The Creative Enterprise of Mathematics Teaching Research
Elements of Methodology and Practice – From Teachers to Teachers
Bronislaw Czarnocha, William Baker, Olen Dias and Vrunda Prabhu (Eds.)

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The Creative Enterprise of Mathematics Teaching-Research presents the results and methodology of work of the teaching-research community of practice of the Bronx (TR Team of the Bronx). It has a twofold aim of impacting both teachers of Mathematics and researchers in Mathematics Education. This volume can be used by teachers of mathematics who want to use research to reflect upon and to improve their teaching craft, as well as by researchers who are interested in uncovering riches of classroom learning/teaching for research investigations. This book represents the results of a collaboration of instructors discussing their own instruction research, analyzed through a conceptual framework obtained via the synthesis of creativity research and educational learning theories, based upon the work of Piaget and Vygotsky. The editors see an urgent need for creative synthesis of research and teaching, an example of which is presented in the book.

Two central themes of the book are the methodology of TR/NYCity model and creativity, more precisely, creativity of the Aha moment formulated by Arthur Koestler (1964) in a very profound but little known theory of bisociation exposed in his work “The Act of Creation”. Incorporation of the theory of bisociation into classroom teaching of mathematics provides the key to enable students who may struggle with mathematics to engage their own creativity, become involved in their learning process and thus reach their full potential of excellence.

Creativity in teaching remedial mathematics is teaching gifted students how to access their own giftedness.
The Creative Enterprise of Mathematics Teaching
Research
The Creative Enterprise of Mathematics Teaching Research

Elements of Methodology and Practice – From Teachers to Teachers

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We dedicate this book to Vrunda Prabhu
One of the founders of the
Teaching-Research Team of the Bronx
In gratitude for leaving behind
The Universal Signpost:

Creativity is the way out!
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FOREWORD

This book is about creativity in students working with mathematics in classrooms and teachers developing mathematics teaching for the benefit of their students. It is written for “teachers of Mathematics and researchers in Mathematics Education” by a team of mathematics teacher-researchers, focusing on the education of “underserved” students in the Bronx, USA. The authors write:

We are addressing ourselves to the teachers of mathematics who want to use research to reflect upon and improve their craft practice, and to researchers who are interested in uncovering riches of classroom teaching for research investigations. And most of all we are interested in those educators who see the urgent need for creative synthesis of research and teaching.

The book is eclectic and wide-ranging, drawing on literature, theory and research in Mathematics Education over several decades. It develops a theoretical model for practice referred to as the TR/NYCity model. The authors present the antecedents of the model, the philosophy of practice on which it is based and examples involving practice with teachers and students. They propose that the model addresses several ‘gaps’: the Achievement Gap between different groups of students (e.g., Pisa, 2012), the Teaching Gap between methods of teaching (e.g., Stigler & Heibert, 2000) and the gap between Research and Practice whereby research is undertaken by and reported to academic researchers, having little relevance or interest for teachers for whom ‘teaching practice’ is their central concern (e.g. Hargreaves, 1996).

The model is underpinned by a number of themes: Arthur Koestler’s theory of bisociation, the main instantiation of which is the “Aha moment”; Laurence Stenhouse’s theory of research as a basis for teaching in which a ‘Stenhouse Act’ is both a teaching act and a research act; and theories of creativity, traceable to Poincaré and Hadamard. In particular the model proposes a synthesis of Koestler and Stenhouse theories by which a teacher seeks to create Aha moments of mathematical understanding through Stenhouse acts. Central to the model is the use of student and teacher reflection as part of the act in which they engage. Fostered by the teacher, student reflection enables students to think beyond the procedural means of getting the solution to a problem, to the rationale for procedures they use and the possibilities of alternative solutions. An aim for student learning is that engagement at this meta-level promotes aha moments of understanding (the cognitive) from which students derive pleasure and motivation (the affective). The teacher meanwhile is reflecting in a similar way, looking critically at her own approaches to fostering students’ creative involvement, and seeking alternative approaches to achieving student understanding.

Creativity in the classroom is centred on problem-solving approaches through which students are introduced to mathematics and engage in dialogue with each
other and the teacher. The associated practice of teaching is fundamentally a research process, in which a teacher engages in design of tasks and classroom activity, and the reflective interaction with students that feeds back to the design process. Several chapters include teaching sequences in which this process is exemplified and critiqued. The authors emphasise that the interactive process, fostering aha moments, has a cognitive/affective duality in which students’ reasoning is challenged in ways that reduce their antipathy for mathematics.

While the TR/NYCity model, focuses centrally on students’ learning through critical reflection in problem solving, its more global significance lies in its teacher-researcher dialectic. The TR-act is a research act designed simultaneously to offer a classroom approach (the teaching act) rooted in dialogic mathematical problem solving and to critique the approach (a research act) in and from practice. Both acts are inquiry-based (involving the asking of critical questions), with a meta-level of inquiry (the TR act). The teaching sequences offered are annotated throughout to point out the stages of this reflective critiquing process.

It is hard to do justice, in a short space, to the complexity of the ideas in this book. I encourage the reader to engage with these ideas, taking a critical position and reflecting at the same time on their own teaching/research practices.

REFERENCE


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Equally important are the words of gratitude in the dedication to Vrunda Prabhu, who sadly passed away prior to the publication of this book. We are grateful for her having introduced us to Koestler’s Theories and the teaching practice exploiting their insights.

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Note of information: Concept maps in the volume are usually in color as are photos of contributors, however the original colors can only be seen in the e-version of the book.
INTRODUCTION

The book Creative Enterprise of Mathematics Teaching-Research submitted for readers’ consideration and enjoyment presents the results and methodology of work of the teaching-research community of practice, TR Team of the Bronx. It is directed towards two audiences, teachers of Mathematics and researchers in Mathematics Education. We are addressing ourselves to the teachers of mathematics who want to use research to reflect upon and improve their craft practice, and to researchers who are interested in uncovering riches of classroom teaching for research investigations. And most of all we are interested in those educators who see the urgent need for creative synthesis of research and teaching. The two central themes of the book are the methodology of TR/NYC model of teaching-research and creativity, more precisely, creativity of the Aha moment formulated by Arthur Koestler (1964) in deeply powerful but little known theory of bisociation exposed in his work The Act of Creation. Both themes are introduced in Unit 1 as the basic thematic threads permeating and organizing this exposition. Unit 1 contains also Chapter 1.3 which describes the student population of the Bronx as one of many educationally “underserved” communities in US and in the world plagued by the increasing Achievement Gap in general, and in mathematics learning in particular (Pisa in Focus, #36, February 2014). TR/NYC model has been formulated with the focus on improvement of learning mathematics within “underserved” student population necessary to bridge the Achievement Gap. Thus TR/NYC is a framework of inquiry in Mathematics Education characterized by the substantive quality of Stenhouse (Rudduck & Hopkins, 1985), that is its “acts of finding” are undertaken to benefit directly and in equal measure not only research community, but also others outside of that community, in our case, students in ours and others’ classrooms.

Arthur Koestler defines Aha moment that is bisociation as “the spontaneous flash of insight, which…connects the previously unconnected frames of reference and makes us experience reality at several planes at once…” (p. 45). We define the bisociative framework as the framework composed of two or more “unconnected frames of reference”, which might be joined by the discovery of a “hidden analogy” through Aha moment of bisociation (p. 179). Since teaching-research is composed of two generally, and unfortunately, separate “frames of reference”, teaching practice and education research, ripe in our opinion with “hidden analogies”, TR/NYC Model is a bisociative framework of inquiry with enhanced possibility to facilitate creative Aha moments both for students and teacher-researchers.

The first coordination of teaching-research practice with Koestler theory of bisociation is done by Vrunda Prabhu (Unit 2) to whom this volume is dedicated. Unfortunately she passed away during her work on Koestler creativity in the
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classroom. Prabhu coordination and recognition of TR/NYCity as a bisociative framework has profoundly transformed our view on the methodology itself and its role in Math Education, especially on the role of student creativity in the improvement of learning mathematics among “underserved” student population and, as a consequence, in bridging the Achievement Gap. One of the central aspects of Prabhu’s work has been the exploitation of cognitive/affective duality characteristic for Aha moments (Czarnocha, 2014) in service of eliminating negative habits and attitudes of students to mathematics. She found support for her quest in Koetler’s assertion that “The creative act…is an act of liberation – the defeat of habit by originality” (p. 96).

The recent papers of the team collected in references of the Epilogue show the further directions of investigations into implications and role of Koestler creativity in mathematics classrooms.

TR/NYCity model finds its “niche” within the inquiry frameworks formulated by Eisenheart (1991) and re-introduced into Mathematics Education by Lester (2010). These authors postulate existence of three different framework of inquiry: theory-based framework, practice-based framework and a conceptual framework. Whereas the first two frameworks are in general separate from each other, the third one incorporates elements of both, while pointing to the essential role of justification in this framework. Our insistence on the balance in the work of TR/NYCity model between research knowledge of the profession and craft knowledge of the teacher finds its expression in the conceptual framework of Eisenheart and Lester with the bisociation leading to creative Aha moments as its central justification.

Incorporation of bisociation into the definition of TR/NYCity model allows to understand Stenhouse acts which are “at once educational act and a research act” as bisociation-in-action (Rudduck & Hopkins, 1985). Important examples of classroom pedagogies, which have the quality of Stenhouse acts are discussed in Unit 3 Tools of Teaching-Research.

We can formulate now the new definition of TR/NYCity Model as the conceptual bisociative framework of Design Research conducted by the classroom teacher, whose aim is to improve the process of learning in the classroom, and beyond – the characteristic of the “substantive nature” of teaching-research. The details of this methodology are obtained as consequences of the definition. The original TR/NYCity definition, which led us to contemporary understanding, is placed at the opening of Chapter 1.1.

Our exposition in the volume is divided into five units, which introduce the reader into practice of TR/NYCity methodology, as well as into results obtained with its help. Following Unit 1, which sketches the main thematic threads of the volume, we present Unit 2 Creative Learning Environment (CLE). Unit 2 contains, in its majority, a collection of TR reports of Vrunda Prabhu describing the process of her search for CLE in classrooms of mathematics, which culminates in the coordination of her practice with Koestler theory in Chapter 2.4. Coordination of
teaching practice with an appropriate theory is an essential bisociative act within TR/NYCity model. We expand the discussion of this process in Chapter 3.2.

Unit 3, Tools of Teaching-Research introduces readers to methods and techniques utilized later in Unit 4. It is composed of two parts, Chapters 3.1–3.3, which focus on preparation, conduct and assessment of a classroom teaching experiment, and Chapters 3.4–3.9 which focus on the chosen set of pedagogical strategies: Teaching Research Interviews, Concept Maps and Discovery method of teaching. All three strategies support the formation of Stenhouse acts and they express the bisociativity of TR/NYCity model. We direct this unit to the attention of teachers of mathematics as a point of entry into mathematics teaching-research. The TR tools introduced here open the pathway through Unit 4 of TR Design, which together with Unit 2 constitute the central nucleus of the presented work.

Unit 4, Teacher as Designer of Instruction: TR Design invites the reader to the exploration of the principles and practice of the Teaching-Research Design as one of the methodologies of Design Research in Mathematics Education.

The types of TR Design are discussed in the extensive introduction to the unit. The introduction continues the discussion of different frameworks of inquiry within Design Research initiated in Chapter 1.1 and applies it to the characterization of the types of TR Design and related with it, classification of Stenhouse acts. The collection of teaching experiments and teaching-research investigations in Unit 4 contains examples for all three types.

Unit 4 starts with the discussion of Koestler’s theory in the context of problem solving in Chapter 4.1 and leads toward synthesis of bisociation with Piaget theories of conceptual development. It demonstrates that processes of reflective abstraction such as constructive generalization and interiorization may be built on bisociative foundations. Although each chapter in Unit 4 typically has its own theoretical framework, Chapter 4.1 is designed to unify these different frameworks of concept development with Koestler’s theory of creativity.

The chapters of the unit present three modes of teaching-research activity. Chapter 4.2 presents our “daily” TR activity in the context of teaching rates and proportions, Chapter 4.6 reports two teaching experiments focused on the iterative classroom design of learning trajectories, and last two chapters present two teaching experiments of different TR Design types.

Unit 5 Teaching-Research Communities illuminates TR/NYCity model from the point of view of teaching-research community of practice and its development. Whereas different aspects of the TR community of practice were touched upon in different chapters of the book, here we focus on the expansion of the TR community to new academic disciplines and cultural/educational environments. The introductory review of learning communities sets the ground for the first two chapters of the unit, which report on the expansion of TR community of practice from its original domain of mathematics into mathematics/English interphase (Chapter 5.1), and expanding further to the learning community of three courses “linked” together through the
mathematically-based theme Part of a Whole (Chapter 5.2). The closing chapters of the unit as well as of the whole book address the professional development of teacher–researchers (PDTR) in two very different cultural educational environments: amongst the teachers of community schools in Dalit villages of Tamil Nadu, India and among the mathematics teachers of five European countries participating in the international project supported by the Socrates program of the European Community. Chapter 5.3 signals further development of TR/NYCity Model into TAR, that is Teaching-Action-Research composed of Teaching-Research in the school and Action Research in the surrounding it village community conducted by the teachers of the school. This development took place during the PDTR in Tamil Nadu, India.

The complexity of the mathematical classroom has been recognized since the works of Anne Brown (1992), Collins (1992) and Wittman (1995) introduced the principles of Design Research into Mathematics Education. The variety of themes addressed by TR/NYCity Model conveys the degree of complexity encountered in the mathematics classroom of “underserved” student population. Consequently, the book and its story can be accessed in several ways: through particular articles and their specified themes, through the inquiries into specific themes e.g. interaction between learning mathematics and language, development of proportional reasoning as a gateway to algebra, problem solving in the context of Koestler theory, the affective role of creativity in transforming negative attitudes to the subject e.t.c. It can also be read as the guide or the handbook facilitating individual or team entry into teaching-research.

