples of the Mediterranean. The theories that made it possible to know the correct time for planting emerged, subordinated to tradition. Recognizing the seasons and celebrating their arrival, like an appeal and subsequent acknowledgment to the one responsible for the regularity, a deity, marked the first moments of worship and religion. The association of religion with astronomy, with agriculture, and with fertility is obvious.

Mathematics began to be organized as an instrument of analysis of the conditions of the sky and daily necessities. I could go on describing how, here and there, in all corners of the planet and at all times, mathematical ideas were being developed that were important in the creation of systems of knowledge and, consequently, behaviors necessary to deal with the environment, to survive and explain the visible and the invisible.

Culture, which is the set of *compatiblized* behaviors and shared knowledge, includes values. In the same culture, individuals provide the same explanations and use the same material and intellectual instruments in their everyday activities. The set of these instruments is manifested in the manners, modes, abilities, arts, techniques – in the ***tics*** of dealing with the environment, of understanding and explaining facts and phenomena, of teaching and sharing all this, which is the ***mathema*** of the group, of the community, of the ***ethno***. That is, it is their ethnomathematics.

In different environments, the ethnomathematics are, of course, different. The Eskimos at the Arctic Circle, when seeking to feed themselves, cannot think in terms of planting, and therefore, never developed agriculture. They dedicated themselves to fishing. Hence, they needed to know the best time to fish. They must fish a lot, perhaps all day long. But the day [light] lasts six months, and the night [dark], six months. Thus, their distribution of time, and the perception they have of the heavens and the forces that influence their day-to-day lives, is quite different from those whose daily lives are spent in the Mediterranean or near the Equator. Their astronomy and their religion are distinct from those that emerged in the Mediterranean or near the Equator, as well as their ways of dealing with their everyday needs. Their ethnomathematics is different.

One of the main things that appears at the beginning of mathematical thinking are ways of counting time. In the history of mathematics (and now I am speaking of academic mathematics, which has its origins in Greece), the great names are linked to astronomy. Geometry, in its origin and in its very name, is related to measurements of land. As Herodoto tells us, geometry was learned from the Egyptians, for whom it was more than a mere measurement of land; it had everything to do with the system of taxation of productive areas. Behind this development, we see an entire system of production and an economic, social, and political structure that requires measurement of land and, at the same time, arithmetic to deal with the economy and with counting time.

While this system of knowledge was developing over 2,500 years ago in civilizations surrounding the Mediterranean, the indigenous peoples of the Amazon forest were also trying to learn about and cope with their environment, developing systems of production and social systems which, in the same way, required measurements of space and of time. The same was true of the Eskimos, the Andean civilizations, and those of China, India, and sub-Saharan Africa – of the entire planet. They were all developing their ways of knowing.

THE EPISTEMOLOGICAL DIMENSION

Knowing what? System of knowledge for what? Knowledge systems make survival possible, but they also respond to fundamental existential questions, such as: Where did I come from? Where am I going? What is my past and the past of my people? What will the future be, mine and that of my people? How to go beyond the current moment, delve into my questioning and objectives, in the past and in the future? How to transcend the here and now?

Knowledge systems are sets of responses that a group gives to its drives for survival and transcendence, inherent to the human species. How are knowledge and practices related?

CHAPTER 2

The great controversy of the history of science is the relation between the empirical and the theoretical, which can be summarized in three direct questions:

1. How do we move from ad hoc observations and practices to experimentation and method?
2. How do we move from experimentation and method to reflection and abstraction?
3. How do we proceed toward inventions and theories?

This sequence serves as the basis for explaining the evolution of knowledge, i.e., for a theory of knowledge, or epistemology.[8]

My criticism of epistemology is that it focuses on knowledge already established, according to the accepted paradigms of the time and of the moment. But the dynamic of the generation of knowledge, of its intellectual and social development, of its diffusion and, consequently, of the returns of this knowledge to those responsible for its production, constitute an integral cycle, and attempts to study this cycle by isolating its components is inappropriate for non-Western systems of knowledge. This becomes very clear when one seeks to focus on theories for ethnomathematics. As Eglash has said, [Western] mathematics is seen as the culmination of a singular, sequential development of human thought. This perception, which he classifies as mythology, is confused with the predominant epistemologies. [9]

My proposal for an epistemology that is appropriate for understanding the cycle of knowledge in an integrated manner can be synthesized in Figure 1.