Many of the chapters contain teaching sequences whose effectiveness has been investigated through several iterations. They can be adapted to particular conditions of the classroom, expanded and experimented with at will. Generally, presented teaching sequences are accompanied by teacher annotations made either on the basis of professional craft knowledge or appropriate theory or both. The aim of annotation is to bring the reader closer to our work, and in particular to the “technology of thinking” (Chapter 1.1) – the process of integrating the teaching craft knowledge with theories of learning, conceptual development or Koestler creativity. Stenhouse acts are the products of that integration.

The work presented here has taken 15+ years of classroom investigations, designs, reflections and re-designs within the community of TR Team of the Bronx. With this volume we take the voice in the discussions on the role of teachers in research as well as on the role of researchers in the classroom; more generally on the teaching practice-research divide. The highlights of the discussion show surprising absence of knowledge of, and respect for the work of the teacher. Our colleague, Erich Wittmann (1999), in his effort to convince research community to take on research upon teaching sequences, which hasn’t been noticed till now, observes that

…the design of teaching has been considered as a mediocre task normally done by teachers and authors of textbooks. To rephrase Herb Simon: why should
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anyone anxious for academic respectability stoop to designing teaching and put himself on one level with teachers? The answer has been clear: he or she usually wouldn’t.

One can similarly ask, what self-respecting master teacher of mathematics would be interested in collaboration with the academician who needs to “stoop” to collaborate and think together? The answer has been clear: he or she usually doesn’t. The result is a deep divide between research-and teaching practice which undermines any effort to improve learning, and to large degree it is rooted in the issue of the “academic status.”

Interestingly enough, 5 years later, there appear calls for the active participation of teachers in the design and construction of Learning Trajectories in the context of Common Core effort in US. Clements and Sarama (2004, p. 85) note “that learning trajectories could and should be re-conceptualized or created by small groups or individual teachers, so that there are based on more intimate knowledge of the particular students involved…”. Since teaching sequences are the essential component of learning trajectories and their design depends on the level of knowledge of particular cohorts of students, teachers’ designs are necessary for any planned improvement of learning. However, as Susan Empson (2011) notes “We know very little about how teachers do these things, in contrast to what we know about children’s learning…” “these things” being to “understand, plan, and react instructionally, on a moment-to-moment basis, to students’ developing reasoning” and coordinate these interactions with learning goals (Battista, 2010) of the learning trajectory.

The present volume responds to these concerns by demonstrating that teachers of mathematics, if freed a bit from the negative constraints of academic respectability, can design learning trajectories as they always did, and investigate their design through the iterations natural to teachers’ semester or yearly work using JiTR methods. It shows how we, teachers, think, how we design and how we teach, and investigate at the same time with the help of TR/NYCity methodology, especially how we “understand, plan, and react instructionally, on a moment-to-moment basis, to students’ developing reasoning” (Empson, 2010) and coordinate these interactions with learning goals called for by Battista (2010). Thus the answer to Steffe (2004, p. 130) who asks “Whose responsibility is it to construct learning trajectories?” is: it is the responsibility of teacher-researchers working with the TR/NYC City Model as the conceptual bisociative framework of classroom Design Research. The TR/NYC City Model’s emphasis on the balanced relationship between research and craft knowledge of teacher, eliminates the issue of “academic status” and substitutes it by the bisociative creativity of Aha! Moments.
UNIT 1

THE MAIN THEMES OF THE BOOK

INTRODUCTION

The aim of this introductory unit is to present the foundations of the work, which are based upon two conceptual frameworks, TR/NYCity Model and Koestler theory (1964) of the Act of Creation. The coordination between teaching practice and theory (here Koestler theory) – a fundamental component of teaching-research has taken place during the teaching experiment by Vrunda Prabhu and became one of the thematics threads throughout the book. The next two chapters present each of the frameworks separately, and the last chapter throws light upon the theme that appears in many chapters of the book, namely the cognitive and affective learning challenges, encountered in education of the "underserved" student population.

Chapter 1.1 presents the “skeletal” (Eisenheart, 1991) structure of TR/NYCity model, which underlines the full volume. We see here TR/NYCity model as a synthesis of practical and theoretical frameworks of inquiry reaching its completion as a bisociative conceptual framework. The bisociative nature of TR/NYCity reveals itself, with the help the Koestler’s theory of the Act of Creation, as the natural environment to support the creative teaching and learning. We lead the reader through its historic development as well as the constructive connections with work of Lewin, Vygotsky, Stenhouse and Eisenheart, each contributing to the formation and versality of the approach culminating with the Act of Creation of Koestler. It is the natural connection of TR/NYCity methodology with creativity that makes it a strong candidate as a methodology to close the Achievement Gap characteristic for the underserved student population.

We expand upon the TR Cycle and point out that its iterative nature joins the practice of research and the practice of teaching-research. We formulate the concept of Just-in-Time Research (JiTR) in analogy to (JiTT) Just-in-Time Teaching as a feedback loop between classroom challenges of the teacher and research knowledge of the profession. We argue that such a bottom – up relationship of teaching practice with research is necessary for research to be accepted and incorporated in the classrooms. Two themes complete the presentation of the methodology: the discussion of characteristic tools of the methodology (1) Discovery method of teaching and (2) development of thinking technology, which lead to Stenhouse TR acts as one of their expressions. The Discovery theme, while present in most of our TR investigations is discussed in Chapter 3.9, while Stenhouse TR acts we encounter again in the Unit 4.
Chapter 1.2 presents an overview of contemporary research on creativity as a background to Koestler’s theory of creativity, the second main theme of the book. A central concept of creative research and Koestler’s theory turns out to be the affective-cognitive duality of the Aha moment (Czarnocha, 2014). The affective component of the duality suggests that facilitation of Aha moments might be a powerful pedagogical tool in changing student attitude to mathematics as well as to themselves as mathematicians. Since the experience of Aha moment is known to members of general population, its facilitation in any regular classroom creates the possibility of opening the research on, and practice of mathematical creativity by all students, in addition to gifted ones. The chapter discusses the issue in the section the Transition from Genius to the Classroom focusing subsequently upon bisociation theory of creativity by Koestler (1964). The author, Bill Baker expands the discussion to delve into the cognitive-affective duality where the mechanism of bisociation enters into Piagetian framework. This discussion is continued in Chapter 4.1 in the context of problem solving. The affective aspect of the duality is discussed within Goldin (2009) work in the area. However it is the Unit 2 of Vrunda Prabhu that needs to be read in conjunction with this chapter as the development-in-practice of the very same duality.

Chapter 1.3, the last chapter of Unit 1 sketches the socio-economic-ethnic background of student population we teach in the Bronx together with the main academic difficulties our student experience. The chapter points to the fact that other similar centres of “underserved” student population in US, Europe and Asia demonstrate similar characteristics, so that the dydactic designs and artefacts formulated in the context of Bronx student population can work elsewhere, can be generalized. The issue of generalizations of the methodology is further developed in Chapters 1.1 and 2.3. The author addresses critical pedagogical issue of what works among the population and shows that the focus on creativity through facilitating Aha moments might indicate the correct route to close the Achievement Gap which at present can be detected throughout the world. The role of community of TR practice for that process is developed in the Introduction to the Unit 5.
1.1. TEACHING-RESEARCH NEW YORK CITY MODEL (TR/NY CITY)

TR/NYCity Model is the methodology for classroom investigations of learning, which synthetizes educational research with teaching practice. It is conducted simultaneously with teaching and it aims at improvement of learning by the teacher of the class in the same classroom, and beyond.

INTRODUCTION

TR/NYCity Model is based on the careful composition of ideas centred around Action Research (Lewin, 1946) with the ideas centred around the concept of the Teaching Experiment of the Vygotskian school in Russia, where it “grew out of the need to study changes occurring in mental structures under the influence of instruction” (Hunting, 1983). From Action Research we take its focus on the improvement of classroom practice by the classroom teacher and its cyclical instruction/analysis methodology, and from Vygotsky’s teaching experiment we take the idea of the large-scale experimental design based on a theory of learning and involving many sites – different classrooms (Czarnocha, 1999; Czarnocha & Prabhu, 2006). Vygotsky teaching experiment methodology introduced the possibility of viewing the classroom teacher as a member of a collaborative research team investigating the usefulness of research based classroom integration. The integration of these two distinct frameworks re-defines the profile of a teacher-researcher:

1. as an education professional whose classrooms are scientific laboratories, the overriding priority of which is to understand students’ mathematical development in order to utilize it for the betterment of the particular teaching and learning process;
2. who as a teacher can have the full intellectual access to the newest theoretical and practical advances in the educational field, knows how to apply, utilize and assess them in the classroom with the purpose of improving the level of students’ understanding and mastery of the subject;
3. who as a researcher has a direct view of, and the contact with the raw material of the process of learning and development in the classroom, acts as a researcher in the context of the daily work and uses that process to design classroom improvement and derive new hypotheses and general theories on that basis.
The implicit vision underlying the profile is the conceptual and practical balance between researches and teaching, where both components of the educational profession are given equal value and significance; both the research knowledge of the researcher and the craft knowledge of the teacher are resources for the teacher-researcher.

Admittedly, the proposed profile is ambitious, yet it’s doable, especially in the context of community colleges whose full time mathematics faculty have PhD level experience in mathematics, physics or engineering research and can relatively easily transfer those skills into classroom-based investigations of learning. On the other hand, given the progressing collapse of public education in US, the majority (80%) of freshman students who enter every semester into our colleges require remediation to be able to get to college level courses. The remediation starts on the level of arithmetic through algebra it constitutes 80% of our “bread and butter” courses. The placement into, and exit from remediation is decided by the university wide – standard exam. Consequently, the mathematics faculty of community colleges are intimately familiar with the issues of school mathematics. The composition of research skills with the craft knowledge of teaching elementary mathematics is at the basis of the formulation of TR/NYCity Model.

HISTORICAL BACKGROUND AND DEVELOPMENT OF TR/NY CITY MODEL

**Stenhouse TR Acts**

TR/NYCity owns its formal origins to Action Research of Kurt Lewin (1946) and Teaching Experiment methodology of Vygotsky. TR/NYCity model finds its completion in the bisociation of Koestler (1964) leading to the Stenhouse TR acts (Rudduck & Hopkins, 1985).

Lewin proposed the Action Research methodology in the context of the quest for improvement of “group relations”, a euphemism for interracial relations in US of 30ties and 40ties. He saw it as “…a comparative research on the conditions and effects of various forms of social action, and research leading to social action.” His Action Research cycle consisted of the stages (or steps) of diagnosis with plan for action, implementation of action, its assessment providing at the same time the basis for “modifying overall plan” and leading to the next cycle. It was however Stenhouse who introduced Action Research methodology into education profession as teaching-research in the inaugural lecture at the University of East Anglia in 1979 presentation “Research as basis for teaching” – a theme whose importance has steadily grown till contemporary times. Already in early seventies of the last century he recognized that one of the possible explanations for the failure of research

…to contribute effectively to the growth of professional understanding and to the improvement of professional practice… was the reluctance of educational researchers to engage teachers as partners in, and critics of, the research results. (Rudduck & Hopkins, 1985)
The extracts from the transcripts of seminars with the part-time MA students reveal his understanding of Action Research in terms closely related to TR/NYCity model arrived at spontaneously through our work. He understood Action Research primarily as “the type of research in which the research act is necessarily a substantive act; that is an act of finding out has to be undertaken with an obligation to benefit others than research community” (p. 57), in our case, students in ours, and other classrooms. However, it’s the concept of “an act [which is] at once an educational act and a research act” (p. 57), that completes a stage in our development of thinking technology, that is the process of integration of research and learning theories with the craft knowledge of the profession anchored in practice. The bisociative framework (see below) of TR acts produces new mental conceptions, the product of thinking technology. These conceptions (e.g. schema, ZPD, hidden analogy, bisociation) become part of the discourse within the community of teacher-researchers, tools to design methodology for improvement of classroom craft and for deepening one’s research interest.

It is surprising Stenhouse did not utilize Action Research cycles. It could be because the curriculum research he envisioned as conducted by teachers, apart from case studies, was to test hypotheses arrived at by curriculum research outside of the teacher’s classrooms (p. 50).

The second root of our methodology is anchored in the methodology of the Teaching Experiment of Vygotsky, which had a professional research team together with teachers investigate the classroom and was conducted “…to study changes occurring in mental structures under the influence of instruction” (Hunting, 1983). Interestingly, introduction of professional research into classroom by Vygotsky and his co-workers in the thirties was the fulfilment of the first part of the Stenhouse’s vision of the seventies who demanded “In short, (1) real classrooms have to be our laboratories, and (2) they are in command of teachers, not researchers” (p. 127). For the second part of Stenhouse vision we propose classrooms, which are in the command of teacher-researchers as the synthesis of both methodological efforts.

The Teaching Experiment methodology reappeared in the work of Steffe and Cobb (1983) as a constructivist teaching experiment, which was appropriated by Czarnocha (1999) for teaching purposes in high school class of mathematics, already as a tool of a teacher. Czarnocha (1999) realized that the constructive teaching experiment can easily become a teacher’s powerful didactic instrument when transformed into guided discovery method of teaching.

**Design Science**

The interest in the work of the professional practitioner of whom teacher is but one particular example has been steadily increasing in the second half of the previous century since the work of Herb Simon (1970), the Design of the Artificial. His work proposes the design as the “principal mark that distinguishes the professions from sciences” (pp. 55–58). Kemmis and McTaggart (2000) developed the principles
of Action Research, while Schon (1983) investigated the concept of a Reflective Practitioner through the process of reflection-in-action. Both frameworks had found applications in the work of teachers and researchers through joint collaborations, however the research/practice gap hasn’t been yet bridged.

The terms Design Experiment, Design Research or the Science of Design are often interchangeable and they refer to the professional design in different domains of human activities. It was introduced into research in Math Education by Ann Brown (1992), Collins (1992), and Whittmann (1995). Anne Brown had realized during her exceptional career that psychological laboratory can’t provide the conditions of learning present in the complex environment of a classroom and transformed her activity as a researcher directly into that very classroom as the leading co-designer and investigator of the design in the complex classroom setting. In her own words: “As a design scientist in my field, I attempt to engineer innovative classroom environments and simultaneously conduct empirical studies of these innovations” (A. Brown, 1992). She provided this way one of the first prototypes of design experiments which, theoretically generalized by Cobb et al. (2003), “entail both “engineering” particular forms of learning and systematically studying those forms of learning within the context defined by means of supporting them…” The profession has followed her lead seeing the classroom design experiments as theory based and theory producing. Paul Cobb et al. (2003) assert that Design Experiments are conducted to develop theories, not merely to empirically tune what works. Design research paradigm treats design as a strategy for developing and refining theories (Edelson, 2002). Even Gravemeyer (2009) who defines “the general goal of Design Research to investigate the possibilities for educational improvement by bringing about and studying new forms of learning” hence stating it closer to substantive quality formulated by Stenhouse, yet he warns us that “great care has to be taken to ensure that the design experiment is based on prior research…” eliminating this way the designs anchored in prior practice. Unfortunately, the educational research profession cuts itself off by these restrictions from the source of profound knowledge contained in the tacit and intuitive craft knowledge of the teachers. Clearly, if the goal is improvement of learning, a more general framework is needed which recognizes both education research and teaching practice as two approaches of comparable significance, value and status.

Frameworks of Inquiry and the Unity of Educational and Research Acts

We find such a framework within the three frameworks of inquiry identified by Margaret Eisenhart (1991): theoretical, practical, and conceptual (Lester, 2010). Following Eisenhart, Lester (2010) posits three types of frameworks used in Math Education, first, the theoretical framework based upon theory i.e. the constructivist, radical constructivist and social constructivist theories discussed second, a practical framework, “… which guides research by using ‘what works’ … this kind of research is not guided by formal theory but by the accumulated practice knowledge
of practitioners and administrators, the findings of previous research, and often the viewpoints offered by public opinion” (p. 72). The third is a conceptual framework that can pull from various theories as well as educational practice.

The theoretical framework guides research activities by its reliance on a formal theory; that is, a theory that has been developed “on the theoretical, conceptual, and philosophical foundations” (Lester, 2010) by using an established, coherent explanation of certain sorts of phenomena and relationships—Piaget’s theory of intellectual development and Vygotsky’s theory. However, as soon as such a theory-based design undergoes a TR cycle, the initial determinative role of theory changes into the JiTR-approach (Just-in Time-Research; see below), which allows for the participation of craft knowledge based on the teaching experience in equally significant manner.

The Practical Framework is employed in what we refer to as ‘action research’ and as discussed, it has some common components with teaching-research.

For Scriven, [quoted in Lester (2010)] a practical framework guides research by using “what works” in the experience of doing something by those directly involved in it. This kind of framework is not informed by formal theory but by the accumulated practice knowledge of practitioners and administrators, the findings of previous research, and often the viewpoints offered by public opinion. Research questions are derived from this knowledge base and research results are used to support, extend, or revise the practice. (Lester, 2010)

However, the distinction that we make with Lester’s description of a practical framework and a framework for teaching research is that we, as researchers, view the goal of teaching-research to inquire into how theory and models of learning reflect upon what the teacher and student experience in the classroom. The question for the teacher researcher and supportive TR community is what needs to be transformed or changed in the existing theories or models in order to improve the fit between these frameworks and classroom practice?

The third and final framework considered by Lester is that of

a conceptual framework [that] is an argument that the concepts chosen for investigation, and any anticipated relationships among them, will be appropriate and useful given the research problem under investigation. Like theoretical frameworks, conceptual frameworks are based on previous research, but conceptual frameworks are built from an array of current and possibly far-ranging sources. The framework used may be based on different theories and various aspects of practitioner knowledge. (Lester, 2010)

We argue that amongst the three frameworks for research present in philosophy of education research only the conceptual framework allows for the possibility of bisociative synthesis between teaching and research through Stenhouse TR acts.

Of special importance in working with conceptual frameworks is the notion of justification. A conceptual framework is an argument including different points
of view and culminating in a series of reasons for adopting some points and not others. The adopted ideas or concepts then serve as guides: to collecting data, and/or to ways in which the data from a particular study will be analysed and explained (Eisenhart, 1991).

According to Lester (2010) “…too often educational researchers are concerned with coming up with “good explanations” but are not concerned enough with justifying why are they doing what they are doing…” (p. 73).

Our insistence on the balance between research and teaching practice, the basis for the unified Stenhouse TR acts, finds its justification and fulfilment in the bisociation of Koestler (1964) that is in “a spontaneous leap of insight which connects previously unconnected matrices of experience” (p. 45). A bisociative framework is the framework composed of “two unconnected matrices of experience” where one may find a “hidden analogy” – the content of insight (Chapter 1.2). Given the persistent divide and absence of deep connections between research and teaching practice, TR/NYCity constitutes a bisociative framework composed of “unconnected [in general] matrices of experience” of teaching and research, within which one can expect high degree of creativity on the part of the teacher-researcher through leaps of insight leading to the unified Stenhouse acts defined above. The process of coordination of TR/NYCity with Koestler bisociation theory is the guiding theme of Unit 2: Creative Learning Environment. Unit 2 presents the search for classroom creativity by Vrunda Prabhu during which this coordination has taken place revealing “hidden analogy” between Koestler theory and Prabhu’s teaching practice.

We can state now a new definition of TR/NYCity methodology:

**TR/NYCity Model is the conceptual bisociative framework of Design Research conducted by the classroom teacher, whose aim is to improve the process of learning in the classroom, and beyond – the characteristic of its “substantive nature”**.

TR bisociative framework facilitates integration or, still better, synthesis of practice and research through instances or sequences of instances of Stenhouse acts which are “at once an educational act and a research acts” (Rudduck and Hopkins, p. 57). In what follows we will call them Stenhouse TR acts. The Stenhouse TR acts are the foundation stones of “thinking technology” discussed below within which their unity is naturally positioned. The facilitation of longer or shorter instances of Stenhouse TR acts can be reached from either teaching practice or from application of research to practice, as well as from both simultaneously. The “skeletal structure” (Eisenhart, 1991) of the TR/NYCity conceptual framework can be obtained as requirements and conclusions from the definition.

We discuss different designs of teaching experiments and TR investigations in Unit 4, The Teacher as a Designer of Instruction: TR Design, while in Chapter 3.2 we discuss “nuts and bolts” of classroom teaching experiment. The Introduction to Unit 4 develops the “skeletal structure” of TR/NYCity as the consequence of the definition.
TEACHING-RESEARCH CYCLE (TR CYCLE)

Just-in-Time Teaching (JiTT) and Just-in-Time Research (JiTR)

Teaching-Research cycle is the fundamental instrument in our work, which allows for the smooth integration of research and teaching practice within our conceptual framework. The difference from other similar cycles of Action Research or of the Design Experiment (Cobb et al., 2003) is simple: it allows the teacher-researcher to enter the classroom investigation from either of both directions, from research and from teaching. There is however, an important methodological trade off: whereas a Design Experiment researcher prepares the design of classroom intervention on the basis of prior research, the teacher-researcher uses Just-in-Time approach, that is research literature consultation takes place during the TR cycle, generally at the Analysis and Refinement nodes, when we either compare the results to assumed theory of learning, or when we search for adequate theoretical framework to understand the learning situation, or in any other unclear classroom situation.

Just-in-Time Teaching (JiTT) as expressed by Novak et al. (1999) is a teaching and learning strategy based on the interaction between web-based study assignments and an active learner classroom. Students respond electronically to carefully constructed web-based assignments which are due shortly before class, and the instructor reads the student submissions “just-in-time” to adjust the classroom lesson to suit the students’ needs. Thus, the heart of JiTT is the “feedback loop” formed by the students’ outside-of-class preparation that fundamentally affects what happens during the subsequent in-class time together. JiTT has been used well together with Peer Leader methodology (Mazur & Watkins, 2009).

Analogically, Just-in-Time Research (JiTR) is research and teaching strategy based on the “feedback loop” formed between the didactic difficulties in the classroom encountered by a teacher, and educational research results that may throw light into the nature of these difficulties. At this moment, the classroom teacher makes contact with the bisociative framework of TR/NYCity model.

Anchoring TR in TR cycle

It is in the introduction of educational research into the classroom that we differ from Action Research. The JiTR approach differs from standard educational research in that theory is repositioned from being a required foundation to the Just-in-Time solution for didactic difficulties in the mathematics classroom.

William J. Harrington, describing his work of a teacher-as-researcher in Laura R. Van Zoest (2006) states that, “Teachers do informal research in their classrooms all the time. We try a new lesson activity, form of evaluation, seating arrangement, grouping of students, or style of teaching. We assess, reflect, modify, and try again, as we consider the perceived consequences of changes we made.” Hence, there is a natural pathway that extends these informal activities into systematic research, offered by the
TR/NYCity model that successively progresses along Teaching-Research (TR) cycles of diagnosis, design of instruction in response to diagnosis, collection of relevant data and its analysis, and, ultimately, with the help of relevant external research results through JiTR approach, the redesign of interventions. The TR cycle, the explicit generalization of Action Research principles in the classroom, is particularly well fit into our work because of our work’s naturally cyclic structure via semesters or academic years. Since every teacher has an option of repeating to teach the same course to a new cohort of students, the TR cycle allows for the continuous process of classroom investigations of the same research question during consecutive semesters. The sequential iteration of TR cycles is one of the main methodological research tools of the TR/NYCity Model facilitating the process of integration of teaching and research into a new unit of professional classroom activity, teaching-research.

TR/NYCity requires a minimum of two full TR cycles within a context of a single teaching experiment to fulfil the requirement of improvement of instruction. In its insistence on the improvement of learning through cycle iteration, TR/NYCity incorporates and generalizes the principles of Japanese and Chinese Lesson studies (Huang & Bao, 2006).

Consequently, every teaching experiment of the TR/NYCity Model has a main teaching-research question, composed of two sub-questions:

- What is the state of the students’ knowledge under the impact of the new intervention?
- How to improve that state of knowledge?

The duration of the TR cycle can vary depending on intervention. In can last a year, a semester, and a couple of days or even one class. In its rudimentary form we can find it even in teacher-student inquiry dialogs (see example in Chapter 4.1).
The bisociative creativity of the teacher reaches its fulfillment during this period of reflection and redesign spurred by the simultaneous consideration of data analysis results, relevant teaching experience, relevant JiTR results from professional literature and appropriate theories of learning or conceptual development. It is precisely at this moment when the new teaching-research hypotheses are formed, leading to new theories and investigations. The focus of this teaching-research activity is the investigation of student learning followed by the design of teaching, whose effectiveness is often investigated in the subsequent TR cycle.

Instructional Adaptability of the TR/NYCity Model via TR Cycle

The increased degree of flexibility created by this integration of teaching and research within a single “tool box” helps teachers reach new levels of instructional adaptability to student learning needs. In fact, the comparison of the adaptive instruction described by (Daro et al., 2011) with the TR cycle reveals a very high degree of correspondence:

For that [success of LT framework] to happen, teachers are going to have to find ways to attend more closely and regularly to each of their students during instruction to determine where they are in their progress toward meeting the standards, and the kinds of problems they might be having along the way. Then teachers must use that information to decide what to do to help each student continue to progress, to provide students with feedback, and help them overcome their particular problems to get back on a path toward success. This is what is known as adaptive instruction and it is what practice must look like in a standards-based system.

Every TR cycle consists of the following components:

• (1) The design of the instruction/intervention, in response to the diagnosis of student knowledge,
• (2) Classroom implementation during an adequate instructional period and collection of data; this incorporates problem-solving, guided discovery classroom discourse and design of interventions for diagnosed difficulties,
• (3) Analysis of the data, in reference to existing experimental classroom data, appealing to the general theory of learning through J-i-T approach and the teacher-researcher’s professional craft knowledge,
• (4→1) Design of the refined instruction based on the analysis of the data obtained in steps 1 through 3, leading to the hypothesized improvement of learning. The symbol “4→1” is intended to convey that the 4th step in the cycle is equivalent to going back to the 1st step in the cycle.

As a result, every such 1→2→3→4→1 is an instance of adaptive instruction – finding the level of students’ understanding through tests, homework assignments and one-on-one interviews, responding to the difficulties by the re-design of the
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intervention, implementation and assessment. Consequently, the TR cycle is called for, as the theoretical framework of the teacher’s work in a mathematics classroom driven by the Common Core Standards. Transformations of the teacher’s pedagogy and improvements, based on research and evidence, have to take place exactly within such a framework. Chapters 4.2, 4.9 and 4.10 provide detailed examples of two (or more) full cycles of such an approach.

Generalization in TR/NYC Model

One of the central questions asked of frameworks related to action research is the question about the generality of our assertions. How general is TR/NYC? Why and how that what we understand in the Bronx, has any bearing anywhere else? In terms of the original definition at the beginning of the chapter, what is the nature of the word “beyond” in that definition?

TR/NYC has three ways to generalize its findings:

1. By coordination with a theory whose correctness has been asserted in the profession. If we coordinate our findings with a theory, then they acquire degree of generality afforded to the theory, that is one can draw conclusions from the findings in terms of the coordinated theory of learning. These conclusions might be relevant, with proper modifications to any classroom situation to which that theory applies.

2. By running an artefact used in a TR investigation through many iterations with different cohorts of students. As a result, the artefact acquires large degree of generality, which provides the basis for its application to different new situations (Chapter 2.2).

3. A special window of generalizations opens up when we consider student populations with similar socio-economic status to the one in the Bronx. The similarity of the socio-economic status results in similar cognitive/affective challenges experienced by students to which similar adaptive interventions are needed (Kitchen et al., 2007) The successful generalization of TR/NYC artefacts has been reached amongst Indian Dalits (downtrodden) of Tamil Nadu (Chapters 2.2 and 5.3) and in Poland amongst rural students of Southern Poland (Czarnocha, 2008).

The discussion of artefacts in the context of Design Research (Unit 4) brings forth an important clarification that its generalization can be obtained by expanding its application to similar student populations.