The fragmentation of this cycle is absolutely inappropriate for understanding the cycle of knowledge. The historiography associated with the fragmentation of the cycle cannot lead to an integral perception of how humanity evolved. Fragmentation is particularly inappropriate for analyzing the mathematical knowledge of peripheral cultures. [10]

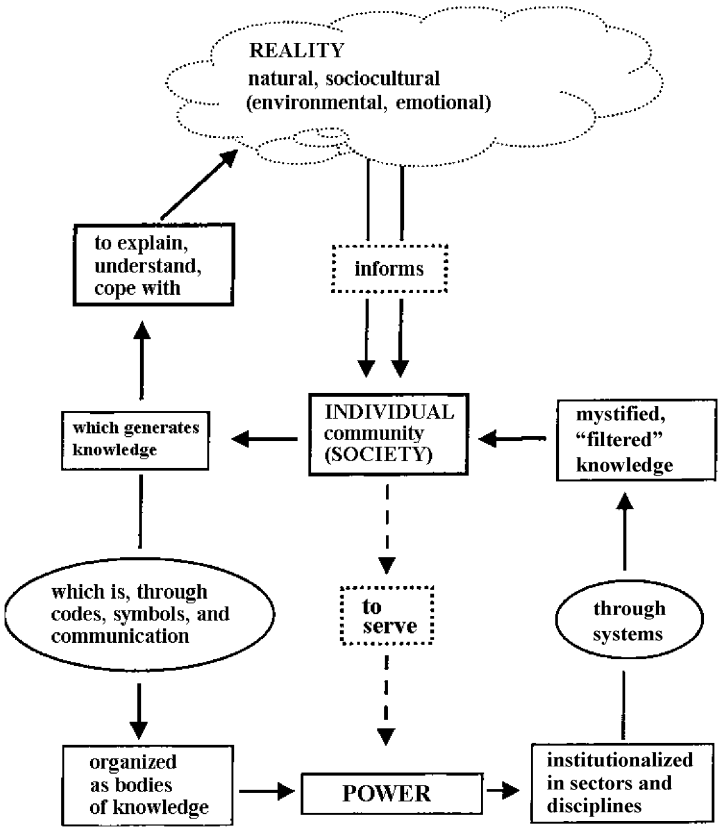

Figure 1.

THE POLITICAL DIMENSION

Nearly 2,500 years ago, a change of power emerged in the region of the Mediterranean. Egyptians and Babylonians, who had based their hegemony on the subordination of their knowledge and behavior to a broad polytheism, were challenged by the great innovation, proposed by the Jews, of a single, abstract god.

The Greeks, and soon thereafter, the Romans, polytheist pagans, expanded the domain of the Mediterranean eastward, conquering thousand-year-old civilizations, such as those of Persia and India, and toward northern Europe, conquering the barbarian peoples. Greece and Rome, which imposed their knowledge systems and social and political organization on the conquered peoples, were

equally challenged by the idea of a single, abstract god, above all, by the emergent idea of Christianity.

With the adoption of Christian monotheism in the 4th Century, Rome imposed not only its politics, science, technology, and philosophy, but primarily, the new religion on a large part of Eurasia above the Tropic of Cancer.

The Roman Empire, imposing its own ways of responding to its drives for survival and transcendence, proved to be efficient in its encounters with other cultures, being successful in its conquests, religious conversion, and consequently, the expansion of its power.

The peak of this success occurred during the transition from the 15th to the 16th Centuries. In about 25 years, navigators from Spain and Portugal circumnavigated the globe. They were soon accompanied by other European nations, and by sea, traveled to the North, South, East and West, in every direction, conquering peoples and carrying with them their explanations and ways of dealing with the environment, modes and styles of production and of power. The process of globalization of the planet had begun.