Thinking Technology

The dictionary definition of technology is “the application of scientific knowledge for practical purposes, especially in industry.” Thinking technology in TR/NYC model is the process of integration of research results and framework with craft
knowledge of the teacher. This spontaneous process inherent for TR/NYCity model finds its elegant expression in Koestler bisociation theory and Stenhouse TR acts. It is a very subtle process, in which scientific concepts such as “hidden analogy” of Koestler become the critical tools, metaphors with the help of which we start to identify classroom situations, the term becomes a phrase with the help of which we, members of the TR team start communicate with each other in our own new language. In fact, by making the connection between scientific meaning and classroom situation we create the analogy between two generally separate matrices of thinking – hence the connection itself is a new bisociation, a possibility of new meaning.

One could conjecture that any process of coordination (as distinct from application) of a theory of learning with elements of teaching practice is the bisociative creative process during which new connections and therefore new meanings are made.

The process of coordinating research and teaching practice is facilitated by the duality inherent in the teacher-researcher work (Malara & Zan, 2002). The practice of teaching-research duality creates a new mental attitude promoting a novel design of instructional methodologies while, at the same time, requiring an investigative probe into student thinking, on the basis of which consequential teaching and research decisions are made. This duality is explored deeper in Units 2 and 4. The exploration together with utilization of the duality is conducted by the classroom teacher-researcher. In this process, teachers are not solely engaged in research on learning, they are also engaged in the transformation of teaching on the basis of, and through that research. This means that they do not simply incorporate the results of research into their teaching practice but rather allow methods of research to become the methods of teaching leading to Stenhouse TR acts. Thus the route towards Stenhause TR acts is through the process of integrating research knowledge and craft knowledge in practice of teaching. In this process, teachers do not switch into a role of researcher, instead, they oscillate between the role of a teacher and the role of a researcher and fuse their efforts toward a new unit of professional activity – bisociative teaching-research with its Stenhouse TR acts.

TR/NYCity and the Discovery Method of Teaching

The discovery method of teaching has been the preferred instructional method by the teacher-research team working with and developing TR/NYCity methodology since its inception. The Discovery method of teaching has a fundamental role in the TR/NYCity model. This method was introduced into TR/NYCity via the Texan Discovery method created and formulated by R. L. Moore, a topologist brought up by the Chicago school of mathematical thought of the thirties. B. Czarnocha and V. Prabhu adopted this method during their NSF grant in calculus 2002–2006. However, our understanding of its role in TR classrooms came with time through
many TR investigations and teaching experiments. Using different approaches such as a “guided discovery method”, “inquiry method” or “inquiry leading to discovery”, it has appealed to our imagination and practice as teacher-researchers because with its help we could lay bare student authentic thinking for our investigations.

On the one hand, from the educational aspect Discovery method provides a learning environment best suitable for facilitation of bisociation. According to Koestler (1964) subjective, individual bisociation are more often encountered in the condition of “untutored learning”. The Discovery method is one of the closest classroom approximations of this condition. This approach to teaching relies on designing situations and using techniques, which allow the student to participate in the discovery of mathematical knowledge. These are authentic moments of discovery with respect to student’s own knowledge, which in the further development of methodology are related to subjective Aha! Moments of Arthur Koestler (Chapter 1.2).

On the other hand, from the research point of view, it is the best instrument, which opens the nature of student thinking to us, teacher-researchers for investigation through careful interaction. It allows us to investigate and to extend the scope of students’ ZPD, to help in eliminating misconception as well as in facilitating bisociations. Thus the process of TR together with Discovery method of teaching constitutes an extended in time Stenhouse TR act.

**Creativity: From Bathos to Pathos – From Habit to Originality**

The institution of creativity as the structural component generated within the learning environment provided by teaching-research has significant consequences beyond its cognitive importance.

Vrunda Prabhu has found out (Chapter 2.4) that student success in her classroom depended on three closely connected components of (i) cognition, (ii) motivation and (iii) self-regulated student learning (Prabhu, 2006). More specifically, when creativity is explicitly nurtured and facilitated in a mathematics classroom in the context of such an integrated learning environment, it can transform the habit of distaste toward mathematics into mathematical originality supporting Koestler’s assertion that “creativity means breaking up habits and joining the fragments into new synthesis” (p. 619). Moreover, according to Koestler:

The creative act, by connecting previously unrelated dimensions of experience, enables him [the inquirer] to attain a higher level of mental evolution. It is an act of liberation – the defeat of habit by originality.

Habitual dislike of mathematics is, at present, one of the main student obstacles for success in mathematics learning that could be eliminated with the help of that “act of liberation” providing a pathway from Bathos to Pathos, using Koestler metaphor (p. 96).
Summary of the Argument

To summarize the argument, TR/NYCity is the generalization of Action research and of the Design experiment methodology (Design experiment methodology is seen here as the further development of the Teaching Experiment of Vygostsky school in Russia). In its original vision it was seen as the bridge between the two methodologies, which eliminates the limitations of both – a new integrative conceptual framework. By the same token, TR/NYCity is designed specifically to bridge the gap between research and teaching practice – one of the fundamental obstacles in the effective transformation of mathematics education. The need for such a bridge was indicated by the report of US National Research Council, How People Learn-Bridging Research and Practice (Donovan et al., 1999). We review below essential components of the research/teaching practice gap in our profession as seen by contemporary reports.

GAP BETWEEN RESEARCH AND PRACTICE

English (2010a) notes that the complexity of educational theory and philosophy, has lead to a gap between educators and researcher based upon concerns about the relevancy of such philosophies to educational practice,

The elevation of theory and philosophy in mathematics education scholarship could be considered somewhat contradictory to the growing concerns for enhancing the relevance and usefulness of research in mathematics education. These concerns reflect an apparent scepticism that theory-driven research can be relevant to and improve the teaching and learning of mathematics in the classroom. Such scepticism is not surprising…claims that theoretical considerations have limited application in the reality of the classroom or other learning contexts have been numerous…it remains one of our many challenges to demonstrate how theoretical and philosophical considerations can enhance the teaching and learning of mathematics in the classroom… (p. 66)

Harel (2010) and Lester (2010) both note that government funding agencies and panels created to direct government research efforts are increasing restricting their attention to quasi experimental-control group efforts with a goal of what works i.e. action research. They advance the hypothesis that more attention to research frameworks would perhaps counter the ideology that all research should be practical-statistical i.e. scientific based methodology based upon a p value indicating success or failure i.e. ‘what works.’ Harel’s (2010) claim that attention to frameworks is lacking in educational research is due in part to his belief that there exists “…a feeling on the part of many researchers that they are not qualified to engage in work involving theoretical and philosophical considerations” (pp. 88–89). The issue that arises for those of us advocating for a more active role of teachers in integrating educational research and craft is that, if researchers feel they are not qualified then
how much more likely those teachers feel unqualified. That is, how can practical research methodology such as that used in action research be expected to integrate theory and practice in a meaningful way when its practitioners may feel unqualified to engage in theoretical considerations? This question is particularly relevant to us because we strongly believe in order for reform efforts, indeed, any research based pedagogy to actually improve education there must be a sustained effort in the school and that any such effort must involve the teacher and the researcher working together or a teacher-researcher to determine what works as well as to reflect upon why it does or does not work from both a practical craft level as well as through the lens of theoretical framework.

Another reason reform effort to improve mathematics education through theoretical considerations has floundered is that mathematical education theories are often appear impractical to the craft practitioner to implement i.e. theories that provide little guidance for instructional design but within the research community there are often contradictory positions about such efforts. The result is that reform efforts and counter reactionary movements tend to arise and disappear like last year’s fashion statements. Sriramen and English (2010) comment on an early attempt by mathematicians to change traditional mathematics called New Math which in the 50’s and 60’s tried to change the rigidity of traditional mathematic through a top down approach to pedagogical change. “One must understand that the intentions of mathematicians such as Max Beberman and Edward Begle was to change the mindless rigidity of traditional mathematics. They did so by emphasizing the whys and the deeper structures of mathematics rather than the how’s but in hindsight…it seems futile to impose a top-down approach to the implementation of the New Math approach…” (p. 21). Goldin (2003) notes how behaviourism led to a back to basics counter movement within mathematics education: “behaviourism was fuelling the ‘back to basics’ counterrevolution to the ‘new mathematics’, which had been largely a mathematician-led movement. School curricular objectives were being rewritten across the USA to decompose them into discrete, testable behaviours” (p. 192). Goldin (2003) also notes that constructivism has more recently displaced this back to basic reactionary movement. “Radical constructivism helped overthrow dismissive behaviourism, rendering not only legitimate but highly desirable the qualitative study of students’ individual reasoning processes and discussions of their internal cognitions” (p. 196). Yet he warns that the excessive of radical constructivism will render it impractical and unsuitable “Constructivists excluded the very possibility of ‘objective’ knowledge about the real world, focusing solely on individuals’ ‘experiential world’” (p. 193).

The point being that a top-down approach to educational reform by research experts has not succeeded and we venture will never succeed without first teacher buy in, but this is not near enough, in order for the craft practitioner to continue to implement reform methodology and to design instruction based upon theory, when the researcher goes back to academia the teacher must internalize the theory
and even more how such theory relates to design of instruction. Yet we consider that even this is not enough to sustain reform efforts especially with underserved populations that demonstrate serious negative affect with mathematics. The approach to educational research in which experiments have a beginning and an end is founded upon an underlying assumption that some truth can be found that will dramatically change educational practice. This assumption needs to be re-evaluated if educational craft practice is to actualize the benefits of research. We consider that a constant collaboration between educational researchers and teachers is needed and provides the best hope of actualizing change in educational practice to close widening gap between research and theory and the scepticism it has caused. Boote (2010) comments on the need for continual teacher development based upon design research in improving educational practice: “Indeed, the professional development of all participants may be more important and sustaining than the educational practices developed or the artefacts and knowledge gained” (p. 164). Examples of such an international professional development of teacher-researchers based on TR/NYCity methodology are discussed in the Unit 5.

THE COMPARISON BETWEEN TEACHING-RESEARCH AND DESIGN-BASED RESEARCH

The discussion in this section is the continuation of the theme found in the section Frameworks of Inquiry and the Unity of Educational and Research Acts, which gets further clarification in the Introduction to Unit 4. Our aim here is to provide a detailed comparison between theoretical and practical frameworks as seen from the point of view of TR/NYCity, which we see as the conceptual framework creating the bridge between the two via TR cycle.

| Research, in particular, design-based research | Teaching-research, in particular TR/NYCity model |
|-----------------------------------------------|-------------------------------------------------
| Theory driven: (EDUCATIONAL PSYCHOLOGIST, 39(4), 199–201 Copyright © 2004, Lawrence Erlbaum Associates, Inc. William A. Sandoval, Philip Bell Design-Based Research Methods for Studying Learning in Context: Introduction.) Design-based research can contribute to theoretical understanding of learning in complex settings. Each of the articles by Sandoval, Tabak, and Joseph reveal how the design of complex interventions is an explicitly theory-driven activity. | Practice driven: (Professional Development of Teacher-Researchers, Rzeszow University, Poland, 2008) (Teaching Experiment NYCity Method, 2004) Teaching-research is grounded in the craft knowledge of teachers that provides the initial source and motivation for classroom research; it then leads to the practice-based design. Its aim is the improvement of learning in the classroom as well as beyond. |
Use of Theories of Learning in Design-Based Research:
(Didacyna Matematyki, 2006, v. 29, Poland, Teaching-Research NYCity Model. B. Czarnocha, V. Prabhu)
The design of innovation enables the teacher-researcher to create the Creative Learning Environment based on teacher’s craft knowledge, which improves learning in the classroom and transforms habits such as misconceptions, into student originality (Koestler, 1964). Learning theories are used as needed to support teachers’ craft knowledge.

Focus of the Teaching Experiment in Design-Based Research:
Cobb and Steffe assert that the interest of a researcher during the teaching experiment in the classroom is “in hypothesizing what the child might learn and finding [as a teacher] ways and means of fostering that learning”.

Use of Iteration in design-based research:
…articulating, refining and validating is an “iterative process of research synthesis and empirical investigations involving” many types of evidence.
Step 1: Meta-research of the concept to create the prototype.
Step 2: Iterative refinement of the prototype

Use of Iteration in TR/NYCity model:
Step 1: Process of iteration, starting with the first iteration designed on the basis of teaching practice.
Step 2: Incorporation of research results as needed in between consecutive iterations.
It is the concept of iteration of the design from semester to semester together with the related refinement that can bring in now relevant research results illuminating the classroom situation or providing help in the design of appropriate set of assignments.
The TR cycle through its natural iteration of teacher’s activity from semester to semester provides the opportunity to move beyond the narrow “chicken or the egg” question of “What is the primary, or the more important realm,—research or practice?” and to creatively integrate design-based practice and design based research (see Unit 4).

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1.2. CREATIVITY RESEARCH AND KOESTLER

An Overview

INTRODUCTION

This section provides an overview of research on creativity in mathematics education: Koestler’s understanding of creativity, the role of creativity with mathematics students, especially those who do not view themselves as gifted and a discussion on the how to support a creative learning environment within the mathematics classroom. Leikin (2009a) notes that, creativity and giftedness are underrepresented topics in mathematics education research; one specific obstacle she specifies is a lack of shared understanding of creativity within the mathematics classroom and ways to promote such creativity. In this present book the authors put forth their work on creativity based upon the definition and supporting theory of creativity due to Koestler (1964) within the framework of teaching research experiments supported by learning communities in an effort to promote creativity research within the classroom-learning environment. At the heart of Koestler’s theory is the view that creativity involves the synthesis of previously unrelated frames of reference, hidden analogies that become apparent through one’s intuition in an often unconscious process during incubation. That process leads to the conscious illumination or realization of this previously hidden analogy. Furthermore, this process has a transformation effect upon an individual’s affect towards the area or subject of thought, in this case mathematics. Following Koestler we consider this process of synthesis to describe both the creative thought of professional-eminent mathematicians as well as the development of cognitive structures in both gifted and ordinary students. We are particularly interested in the benefits for student affect during the process in which an individual discovers for themselves new material or previously hidden analogies with students one might characterize as being resistant to mathematics i.e. those who have a negative affect or understanding of their ability in mathematics. Another central component to the work of Koestler is that there are strands of creative thought that cross the three domains of; humor, scientific and mathematical research and also, literary and artistic endeavours. Koestler refers to this notion as a Triptych. The role of the Koestler’s Triptych in this volume (Chapter 2) is not so much as to argue that creativity crosses domains rather that there
is an affective relationship between creativity in these domains that can be beneficial for students with a negative or resistant attitude towards their mathematical ability.

THE NEED FOR RESEARCH IN CREATIVITY IN EDUCATION

Leikin (2009a) notes that there is a dearth of research on creativity and gifted students and the relationship this has to talent loss,

nurturing mathematical promise is directed at preserving human capital, it allows the community to grow new generations of creative mathematicians, scientists, and high-tech engineers who contribute to the further development of sciences, technologies, and various branches of mathematics…talent loss is a major challenge for every society. (p. 387)

Although we agree completely with Leikin’s thesis on the need for research with gifted individuals our goal is more in line with Silver’s (1997) view that creativity has a psychological aspect i.e. creativity is a disposition that is not restricted to a few gifted students, and as a result creativity should be introduced through-out instruction in mathematics,

persons creative in a domain appear to possess a creative disposition or orientation toward their activity in that domain. That is creative activity results from an inclination to think and behave creatively…this view suggests that creativity-enriched instruction might be appropriate for a broad range of students, and not merely a few exceptional individuals. (pp. 75–76)

Furthermore, we argue that accepting an educational culture in which a majority or significant minority of the population have no experience with the beauty and satisfaction of learning and discovering mathematics for themselves would result in a society with two tiers, in the lower tier are individuals who have access to ever more sophisticated technology yet less and less understanding of the basic principles upon which such technology is based. As eloquently stated by Koestler:

Modern man lives isolated in his artificial environment…his lack of comprehension of the forces…the principles which relate his gadgets to the forces of nature, to the universal order…his refusal to take an interest in the principles behind it. By being entirely dependent on science, yet closing his mind to it, he leads a life of an urban barbarian. (Koestler, 1964, p. 264)

Sriraman et al. (2011) points out that research on creativity within mathematics education with students who do not consider themselves gifted is essentially non-existent, “There is almost little or no literature related to the synthetic abilities of ‘ordinary’ individuals, except for literature that examines polymathy” (p. 120). The issue of polymathy or giftedness across domains will not be directly tackled but is inherent in the work we do following Koestler’s notion of the Triptych and his thesis that there are grey areas between the three domains of humor, science and
art-literacy. Specifically, we consider the role of humor and literacy endeavours in promoting affective aspects of students in their studies of mathematics. Our thesis is that when presenting mathematics to students from an underserved population or to any student with a negative attitude towards mathematics a creative and supportive learning environment must be established in order to transform the habitual negative affect expressed in the commonly heard phrases, “I hate math” or “I such at math” with a positive affect that allows for learning and even appreciation of the mathematical cognitive content. Furthermore, as Vrunda Prabhu was fond of stating, when an instructor begins to establish such a creative learning environment founded upon education research in creativity and cognition it will simultaneously engender creativity on the part of the students.

ROOTS OF CREATIVITY RESEARCH

Creativity plays a vital role in the full cycle of advanced mathematical thinking...Yet it is external to the theory of mathematics...Most mathematicians seem to be not interested in the analysis of their own thinking procedures...only a few explicitly attempt to describe ideas related to mathematical creativity. (Ervynck, 1991, p. 42)

The roots of contemporary research in mathematical creativity focus on the ‘creative person’ (Leikin & Pitta-Pantazi, 2013) and are often traced to the work of H. Poincaré, and J. Hadamard. As noted in Liljedahl (2013), “Hadamard’s treatment of the subject of invention at the crossroads of mathematics and psychology is an extensive exploration and extended argument for the existence of unconscious mental processes. To summarize, Hadamard took the ideas that Poincaré had posed and, borrowing a conceptual framework of Wallas turned them into a stage theory. This theory still stands as the most viable and reasonable description of the process of mathematical invention” (p. 254). Wallas was a Gestalt Psychology and the resulting stage theory of creativity within a problem-solving environment includes: initiation (preparation) incubation, illumination and verification. The first stage is characterized by, “an attempt to solve the problem by trolling through a repertoire of past experiences” (Liljedahl, 2011, p. 52).

A problem-solving situation in which no idea or solution is forthcoming may lead to the next stage of incubation, “one forces oneself consciously to work hard on a new problem or an idea. When no solution is forthcoming, the problem is put aside and one’s mind needs to relax to make the necessary connections” (Sriraman et al., 2011). The importance of the initial-preparation stage is because it, “creates the tension of unresolved effort that sets up the conditions necessary for the ensuing emotional release at the moment of illumination” (Liljedahl, 2013, p. 254). In the view of Poincaré and Hadamard putting aside conscious work when blocked allows the solver to “begin to work on it at an unconscious level” (Liljedahl, 2013, p. 254) during the incubation stage. This creates the possibility of illumination,
“the manifestation of a bridging that occurs between the unconscious mind and the conscious mind…a coming to the conscious mind of an idea or solution.” Colloquially this is often referred to as the ‘Aha’ experience (Liljedahl, 2013, p. 255).

Koestler studies the creative experience of many eminent mathematicians and scientists he comments that Poincaré believed in divine influence or unconscious intuition that during incubation selects out of countless combinations of patterns and thoughts only the relevant or beautiful ones i.e. “the aesthetic sensibility of the real creator” (Koestler, 1964, p. 163). The following quote from the musing of Poincaré is used by Koestler to demonstrate what he considers the two essential characteristics of creativity first, an affective component in which, “the self is experienced as being a part of a larger whole, a higher unit-which may be Nature, God, Mankind, Universal Order…an abstract idea…the participatory or self-transcending tendencies” (p. 52). The second component is the synthesis of two frames of references previously considered independent:

One evening, contrary to my custom I drank black coffee and could not sleep, Ideas rose in crowds; I felt them collide until pairs interlocked, so to speak, making a stable combination. By the next morning I had established the existence of a class of Fuchsian functions, those that come from hypergeometric series. I had only to write out the results, which took but a few hours…and I succeeded without difficulty in forming the series I have called theta-Fuchsian. Then, I turned my attention to the study of some arithmetical questions apparently without much success…disgusted with my failure I went to spend a few days at the seaside, and thought of something else. One morning, walking on the bluff, the idea came to me…with just the same characteristics of brevity, suddenness and immediate certainty, that the arithmetic transformation of indeterminate ternary quadratic forms were identical with those of non-Euclidean geometry. (pp. 115–116)

THE TRANSITION FROM GENIUS TO CLASSROOM

Liljedahl (2013) points out that implicit in this view of creativity as an original product is that the discovery can be, “assessed against other products within its field, by the members of that field, to determine if it is original and useful” (p. 255). Liljedahl (2013) comments that as a result of Koestler and other’s treatment of creativity is that, “…creative acts are viewed as rare mental feats which are produced by extraordinary individuals who use extraordinary thought processes” (p. 255) i.e. “… creative processes are the domain of genius and are present only as precursors of the creation of remarkably useful and universal novel products” (p. 256). We note that the sentiment that creative mathematics is tied to mystical geniuses has been linked to limiting creativity in the math class. For example, Silver’s rather striking comment about the ‘genius’ view of creativity and how it has limited research into creativity in mathematics education:
The genius view of creativity suggests both that creativity is not likely to be heavily influenced by instruction and that creative work is more a matter of occasional bursts of insight than the kind of steady progress towards completion which tends to be valued in school. Thus, there have been limited attempts to apply ideas derived from the study of creativity to the education of all students. (Silver, 1997, p. 75)

We note that any instructor of a remedial or first-year mathematics course frequented by non STEM (science, technology, engineering and mathematics) majors can probably relate to the frustration of having students equate their lack of mathematics fluency with a lack of God given talent and their strong conviction, that it is therefore not worth making a concerted effort.

Leikin and Pitta-Pantazi (2013) are even blunter than Silver in their critique of this so-called genius approach to creativity as they comment on how the perception of creativity has changed with the dawn of research into creativity in the classroom: “Initially creative ideas were considered to be generated mystically…subsequently, the mystical approach was replaced by a pragmatic approach which was mainly engaged in ways of developing creativity” (p. 160).

Although we completely agree with the thesis put forth by Silver, Liljedahl and Leikin and Pitta-Pantazi that assert creativity should be part of every student’s experience in mathematics, we note that Liljedahl’s comment that Koestler’s treatise on creativity lends itself to the genius approach obscures the fact that both Koestler and Liljedahl emphasise the affective component of the creative experience, regardless of whether the product is original to the mathematical community or only novel to the individual solver. “Minor, subjective bisociative processes do occur on all levels, and are the main vehicle of untutored learning” (Koestler, 1964, p. 658).

We also note that Koestler’s work provides a precise description of a mechanism that underlies the transformation between incubation and illumination; a mechanism that appears to be lacking in existing mathematical educational literature on creativity. This mechanism for the synthesis of planes of reference allows for a theoretical framework to study creativity within the social (classroom) situation as students create meaning of mathematics. That is when the individual discovers a product-result that is new to themselves but known to the instructor, tutor etc.

The transformation of creativity from an analysis of eminent mathematicians to the classroom i.e. the transformation from the creative person and creative process in the field of mathematics to the creative person, creative process and creative environment (in the classroom) has led to a multitude of definitions and approaches to studying creativity.

Definitions of Creativity in Mathematics Education Research

The significance of creativity in school mathematics is often minimized because it is not formally assessed on standardized tests, which are designed to
measure mathematical learning. The problem with relating to students’ work as ‘creative’ is rooted in the definition of creativity as a useful, novel, or unique product...Although according to the traditional view of creativity students’ work would not be considered as creative, the researchers agree that students’ discovery may still be considered creative if we examine the issue of creativity from a personal point of view, namely, whether the students’ discoveries were new for them. (Shriki, 2010, p. 162)

As noted by Shriki, “Eminent mathematicians such as Jacques Hadamard and George Pólya argued that the only difference between the work of a mathematician and that of a student is their degree” (p. 161). The integration of creative pedagogy within classroom mathematics for gifted or ordinary students of mathematics raises the issue of how does one define and then using this definition measure creativity. Sriraman et al. (2011) like Shriki propose that, “a differentiation be made between creativity at the professional and school levels” and that creativity at the school level should include:

1. The process that results in unusual (novel) and/or insightful solutions(s) to a given problem
2. The formulation of new questions and/or possibilities that allow an old problem to be regarded from a new angle (p. 120).

We are entirely in agreement with Sriramen’s intention to extend the range of creativity to include classroom mathematics. However, the practical and theoretical issue that arises is not whether there should be a definition for creativity in educational mathematics but the plurality of definitions. Mann (2006) notes that educational research included over 100 definitions of creativity he claims that, “the lack of an accepted definition for mathematical creativity has hindered research efforts” (p. 238). R. Leikin has done much work on definitions and assessment of creativity focused on gifted students. For example, Leikin (2009b) notes two formulations of creativity in mathematics educational research that have been used to assess an individual’s propensity for creativity. The first is the ability for convergent and divergent thinking due to J. Guilford. “Convergent thinking involves aiming for a single correct solution to a problem, whereas divergent thinking involves the creative generation of multiple answers to a problem or phenomena, and is described more frequently as flexible thinking.” Her review also notes the definition suggested E. Torrance i.e. the capacity of an individual for flexibility, fluency, novelty and elaboration.

Fluency refers to the continuity of ideas, flow of associations, and use of basic and universal knowledge. Flexibility is associated with changing ideas, approaching a problem in various ways, and producing a variety of solutions. Novelty is characterized by a unique way of thinking and unique products of a mental or artistic activity. Elaboration refers to the ability to describe, illuminate, and generalize ideas. (Leikin, 2009b, p. 129)
Silver (1997) did not like the focus on spontaneous illumination if it meant any diminishing of attention to the need for hard work and instruction, “creativity is closely related to deep flexible thinking in content domains; is often associated with long periods of work and reflection rather than rapid, exceptional insights; and is susceptible to instructional and experimental influences” (p. 75). Silver went on to propose that creativity is an essential component of problem solving and problem posing. We note that most of these definitions and the research based upon them are designed to separate gifted-creative students of mathematics from those who are not necessarily gifted. That is to distinguish between those students who are fluent-proficient and flexible problem solvers capable of understanding multiple representations that often diverge from the instructor presented approach. Silver (1997) admonishes educators to consider that a creativity-enriched instruction is appropriate for students not considered gifted and that instruction is an important component of establishing a creative learning environment, which we refer to as the attempt to democratizes creativity (Prabhu & Czarnocha, 2014). Shriki (2011) goes a step further stating that creativity should be part of all students learning experience:

It is widely agreed that mathematics students of all levels should be exposed to thinking creatively and flexibly about mathematical concepts and ideas. To that end, teachers must be able to design and implement learning environments that support the development of mathematical creativity. (p. 73)

It is important to point out here that both flexibility and fluency may not reinforce student creativity but diminish it. Koestler (1964) points out that that flexibility as a component of a “rigid and flexible variations on a theme” (p. 660) contributes to the formation of a habit, hence it may diminish originality and with it, creativity of an individual. Since “The creative act is…the act of liberation – the defeat of the habit by originality” (p. 96). Thus, acting out of a habit diminishes originality. The possible decrease of originality due to flexibility had been noted by Leikin et al. (2013) and by Kyung Hee Kim (2011). The theoretical issue that that arises, is whether Koestler’s definition can provide for a foundation to focus on the creative aspect of student originality, even with students who do not demonstrate fluency and whose flexibility and divergent thinking patterns are intermittent. In other words, how can creativity be integrated into a student’s attempt to give meaning to material presented in the classroom. A constructivist perspective would argue that all learning is essentially the self-discovery of what is known by others. For example, Sriramen et al. (2011) points out “The ability to create an object in mathematics is an example of mathematical creativity” (p. 121).