When speaking of conquest, we are, of course, recognizing the existence of a conqueror and the conquered. The conqueror cannot allow the conquered to manifest themselves. The fundamental strategy in the process of conquest adopted by a [dominant] individual, group, or culture, is to maintain the other [dominated] individual, group, or culture in an inferior position. One very efficient way to keep an individual, group, or culture inferior is to weaken their roots, removing the historical ties and historicity of the dominated. This is the most effective way to carry out a conquest.

Removing the historicity implies removing the language, production, religion, authority, recognition, land and nature, and the systems of explanation in general. For example, practically any indigenous Brazilian can recite the Lord's Prayer and the Ave Maria and believes in God and in Christ, even though this entire system has nothing to do with their traditions. Upon seeing the system of production that sustains them destroyed or modified, the dominated begin to eat -and to like - what the dominator eats.

Thus, the strategies of the dominated for survival and transcendence are eliminated and substituted. In some cases, the conquered individual himself was eliminated, in an evident act of genocide.

Over a period of 300 years, not only was the culture eliminated, but the individuals of this culture, as well, as occurred with the

indigenous peoples of the Atlantic Coast of the Americas and the Caribbean, who were exterminated. In other regions of the planet, many individuals survived. They remained in marginalized and excluded cultural groups, or were co-opted and assimilated into the culture of the dominator.[11] However, a latent culture, often disguised or clandestine, maintained itself during the period of colonization.

The importation of Africans for slavery in the Americas should be given particular emphasis. The New World underwent, and continues to undergo, great transformations in the conjunction of the indigenous, African, and European cultures. The transfer of African cultures to the New World, and their preservation, incorporating and modifying linguistic, religious, artistic, and above all, scientific traditions, has as yet been analyzed very little by historians.[12]

A similar situation takes place in the schools. The school has broadened its reach, taking in youth from the villages, to whom it offers the possibility of social access. But this access occurs as a function of the results, which is one modality of co-optation. Systems for choosing who will merit access are created and justified by convenient theories of behavior and learning. One very important instrument of selection is language. Latin was the standard, later substituted by the cultivated norm of language. Even today, many children feel inhibited to speak because they know they speak incorrectly and, since they are unable to speak correctly, they remain silent. Later, mathematics assumed the role of instrument of selection. And we know that many children are still punished for counting with their fingers!

How to explain what occurs with peoples, communities, and individuals when they encounter the strange, the different, the unfamiliar. Every individual carries with him/her cultural roots that come from their home, from the day they were born. They learn from their parents, friends, neighbors, community. The individual spends many years acquiring these roots. Upon arriving in school, there is normally a process of refinement, transformation, and substitution of these roots, very similar to that which occurs in the process of religious conversion.

The moment of cultural encounter has a very complex dynamic. This encounter occurs between peoples, as occurred in the conquest and colonization, between groups. It also occurs in the encounter between the young man or woman, who have their own cultural roots, and the other culture, the culture of the school, with which the

teacher identifies. The civilizing process, and we could say the same of the school process, is essentially the management of this dynamic.

The school dynamic could also have creative and positive results, which manifest themselves in the creation of the new. But generally, negative and perverse results are noted, which manifest themselves, above all, in the exercise of power and the elimination or exclusion of the dominated.

Conversion depends on the individual forgetting or even rejecting her roots. But an individual without roots is like a tree without roots or a house without a foundation: it is blown over with the first wind. Individuals without solid roots become more fragile, and do not resist harassment. The individual needs a reference, which is situated not on the roots of others, but on their own roots. Without roots, he grasps onto the other when he falls, and embarks on a process of dependence, which is fertile ground for the perverse manifestation of power of one individual over another.

We are witnessing this process in school systems and in society. It is the power of those who know more, who have more, who *can* more. What feeds the power of the dominator? This power can only have continuity if someone depends on him, the dominator grasps onto to him. And who is going to grasp onto him? Without a doubt, those who have no roots.

This was the efficient strategy adopted by the colonizer: eliminate the historicity of the conquered, i.e., eliminate their roots. The process of de-colonization, which celebrates with the adoption of a flag, a hymn, a constitution, is incomplete if the cultural roots of the colonized are not recognized.