One of the tenets of this volume is that creativity is needed to reach all students of mathematics especially, those students who are resistant or do not consider themselves gifted in mathematics. Moreover, for sustained implementation of a creative learning environment teachers need to be involved in all phases of research; in our view the underlying deficiency in mathematics education is that researchers continue to believe that a holy grail can be found in a new pedagogy based upon a
learning theory that will be designed, implemented and assessed with input from teachers only at the implementation phase. This model has not worked in changing mathematics pedagogy from the traditional model of an active instructor lecturing to passive students to a creative learning environment because it cannot be sustained without direct input from teachers in design, implementation, assessment and a refinement cycle. Creativity and originality in the classroom must come from both the instructor and the students. Students must take responsibility for their own learning and instructors must take responsibility for researching that learning process to improve it while exploring theoretical aspects of their craft. In this situation the students will pick up the enthusiasm and motivation of their instructor and break their habits of learned failure through the creative originality. In turn their attempts to overcome their limitations at whatever level they are will inspire the instructor. In like manner educational research will benefit from the craft knowledge of instructors relating their work and findings to theories of learning and creativity. Another basic tenet of this work is that a sustained effort to implement a creative learning environment in the classroom requires a learning community of instructors, a support network of other like minded teacher-researchers to share the many joys, disappoints and the assessment of what works and does not work and to assist each other relate their work and findings to theoretical aspects of learning and creativity.

Question that arise and that will be addressed include; what is the nature of a creative learning environment, what is the relationship between creativity and conceptual or critical thinking? How does creativity enter into theories of learning? Certainly all students can be original and this original thought can diverge from what is presented in the classroom, students have intuition that comes into play in their effort to understand mathematics. The question of how to design, implement and assess such an environment using Koestler’s work underlies much of the material presented.

CREATIVITY AND KOESTLER

Koestler (1964) considers creativity as situated within a problem solving environment which he defines as, “bridging the gap between the initial situation and the target” (p. 649). The initial effort or preparation to bridge the gap occurs while, “keeping my eyes both on the target and on my own position” the search for a solution is characterized as, “searching for a matrix, a skill which will bridge the gap” (p. 651).

Bisociation

Koestler (1964) describes the main mechanism of creativity in terms of an analogy between two or more previously unrelated frames of reference: “I have coined the term bisociation in order to make a distinction between the routine skills of thinking on a single plane as it were, and the creative act, which … always operates on more than one plane” (Koestler, 1964, p. 36).
The terms matrix and code are defined broadly and used by Koestler with a great amount of flexibility. He writes, “I use the term matrix to denote any ability, habit, or skill, any pattern of ordered behaviour governed by a code or fixed rules” (p. 38). He uses the same definition later, substituting the phrase pattern of activity in place of pattern of ordered behaviour. The encompassing nature of these phrases allows one to include most processes used in the mathematics classroom, the caveat being that there must be some underlying order to the patterned activity. Indeed, as Koestler states, “all coherent thinking is equivalent to playing a game according to a set of rules” (p. 39). It follows that the term matrix can be applied to all coherent, logical or rule-based thought processes employed by an individual learning mathematics:

The matrix is the pattern before you, representing the ensemble of permissible moves. The code which governs the matrix…is the fixed invariable factor in a skill or habit, the matrix its variable aspect. The two words do not refer to different entities; they refer to different aspects of the same activity. (Koestler, 1964, p. 40)

Thus, for Koestler (1964), bisociation represents a “spontaneous flash of insight…which connects previously unconnected matrices of experience” (p. 45). That is a, “transfer of the train of thought from one matrix to another governed by a different logic or code” (p. 95). Examples of bisociation in Koestler’s work abound, and range from humour to some of the most significant scientific discoveries. He describes one such humorous illustration in a story about a student cutting and replacing the legs of a pompous science teacher’s chair. In this case, the matrices were the professor’s attitude of absolute authority and the law of gravity the science teacher was lecturing about, which the student understood well enough to apply in his prank. Bisociation is also used by Koestler to describe original inventions; for example, when Gutenberg fused together two matrices to invent the printing press, “the bisociation of the wine-press and seal, when added together, became the letter-press” (p. 123).

The focus of the Act of Creation Theory is on the bisociative leap of insight, that is, an Aha! moment, or a moment of understanding,—a phenomenon that can be observed amongst the general population, and hence emphasizes, for example, the creativity of all students in a classroom setting or if one prefers the joy of learning. In this sense Koestler’s theory can be seen as a foundation for understanding educational research on creativity for all students including those who do not necessarily consider themselves gifted in mathematics.

Incubation and Illumination

Max Plank the father of quantum theory wrote…that the pioneer scientist must have a vivid intuitive imagination for new ideas not generated by deduction, but by artistically creative imagination. (Koestler, 1964, p. 147)
The concept of the Incubation period embodies the belief that, “after attempting to solve a problem which needs wider knowledge and insight…a solution is more achievable if work on the problem is interrupted” (Sriraman et al., 2011, p. 123). It has been noted that there is a lack of research on the gestalt theory leading to a Eureka moment in the classroom:

The period of incubation eventually leads to an insight on the problem, to Eureka or the ‘Aha’ moment of illumination…Yet the value of this archaic Gestalt construct is ignored in the classroom. This implies that it is important that teachers encourage the gifted to engage in suitably challenging problems over a protracted time period thereby creating the opportunities for the discovery of an insight and to experience the euphoria of the “Aha” moment. (Juter & Sriraman, 2011, p. 48)

Koestler’s mechanism of bisociation could provide a useful tool for clarifying if not the process of incubation then instead the mechanism leading to illumination. Sriraman et al. (2011) note that the mechanism for incubation is poorly understood and advocates that a study of this mechanism would be beneficial for mathematics education. He reviews several theories of incubation and characterizes mechanisms underlying incubation as vague and complex. Liljedahl (2009) reviews mathematicians whose most original discoveries ‘Aha’ moments come to them in non-mathematical activities such as, “showering, walking, sleeping, talking, cooking, driving, eating, waking, and riding the subway” (p. 65). Moments of illumination were reported through intuition in states between consciousness awareness and unconscious sleep and through the use of imaginative visual thought with pictures. Eureka moments were reported during mathematical activities involving the review and questioning of previous work especially where they notice gaps in their understanding. Sriramen notes the importance of illumination is that it can lead to insightful or creative thinking. “One theoretical reason for studying incubation is because it is closely associated with insightful thinking… understanding the role of incubation period may also allow us to make use of it more efficiently to foster creativity in problem solving, classroom learning and working environments” (Sriramen et al., 2011, p. 125).

Liljedahl (2009) highlights the affective aspect of illumination among prominent mathematicians notes that:

The prevalence of anecdotal comments pertaining to this strong emotional response along with the uniform absence of any mathematical detail in these same anecdotes leads to a final conclusion that the AHA! Moment is primarily an affective experience. That is what sets it apart from other mathematical experiences is not the ideas, but the affective response to the appearance of the ideas. (p. 67)

He reinforces and elaborates on the connection between the cognitive and affective components of creativity in particular illumination:
That is, what sets the phenomenon of illumination apart from other mathematical experiences is the affective component of the experienced, and ONLY the affective component. This is not to say that illumination is not a cognitive experience, for clearly it is. After all, it is the arrival of an idea that, in part, defines the phenomenon...while the affective component of illumination is consequential to the differentiation of it from other mathematical experiences, the cognitive component is not. (p. 264)

In his work with pre-service teachers who were ‘resistant’ to mathematics he notes that illumination or the “unexpected presentation of a solution filled them with positive emotions, precipitating changes in beliefs and attitudes, and encoding the details of the experiences” (Liljedahl, 2013, p. 264).

One might question the hypothesis that the affective component of illumination is of equal value to the cognitive for example the affective response may not last if the insight is determined not be valid or the material one thought one learned cannot be recalled during an exam. Is there a sharp distinction as painted by Liljedhal or a gradual shading of distinction a matter of degrees of affect as well as cognition that mark learning and creative work in math?

DEMOCRATIZING CREATIVITY

Prabhu and Czarnocha decided to bring the idea of teaching research experiments and their interest in creativity in mathematics education to students who had intermittent if any formal education in mathematics in the rural villages of India. This fruitful effort required that children who had limited exposure to or training in mathematics begin to engage in reasoning leading to formal math concepts. To accomplish this Prabhu focused on ‘democratizing’ the research and theory of creative research within the Teaching Research framework to provide a support learning environment that fostered creativity and ownership of ideas for students both women and their children who had limited experience with formal mathematics.

Much of the research on creativity in mathematics education has been focused on gifted students who have the talent to become research orientated mathematicians, the effort to bring such research into underserved communities and ordinary students has received less attention. The following assessment of a contemporary research effort on creativity notes:

Missing is information on what initiatives are in place to develop and facilitate mathematical creativity in underserved and under-identified populations. This type of discussion would be informative to the field of gifted education and counter the criticism that field is not inclusive. (Chamberlin, 2013, p. 856)

Our hypothesis is that, in order to bring such research into underserved communities one needs to address the democratization of creativity i.e. the role of creativity in learning with ordinary even resistant students who do not consider themselves
gifted. Yet much of the assessment and the underlying definitions of creativity focus on gifted students. Leikin and Pitta-Pantazi (2013) review educational research on gifted students which focuses on the affective domain of giftedness, that is the effect upon creativity of ‘personality variables’ such as self-concept and anxiety as well as ‘personal-psychological attributes’ such as: self-regulated learning, self-evaluation, responsiveness to extrinsic rewards, mathematical inclination, self-promotion, and the ability to learn how to play the game as well as risk taking. They note that the literature is divided upon whether creativity is a subset of, or independent of such giftedness (p. 160).

If one considers that creativity is defined or strongly characterized by cognitive abilities such as: flexibility, fluency and originality and affective-personal properties such as motivation, self promotion, risk taking, the ability to be taught etc. then one is more likely to view creativity as a subset of gifted students or eminent mathematicians who display these qualities.

For Koestler (1964) the defining characteristics of creativity are both the affective self-transcendent ‘Aha’ moment and the cognitive synthesis of two previous independent matrices. These two criteria, one affective and one cognitive, both require a high degree of conscious attention what he refers to as the transition from habit to originality. In contrast, fluency for Koestler is related to what he refers to as an exercise in understanding, which like the cognitive theorist Anderson (1995) notion of ‘proceduralization’ all too often has an inverse relationship to conscious attention i.e. it can promote habit: “We may then, somewhat paradoxically, describe awareness as that experience which decreases and fades away with our increasing mastery of a skill” (Koestler, 1964, p. 155). In regards to flexibility, Koestler notes that, “some highly developed, semi-automatized skills have a great amount of flexibility – the results of years of hard training; but their practitioners are devoid of originality” (Koestler, 1964, p. 157). In our effort to bring creativity into the classroom environment with ‘ordinary’ students we, like Koestler, consider that attributes such as fluency and flexibility, which are used to identify gifted students, are some markers of creativity but do not define it. The act of creation, is the spontaneous leap of insight that connects frames, which are disconnect defines creativity and provides a framework for its facilitation. Thus, we have a theoretical foundation to consider creativity as a critical component of any self- learning process that contains both gifted and ordinary or even resistant students of mathematics.

The sentiment that the Eureka experience can be used and beneficial in the mathematics classroom during problem solving and learning with students in mathematics (not only gifted) is expressed by Sriramen et al. (2011):

understanding the role of incubation period may also allow us to make use of it more efficiently to foster creativity in problem solving, classroom learning, and working environment. Educators try to incorporate incubation periods in classroom activity in temporal pauses during classroom discourse or extended time periods for project related assignments...Incubation should
not be neglected in the classroom. Students should be encouraged to engage in challenging problems and experience this aspect of problem solving, till a flash of insight results in the ‘Eureka’ or ‘Aha’ moment and the solution is born...The benefits of incubation are completely evident. (Sriramen et al., 2011, p. 125)

The concept that creativity is a central component of learning is not new, “Indeed, there is a sense in which all reasoned thinking, all genuine acts of figuring out anything whatsoever, even something previously figured out, is a new making, a new series of creative acts” (Paul & Elder, 2008, p. 8). The question that arises then is what is the relationship between learning and creativity? Our analysis of this issue which will involve a review of the learning environment according to Piaget, Koestler, Vygotsky and the cognitive theorist Anderson with a focus on how creativity and creativity research integrates into what Koestler would refer to as progress in understanding and Piaget would refer to as accommodation i.e. how an individual builds structures or schema. Specifically we are interested the relationship between Koestler’s bisociative mechanism of creativity and Piaget’s mechanism of learning ‘reflective abstraction.’

KOESTLER: LEARNING ENVIRONMENT FOR CREATIVITY

Koestler (1964) notes the lack of student engagement in math and science education due to scripted learning pedagogy based upon modelling problems from textbooks followed by students reiterating the techniques presented:

The same inhuman-in fact anti-humanistic-trend pervades the climate in which science is taught, the classrooms and the textbooks. To derive please from the art of discovery, as from the other arts, the consumer-in this case the student- must be made to re-live, to some extent, the creative process. In other words, he must be induced, with some proper aid and guidance, to make some of the fundamental discoveries of science by himself, to experience in his own mind some of those flashes of insight which have lightened its path. (pp. 265–266)

Koestler would allow that bisociation or originality (in the sense of Sriraman) is the essence of untutored learning: “Minor, subjective bisociative processes do occur on all levels, and are the main vehicle of untutored learning” (p. 658). This statement demonstrates two important viewpoints of Koestler that: (1) bisociation is an essential mechanism in the learning process, and (2) the subjective learning environment must allow for and approximate the conditions of ‘untutored learning’. We believe, as did Koestler, that students cannot engage in mathematical problem-solving until they, in some sense, discover mathematics for themselves. This leads us to explore the mechanism of bisociation as foundational to learning mathematics.
For Koestler, bisociation explains the individual’s use of analogies in learning and discovery. The distinction between the Eureka moment of originality and routine thinking is in the degree of novelty or unexpectedness of the analogy used. He writes, “one of the basic mechanisms of the Eureka moment is the discovery of a hidden analogy; but hiddenness is a matter of degrees. How hidden is a hidden analogy” (p. 653).

CREATIVITY AND LEARNING: VON GLASERFELD

Von Glaserfeld’s position expressed here is a paraphrase of his work (Glaserfeld, 1998) and represent in our view a striking parallel to Koestler’s theory of creativity. This supports the hypothesis that, “learning itself may be seen as a creative process in which meaning is constructed by the learner” (Bodin et al., 2010, p. 144). In theorizing how an individual learns new knowledge, Von Glaserfeld postulates that the ability to search for patterns, regulations, groupings and rhythms are innate. This process is accomplished through schemes, which we take to be essentially the same nature as Koestler’s matrix. A scheme from Von Glaserfeld’s view contains three components first, the recognition by an individual of a problem situation second, the association of an activity with this situation and third, an anticipated result from applying this activity. When an individual is presented with a problem that fits a scheme, they recall the appropriate activity and the present situation is assimilated into the existing structure what Koestler refers to as an exercise in understanding. In contrast, progress in understanding or the Piaget’s equivalent of accommodation takes place when the existing situation requires modification of the schema or no schema can be found that applies. Creativity enters the learning process in this last situation. As Koestler would describe it, the solver runs through all available matrices or schemes that may apply, not finding any that fit. Then they go through the incubation period in which selective attention is applied to features or characteristics of the situation that were previously overlooked or not focused on. Von Glaserfeld employs the Piaget terms of perturbation and disequilibrium to describe this. At some point intuition, good fortune or hints from a tutor leads the solver to an analogy between some characteristic of this problem to a previously overlooked matrix or schema that is relevant. What Koestler refers to as the hidden analogy, Von Glaserfeld describes this creative moment as an analogy to a previous situation that allows one to construct a hypothetical rule or concept that explains or sheds new light on the current situation. This hypothetical rule or principle if sufficiently verified becomes the code of the new matrix or scheme a process referred to as accommodation. This creative leap based upon analogy requires the ability to recall previous situations reflect upon their relevancy and compare these previous schemes to the present situation. The simultaneous selective attention required to focus on and compare aspects of the present situation and previous relevant-analogous schemes is for Von Glaserfeld a conceptual step, a generalization. This generalization from a specific problem situation is creative in that the analogous rule is hypothetical that
is a hypothesis created by the solver to be verified. Koestler’s cognitive mechanism underlying bisociation, i.e. the discovery of the hidden analogy, though without the affective component, appears to be similar to Von Glaserfeld description of creativity. The synthesis of two matrices certainly allows for the abstraction of a rule from a hidden matrix to apply to a present situation that result in the development of a new code and this process is certainly creative as well as developmental in Koestler’s model. In this light Von Glaserfeld’s description of creativity presents a foundation one may use to situate bisociation within the context of learning theory based Piaget, which is the content of Chapter 4.1.

Pedagogy: Affect and Creativity

Throughout the collaboration of Prabhu, Czarnocha, Dias and Baker, the TR team collectively reflected upon the challenge of providing an appropriate learning environment for students in remedial classes of mathematics in the City University of New York (CUNY) system. Although these students have had exposure to mathematics in their secondary education they have failed mathematics placement exams and required a review before entry into college level mathematics (open admissions). In this situation their background in mathematics was frequently not the issue but instead the affective issues of lack of consistent motivation, their anxiety and at times apathy for mathematics. These students tend to view mathematics as a rule based experience and interpreted problem solving through an only-one-method-allowed frame in which the instructor as the authority was to provide to them (Woods et al., 2006). Prabhu conceived of the concept of the creative learning environment as methodology to motivate students and promote positive affect. The goal being to engage these students more fully in the learning process to get them to take ownership of their learning process, and to realize that their acceptance of failure is not helpful.

Shiriki’s (2010) claim that students at every level should be exposed to thinking creatively, and the attempt to democratize creativity, even for resistant students results in the need to deal with student affect. Yet research on affect is arguably less available than research on creativity: “Affective issues in mathematics teaching and learning have long been under-represented themes in research” (Cobb et al., 2011a, p. 41). This is perhaps due to the belief among mathematicians and educators that learning mathematics should exclusively focus on cognitive development:

Learning mathematics has been almost exclusively understood as a rational cognitive process of acquisition more or less along the lines the structure of mathematics makes available…Only when learning has failed, when it was difficult to understand why it failed due to some assumed misconception blocking the way, was it necessary and appropriate to think about the social/ emotional aspects of mathematics. (Seeger, 2011, p. 207)
In this work the relationship between affect and creativity is explored in two aspects. The first is the creative learning environment; the social setting that provides support in the classroom for students to feel secure and valued enough to try their best. The creative learning environment is founded upon the social contract or relationship between the instructor and student. The goal of the TR team is a collaborative effort to reflect upon, and analyze pedagogy: classroom discourse, curricula and methodology in order to improve learning in the classroom. Thus, the TR team by supporting one another’s effort to establish a creative learning environment in our classrooms brings about improvement of learning. The second aspect of affect and creativity studies is the relationship between affect and cognition during the creative ‘Aha’ moment of understanding. It has been noted that the creative experience of illumination has a transformative effect on student affect. For example, Sriraman et al. (2011) asserts that the experience of illumination can transform student affect “the ‘Aha’ experience has a helpful and strongly transformative effect on a student’s beliefs and attitudes towards mathematics and their capability to do mathematics” (p. 124). While Leikin’s understanding of illumination and other cognitive based experiences in learning by gifted students is eloquently stated, “the realization of intellectual potential by individuals improves their self-perception, creates a positive emotional background and causes satisfaction through the achievement of goals and by overcoming multiple challenges. The realization of potential determines to a great extent the future of the individual” (Leikin, 2009a, p. 386).

The hypotheses put forth by Leljehdal (2009) and (2011) that cognitive learning in general does not have the same affective component, as the experience of illumination is perhaps as controversial as it is tantalizing. The question that arises is whether the implementation of a creative learning environment that supports both the cognitive and affective aspects of illumination i.e. bisociation can transform students learned habits of failure to willingness to learn and ownership of their learning.

Prabhu’s work is profoundly shaped by her realization that the central problem of learning encountered with underserved populations in the mathematics classrooms is not so much cognitive as affective. It became clear to her that the main issue our students encounter is how to access their own intelligence and talent. Thus, Prabhu realized many students were limited more by their self-identification as failures in mathematics than by actual cognitive deficiencies. This self identity as a habitual failure was built upon painful memories of mathematics. The effort to democratize creativity is founded upon the belief that creativity within the learning process is important for remedial underserved mathematics students: “A remedial algebra student can exhibit creativity as often and as clearly as an advanced calculus student” (Applebaum & Saul, 2009, p. 276). Indeed creativity is not only important for learning it may well be a requirement for learning, as Goldin (2009) states, “An indifferent person cannot be a creator” (p. 184).

We now look at the question of what is the appropriate environment to promote creativity within cognitive development that promote positive affect:
“Affective pathways are likely to interplay importantly with inventive mathematical behaviour-curiosity or puzzlement, together with a sense of courageous adventure, may evoke invention, while feelings of excitement, wonder or dogged determination may contribute to …inventive acts” (Goldin, 2009, p. 187).

Creative Learning Environment: Social Contract

Sarrfay and Novotná (2013) relate Brousseau’s didactic contract to creativity within a mathematics classroom. They note that creativity when viewed as originality (Koestler) or inventiveness (Goldin, 2009), cannot be taught by an instructor: “Obviously a teacher can never teach the ability to invent new solutions (at least not directly); he/she can ask for it, encourage it, but cannot require it. This is one of the fundamental paradoxes of the whole didactical relationship” (p. 281). Sarrfay and Novotná (2013) characterize this didactic contract:

The didactic contract should not be understood as a real contract formulated and signed by the teacher and his/her students, but as a didactical relationship that is established between the teacher and the student show act as if such a contract existed. (p. 283)

Prabhu (Unit 2) interpreted the didactic contract as a handshake and a compromise, in which the student’s role was to live up to their potential, through active engagement in the class discourse. This involves changing the traditional lecture format with its implied didactic contract of a teacher-authority figure displaying the correct, absolute and unalterable truth of mathematical knowledge to passive students with a didactic contract in which students take responsibility for their learning and the reaching of their potential.

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1.3. UNDERSERVED STUDENTS AND CREATIVITY

INTRODUCTION

In this section we discuss the educational issues of immigrant students who frequently make up a sub-cultural often characterized by different language, race and a higher rate of poverty than the predominant culture. In addition to language these students frequently struggle with mathematics both cognitively and in the affect domain, alongside identity issues of what college means to the individual often being the first generation in their family to go to college. We also discuss the role of the instructor in promoting a creative learning environment that can transition students from habits of failure to discovery of their own excellence. A transition that is vital when working with students who come from poverty and frequently lack exposure to the dream and expectations of college.

HOSTOS COMMUNITY COLLEGE AND DEVELOPMENTAL MATHEMATICS

Eugenio María de Hostos Community College was established in the South Bronx an inner city college in the City University of New York (CUNY) system to meet the higher educational needs of people who historically have been excluded from higher education. That is to provide access to higher education leading to intellectual growth and socio-economic mobility. The college student profile indicates that the student body is approximately: 66% Female, 59% Hispanic, 23% Black, with 13% not known. Approximately 85% of entering student require remedial mathematics about 59% of students have HS degrees while 15% have G.E.D and 21% have HS degrees from another country. In general they are young adults, as well as returning students of variety of ages. It should be noted that while the students represent a variety of immigrant and native cultures, language and racial background, a common denominator of the Bronx is poverty. New York State Department of Health data for 2012 indicates that the poverty rate of the Bronx was about 31% compared to 16% in the state as a whole; with about 44% of the children below age 18 living in poverty compared to 23% for the rest of the state. The effect of poverty on completion of a High School (HS) degree is noted in (Kewal Ramani et al., 2011) as being even more important than race. Thus while Hispanic and blacks have almost twice the drop-out rates as white children; low income children are five times more likely to drop out of High School as high income children. Although the Bronx is increasing the home of charter schools the reality of children living in poverty and going to public schools mirrors a nationwide

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trend that has been exasperated by the economic collapse of the global markets in 2008 as reported in Washington Post article (Lyndsey Layton, May 28, 2015).

Tough (2015) notes that two trends in education in the U.S. the first is that many including minorities and immigrants make it to college but drop out all too often with debt and the second is that the wealth of the parent is a huge indicator of college graduation rate. Santiago and Stettne (June, 2013) report that Hispanic students indeed many minority students that do graduate from High School attend community college yet their graduation rate in community colleges is low and a real concern. There are many reasons for low graduation rates among minority, immigrant and low income students the high number of single mothers, students working to survive while trying to simultaneously earn money, court cases and homelessness are factors. Leonhardt et al. (2015) report on a study that documents the importance of the income level of the neighborhood you grow up in on determining ones graduation and success rate. An important academic factor for underserved students who often attend community colleges is the need for remediation in language and mathematics. Colleges place student in remedial mathematics when they are not proficient with H.S. algebra typically use placement exams. As pointed out by Lu (2013) while 60% of students entering community college require remedial courses many of these do not graduate: “Only 28% of two-year college students who took at least one developmental course earned a degree or certificate within 8.5 years, compared to 43% of non-remedial students.” The article goes on to say that states such as Florida, Texas, Connecticut and Colorado have made such courses optional. Indeed, as the recent article in the New York Times article “Is Algebra Necessary?” (Hacker, 2012) demonstrates, U.S. society is struggling with the issue of whether proficiency in algebra is indeed necessary. In addition to algebra placement exams cover pre-algebra or basic arithmetic skills and for those entering at this level of remedial (developmental) need the outlook is even less bright. A study conducted at the City University of New York (CUNY) in 2005 (Akst, 2005) revealed that among those students who start their mathematics developmental sequence with arithmetic only 37% pass the subsequent course in developmental algebra. From this it may be inferred that the central source of their difficulties is what is known as the cognitive gap between arithmetic and algebra (Filoy, Trojano, 1985; 1989). This gap is real factor or barrier in these students’ educational process and subsequent social mobility. The U.S is not alone in this issue a report from PISA 2012 (PISA in Focus, #36, February 2014) documents the difficulties that European Union as well as some Asian countries have with mathematics and the negative effects that a lower socio economic level have on the results of the test.

IMMIGRANT-MARGINALIZED STUDENT GLOBAL ISSUE

The issues of teaching mathematics to diverse students within an inner city are not unique to the South Bronx, USA. Civil et al. (2012) state that, “The underachievement of certain immigrant groups has become a globalized phenomenon in the modern world” (p. 267). Civil and Planas (2011) consider how language differences can
engender, “language policies that are directed to immigrant students which are politically charged” (p. 38). Yet despite governmental pressures they note that second language minority students across the globe “find ways to overcome political restraints” (p. 43). Alrǿ et al. (2009) note that while:

Cultural Diversity has always been present in Denmark—as much as any other society. However, the recent increase immigration of people from non-European, non-Western countries has exacerbated the discussion of cultural difference and multiculturalism, unfortunately the tone of such discussions has not necessarily been positive. (p. 14)

Civil (2010) reviews research on immigrant students in Europe and U.S. she quotes educators who characterize the state of minority students education as underserved: “African American and Latino students and poor students, consistently have less access to a wide range of resources for learning mathematics, including qualified teachers, advanced courses, safe and functional schools, textbooks and materials” (p. 1449). César and Favilli (2005) note that Italy, Spain and Portugal were emigrant countries until the late 70’s at which time African children began to immigrate to these countries with second language issues. They note the special role of teacher education in creating a, “social disposition that facilitates the existence of an inclusive society” (p. 1163). In writing about equality in mathematics education Esmonde (2009) argue that, “mathematics plays a central part in governmental and corporate decision making and these decisions disproportionately affect marginalized people—the very people who are less likely to have access to quality mathematics education: “Mathematical knowledge can therefore be an important component of struggles for social justice at home and abroad” (p. 1008). Tate (2005) furthers this argument stating that, “a disproportionate percentage of African American students are using curricula designed for low ability or non-college bound students” (p. 179).

William et al. (2009) look at immigrant students in a poor neighbourhood of U.K. they note the students who do decide to go to college are often the first and only in their family indeed their block to make that decision. For these students the decision to go to college is typically to either: a) become someone, b) the personal satisfaction of doing what interest them c) to obtain a vocation and hence satisfy the need of making money. For these students, as in many immigrant populations around the world, education is seen as a ticket out of poverty for themselves and often as single mothers for their new family. Difficult with remedial mathematics and entry into college math can be a rite of passage one, which they are often the first perhaps only members of their family to embark on.