Ethnomathematics fits into this reflection about de-colonization and the search for real possibilities of access for the subordinated, the marginalized, and the outcast, or excluded. The most promising strategy for education in societies that are in transition from subordination to autonomy is to restore dignity to their individuals, recognizing and respecting their roots. Recognizing and respecting an individual's roots does not signify rejecting the roots of the other, but rather in a process of synthesis, reinforcing their own roots. This is, according to my thinking, the most important aspect of ethnomathematics.

THE EDUCATIONAL DIMENSION

The ethnomathematical proposal does not signify the rejection of academic mathematics, as suggested by the unfortunate title of an excellent article about ethnomathematics published in the *Chronicle of Higher Education*, "*Good Bye, Pythagoras*", that I mentioned in the Introduction. It is not about ignoring nor rejecting the academic mathematics symbolized by Pythagoras. Due to historical circumstances, like it or not, the peoples who, beginning in the 16th Century, conquered and colonized the entire planet were successful thanks to the knowledge and behavior that was founded in Pythagoras and his companions from the Mediterranean Basin. This is the knowledge and behavior that, incorporated into modernity, guides our daily lives today. This is not about ignoring nor rejecting modern knowledge and behavior, but rather perfecting them, incorporating values of humanity, synthesized into an ethics of respect, solidarity, and cooperation.

Knowing and assimilating the culture of the dominator can become positive as long as the roots of the dominated are strong. In mathematics education, ethnomathematics can strengthen these roots.

From a utilitarian point of view, which cannot be ignored as a very important goal of school, it is a big mistake to think that ethnomathematics can substitute **good academic mathematics**, which is essential for an individual to be an active being in the modern world. In modern society, ethnomathematics will have limited utility, but at the same time, much of academic mathematics is absolutely useless in this society, as well.

When I say **good academic mathematics**, I am excluding that which is uninteresting, obsolete, and useless, which, unfortunately, dominates current programs. It is obvious that we will obtain **good academic mathematics** if we do away with much of what continues to be included in programs with no other justification beyond a damaging conservatism and an unsustainable propeadeutical character. We often hear, “It is necessary to learn **this** to have a basis to be able to learn **that**.” The fact is, the “that” should be dropped, and more still, the “this”.

For example, it is inadmissible today to think about arithmetic and algebra, which privilege quantitative reasoning, without the full use of calculators. Quantitative reasoning made possible the great advances in mathematics, beginning in the late Middle Ages, thanks

to resorting to quantification of the results of experiments which eventually came to dominate mathematics education.[13] Quantitative reasoning was the reason for the existence of calculators and computers. And now, the greatest educational accomplishment of quantitative reasoning, calculus [arithmetic, algebraic, differential, integral] is integrated with calculators and computers.[14]

On the other hand, qualitative reasoning, also known as analytical, which is strongly conceptual and had regained momentum at the beginning of the 17th Century, gained importance in the modern world, giving rise to new fields of mathematics that developed in the second half of the 20th Century, such as statistics, probabilities, programming, modeling, *fuzzies*, and fractals. Currently, one of the most active fields of research, artificial intelligence, aims to incorporate qualitative reasoning into computers.

It may seem contradictory to speak about such sophisticated mathematics as *fuzzies* and fractals when we make the ethnomathematics proposal. But this is precisely the essence of ethnomathematics: incorporating the mathematics of the cultural moment, contextualized, into mathematics education. Fractals are, today, part of the popular imagination and curiosity. They awaken the interest of children, youth, and adults.[15]

Qualitative reasoning is essential to arrive at a new organization of society, as it permits the exercise of criticism and analysis of the world in which we live. It should, without further hesitation, be incorporated into educational systems. This incorporation occurs through the introduction into these programs, at all academic levels, of statistics, probability, programming, modeling, *fuzzies*, fractals, and other new, emerging fields of current science.

Ethnomathematics privileges qualitative reasoning. An ethnomathematical focus is always linked to a larger question, of an environmental nature or of production, and ethnomathematics rarely appears disassociated from other cultural manifestations, such as art and religion. Ethnomathematics fits perfectly into a multicultural and holistic conception of education.

Multiculturalism is becoming the most notable characteristic of education today. With the great mobility of people and families, intercultural relations will become more intense. Intercultural encounters will generate conflicts that can only be resolved based on ethics that result from the individual knowing him/herself and knowing his/her culture, and respecting the culture of the other.

Respect will come from knowing. In any other manner, behavior will reveal arrogance, superiority, and prepotency, which inevitably result in confrontation and violence.

The absolute priority of our mission as educators is to obtain PEACE in future generations. We cannot forget that these generations will live in a multicultural environment, that their relations will be intercultural, and their day-to-day lives will be impregnated with technology. Perhaps humans will co-exist with cloned and transgenic individuals, or even androids. A scenario of fiction, as we see in films such as *Blade Runner* and *Matrix*, can become reality. We do not yet know how to deal with this. It is future generations that will organize the world of the future. Today we do not yet know what to do in a future that reveals itself with facts that are still in the realm of fiction, but that are rapidly becoming reality.

How can we teach them to build their world of peace and happiness? The future will be built by them. What can we offer them so that they can build a future without the evils of the present? The manner in which past generations dealt with the future, anchored in all the knowledge modernity offered, gave us our present; a distressful present, of inequality, injustice, arrogance, exclusion, environmental destruction, inter- and intra-cultural conflicts, wars. This is not the legacy we should leave to our great grandchildren and future generations.

As educators, we can offer the children of today, who constitute the generation that will, in twenty or thirty years, be in decision making positions, a critical vision of the present, and the intellectual and material instruments that we have at our disposal for this critique. We are experiencing a profound transition, with more intensity than in any other period in history, of communication, of economic models and production systems, and of systems of governance and decision making.

Education in this transition cannot focus on the mere transmission of obsolete content, for the most part uninteresting and useless, and inconsequential for the construction of a new society. What we can do for our children is offer them the communicative, analytic, and material instruments for them to live with a capacity for criticism in a multicultural society impregnated with technology. [16]

Mathematics imposed itself as a strong presence in all fields of knowledge, and in all actions of the modern world. Its presence in the future will certainly be intensified, but not in the form practiced

today. It will no doubt be an integral part of communicative, analytic, and material instruments.The dynamic acquisition of mathematics integrated with the knowledge and practices of the future depends on offering students enriching experiences. It is up to the teacher of the future to idealize, organize, and facilitate these experiences. But for this, the teacher must be prepared with another dynamic. As Beatriz D'Ambrosio said, "the future mathematics teacher should learn new mathematical ideas in an alternative way".[17]

I see our great mission as educators to be the preparation of a happy future. And, as mathematics educators, we must be in step with the great mission of the educator. The mathematics educator who fails to perceive that there is much more to his mission as educator than teaching how to add and subtract or solve equations and problems that are absolutely artificial, even though they often appear to refer to real facts, is, in the least, mistaken.

The pedagogical proposal of ethnomathematics is to bring mathematics to life, dealing with real situations in time [now] and space [here]; and, through criticism, to question the here and now. Upon doing so, we plunge into cultural roots and practice cultural dynamics. We are effectively recognizing the importance, in education, of the various cultures and traditions in the formation of a new civilization that is transcultural and transdisciplinary.

As Teresa Vergani said,

> Ethnomathematics knows that an *unitary and plural world* is being generated, and that the undoing of the blockade between cultures begins with tending to the problem of reciprocal 'translatability'.
>
> The first hybrid characteristic of ethnomathematics to take into account is its *engagement in the dialogue between identity (world) and alterity (local), terrain where mathematics and anthropology intersect.*[18]

For all these reasons, I see ethnomathematics as a path to a renovated education, able to prepare future generations to build a happier civilization. To attain this civilization, that I dream of and that I believe can be reached, it is necessary to attain PEACE, in its various dimensions: individual, social, environmental, and military.

The United Nations, through UNESCO, has proclaimed the decade that dawns before us as the Decade for a Culture of Peace and Non-Violence. All educational efforts should be directed toward this priority. Ethnomathematics is a response to this appeal.