MARGINALIZED STUDENTS AND POSITIVE SOCIAL INTERACTIONS

Social-cultural identity plays a major role in student acceptance or rejection of the mathematics classroom environment as well as the teacher acceptance or rejection of student behaviour,
Students who come to school already speaking the school Discourse and behaving as teachers expect are provided more opportunities to learn than their peers are. …Those who do not embody these privileged ideas are often positioned as being deficient and difficult to teach and so report that their teachers do not seem to care whether they learn mathematics. (Edmonde, 2009, p. 1018)

For this reason a major component of reform pedagogy that is often applied to marginalized communities focuses on small group discussion, and discovery learning in which student create meaning working together. Edmonde (2009) reviews literature about the social interaction specifically in small groups of minority and marginalized students. He notes specific social processes that literature supports help with learning and achievement that include, “asking question, discussing problem-solving strategies, observing someone else’s problem-solving strategies, teaching a peer, resolving a disagreement or conflict, and explain one’s thinking” (p. 1016). Other critical thinking skills that Edmonde lists as important for developing conceptual understanding include time spend reviewing and discussing, “multiple strategies and solution paths…explaining their thinking, asking specific questions and making connections” (p. 1028). Clearly the quality of the critical thinking and reasoning is the important factor. In contrast, Civil and Planas (2004) note that when a (middle school) teacher employed pedagogical activities that encouraged student discussion and participation in the mathematics classroom more popular students (those involved in athletics) were the centres of attention until the discussion became more academic when those who were recognized as gifted in mathematics became the focal point. They conclude that social exclusion of certain groups or types may occur during such pedagogy.

Creativity and Marginalized Students

In a review of literature on what stimulates creativity among marginalized students (Haley et al., 2006) suggests several characteristics: authenticity of the themes and tasks, novelty the sense that it was different, the role of a mentor and the freedom to explore. These authors study one program that prompted higher order thinking skills with disadvantaged students through the use of: computers, drama and Socratic thinking i.e. creative and critical-logical dialogue between teacher and pupil that required more than one word answers on the part of the student. The effects of this program included enhanced ability to: explain ideas; engage in conversation, problem-solving skills and increased confidence and motivation.

It was the firm belief of Prabhu that the pathway to reach underserved students was through their creativity; i.e. when students begin to enjoy thinking and reasoning within mathematics they transition from their habitual acceptance of failure to ownership of their learning and excellence of their potential. This creative process is both cognitive as students began to construct meaning of mathematics for
themselves and affective as they struggle with their image of themselves as someone who cannot do math or as a family member of immigrants and/or a racial group that traditionally has not gone to college. Common themes that run through literature on what works with serving the needs of these disadvantaged students include: quality student-teacher relationships, supportive administrations—schools or colleges and a sense of community for students eliminate within the context of high expectation of success. As a learning of community of teacher researchers in the South Bronx this work contains some of the results of our focus on the design of instruction and pedagogy within the mathematics classroom to encourage student creativity and engage them in their learning process.

Prabhu’s realization that underserved students in remedial mathematics must be reached through the affective domain simultaneous with the cognitive domain to complete a transition from habits of failure to excellence highlights the importance of the teachers role in promoting a creative learning environment as well as the relationship between affect and cognition.

The Importance of Affect on Student Cognition

As noted by Furinghetti and Morselli (2009): “The most important problem in research on affect in mathematics is the understanding of the interrelationship between affect and cognition” (p. 72). Studies on affect tend to employ at least the following three components beliefs, attitudes and emotions, “emotions are most intense/least stable, beliefs as most stable least intense and attitudes in between,” (Rosetta et al., 2006). Goldin (2009) attributes the lack of positive affect to be the main reason student drop out of challenging mathematics, “it was the affective dimension that in my view played the primary role” (p. 182). Goldin hypothesizes an internal representation system for an individual’s affect that is central to the relationship to problem solving and mathematics, “human affect serves as an internal representational system, encoding meanings, facilitating communication, and (like the cognitive representational system) contributing to or impeding mathematical power” (pp. 182–183). Goldin goes beyond the hypothesis that creating or illumination has a powerful effect upon an individual’s affect in asserting that “states of emotional feelings do not merely accompany cognition. Rather, the emotional feelings themselves have signification; that is they encode information or carry meanings” (2009, p. 186).

Polya’s first stage of problem solving ‘understanding the problem’ involves both reading the problem and relating or processing the information in a meaningful manner, successful solvers’ often reformulate the problem through, “…gestures, words, pictures, symbols sketches, examples and so on” (Furinghetti & Morselli, 2009, p. 72). Students who cannot associate the problem information with an appropriate matrix or scheme will not be able to assimilate the information into the scheme. Either because they do not recall a relevant matrix or because the scheme they attempt to employ is inappropriate. At this point student beliefs
about their own ability can already be a factor many students chose the simplest strategy available, the one done immediately before by the instructor or focus their attention on superficial problem information that is within their safe zone. They may feel frustrated when their choice of strategy doesn’t work out particularly if they perceive that other students can solve the problem but they cannot. “The frustration may evoke useful problem-solving heuristics leading to…such as trying a simpler, related problem. On other occasions…the feeling may rapidly give way to anxiety or despair, and evoke avoidance strategies” (DeBellis & Goldin, 2006, p. 133).

Thus, the affective experience of frustration during problem solving i.e. blind alleys because there is not available schema or matrix to assimilate the problem information can lead to searching for a new strategy. As noted by Schlöglmann (2009) humans when confronted with a situation that leaves negative affect tend to forget the experience we tend to suppress unpleasant memories and try to avoid such situation in the future. This statement about human nature and affect could help to explain the short memory span and avoidance behaviour of some students for mathematics problem-solving and mathematics classes in general.

The feeling of frustration during problem solving is what Goldin refers to as local affect “the rapidly-changing states of feeling that occur when participating in an activity…engaging in mathematics” (Goldin, 2009, p. 187). Strong emotions such as frustration or elations during problem solving can lead to global affect or the individual’s attitude towards mathematics i.e. math anxiety, the common reframes of “I hate math” or “I hate fractions.” Thus the period incubation when the solver cannot find an readily available matrix or scheme to assimilate the problem information can lead to the creative and transformative affective-illumination and cognitive-accommodation experience or to individuals who question their problem solving ability and a society that questions the importance of mathematics to real life. We now look at the question of what is the appropriate environment to promote creativity within cognitive development that promote positive affect.

TEACHER’S ROLE

“Traditional teaching methods involving demonstration and practice using closed problem with predetermined answers insufficiently prepare students in mathematics. Students leave school with adequate computational skills but lack the ability to apply these skills in meaningful ways. Teaching mathematics without providing for creativity denies all students, especially gifted and talented students, the opportunity to appreciate the beauty of mathematics” (Mann, 2006, p. 236).

We began our consideration of the role of the instructor with some reflections on constructivist’s pedagogy that underlies a shift from a classroom methodology based upon an active instructor lecturing to passive students to an environment with dynamic interaction and active student engagement As Sawyer (2004) notes, “The basic insight of constructivism is that learning is a creative improvisational process” (p. 14). Sarrazy and Novotná (2013) state the constructivist position on
the role of the teacher as bring the guiding consciousness that helps students bring dead math back to life: “To achieve this revival, teachers must create situations in which they can show students the use, the interest, and other aspects of the mathematics” (p. 281). We consider that Koestler would share this view that, the role of education is to bring the thrill of original discoveries to the student by a reconstruction of the discovery process within the class. Bodin et al. (2010) argue that, “the teachers role is the key to creative thinking in the classroom” (p. 145). These authors agree with the constructivist assertion of Von Glaserfeld that creativity is integral to the learning process. That is, an individual is being creative whenever he/she creates new meaning for themselves, “children are being creative… when they produce something new to themselves, as when they construct meaning for symbols, signs and operations, make sense of a mathematical problem, devise a way of solving it.” Yet as noted by these authors pre-service teachers find it, “difficult to be specific about encouraging and assessing creativity in mathematics lessons” (Bodin et al., 2010, p. 144).

Leikin et al. (2013) note that “teachers consider themselves as a key factor in developing mathematical creativity without holding themselves accountable for concurrently hindering creativity…they are more likely to blame the educational system...” (pp. 210–211). Thus, the vagueness or lack of a clear definition of creativity within the learning process for ordinary students and pressure to complete a syllabus focused on procedural skills appears to leave those who search for creativity within the classroom at a loss. Shriki (2010) states that, “Although most teachers would agree that it is important to develop students’ creativity the literature indicates that creativity is not normally not encourage at schools” (p. 161). More to the point Shriki conclude that, “it should be noted that I could not find specific recommendations or guide lines aimed at providing teachers with an assessment tool for evaluating students’ creativity” (p. 162). The lack of a specific commonly accepted definition of creativity and its role in learning and by this we include the affective component results in teachers who teach the way they were taught. Goldin (2009) describe the affective side of the mathematics classroom experience that most of us can relate to:

School mathematics often presents an affective context for mathematics that is not very conducive to trust or intimacy... sometime the teacher seemed to place the highest value on speed and accuracy of routine computations... with painful negative consequences for the children’s self-esteem as they were neither especially fast and accurate nor neat. Opportunities for inventiveness or creativity were relatively infrequent, since mathematics tended to be presented as systems of rules to be learned and procedures to be followed. (p. 182)

Sawyer (2004) describes a creative learning environment based upon socio-constructivist approach as one in which learning within the classroom is a process of co-construction where communication between members of the group provides meaning for new concepts. He describes creativity within the classroom in terms of
a transition from the mindset or didactic contract of, “The teaching as performance metaphor” to one an environment characterized as “improvisational performance” (p. 12). The importance of creative improvisation is that without such openness to student’s thoughts and ideas i.e. when the instructor scripts the discourse by following a text, “students cannot co-construct their own knowledge” (p. 14).

Norton and D’Ambrosio (2008) deliberate on the socio-constructivist viewpoints of Bruner and Vygotsky in which meaning is internalized through classroom discourse. They note that the teacher must be engaged in the assessment and refinement cycle in order to prepare lessons appropriate to support student creativity and learning i.e. “the teacher must continually establish meaning of the students’ language and actions so that the students’ actions guide the teacher…developing new hypotheses about students’; cognition while remaining open…in order to design tasks to provoke creative activity in the students” (p. 225).

Prabhu frequently notes the uniqueness in the Teaching Research cycle of the refinement stage where a bisociation occurs between two frames of reference: that one from the past cycle with the information about its instructional effectiveness with the one to envision for the next cycle, which aims at the elimination of the ineffective components in order to refine curriculum material classroom pedagogy and to reflect upon how the results gleamed reflect upon learning theories. We point out that this process necessitates the involvement of the teacher-researcher synthesizing the role of researcher with their craft experience to bring creativity into classroom and to reflect upon how what occurred in the classroom reflects upon learning theories. Thus, one goal is that an inspired instructor will bring motivation into the classroom and thus, students may experience the energy and enthusiasm for mathematics and the process of learning mathematics engendered by this process. In a teaching research experiment designed to support a creative learning environment the goal is for learning theory to assist teacher-researchers construct creative curriculum and pedagogy with assessment and refinement as part of the process. An equally important goal is to reflect upon the relationship of what occurs in the classroom to educational research on creativity and learning. Cobb et al. (2011) note that: “In my view, the most important contribution that theory can make to educational practice is to inform the process of making pedagogical and design decisions and judgements in particular cases” (p. 112). The guiding philosophy of teaching research to implement a creative learning environment is that teachers-researchers must simultaneously exist in both frames of reference (bisociation) otherwise creativity cannot be sustained, the transformation of students occurs one day at a time, brilliant insights and multi-year projects that provide for teachers to drink from the source of educational literature and theory is not sustainable. In the same article Cobb et al. (2011) states:

The increasing importance that we came to attribute to the teachers’ central mediating role is at odds with the way in which the teacher is backgrounded… the teacher’s initiatives and her responses to students are treated as ancillary to the focus on students’ learning. (p. 114)
A creative learning environment necessitates a creative teacher-researcher not as an ancillary focus but as part of the central focus on the dynamic of the classroom and as an integral part of the research team. For none other than the teacher can bring creativity into the classroom with a sustained effort through assessment and refinement.

The teaching research paradigm for promoting a creative learning environment is founded upon teacher involvement in researching his/her own craft, as does action research (Benke et al., 2008). The distinction being that in teaching research the curricula and pedagogy of the instructor can change with time and circumstance as long as they are committed to a methodology to inspire students engagement in their classroom, reflect in a meaningful way on their actions and results and relate this to educational research on the phenomena way of teaching and student learning.

The effort to encourage student engagement within the classroom requires openness to student ideas, input and suggestions even when they take the instructor off topic, the instructor has to encourage and prod student to engage from day one promoting a social contract in which participation is expected however introductory college classes are all too often characterized by large size and thus students who have negative or low affect towards mathematics tend towards non-participation. In secondary education negative student affect is often observed during the transition from the small class size of primary schools to the large impersonal size of mathematics classes in middle or junior high school. This transition frequently marks the end of student’s positive affect towards mathematics (Athanasiou & Philippou, 2009). Even in small classes with students who are motivated it is frequently the case that one or two students dominate the classroom dialogue while weaker student and those with negative self images wait passively for another student to answer or go to the blackboard. Thus, the expectation that all students will participate is an essential component of the social contract that must be established in day one and reinforced throughout the class in order for students to live up to their potential i.e. their excellence.

The relevant question, at the heart of the social-constructivist argument that the instructor’s role is to create an environment in which knowledge is co-constructed through classroom dialogue is eloquently posed by Norton and D’Ambrosio (2008) “how a teacher can know whether a student has meaningfully imitated an action or whether she has simply mechanically repeated observed action that she was trained to follow. When does a teacher’s assistance generate meaningless habits, and when does it promote development” (p. 221). Student learning especially in social situations can be illusive, active participation in a discussion and supplying steps for the instructor on one day is often followed by ‘the next day effect’ in which blank stares greet an instructor’s attempts to get students to solve the same question.

Norton and D’Ambrosio (2008) suggest that an essential tool in the instructors’ repertoire is scaffolding which they characterize as, “selecting an appropriate task, directing the students attention, holding important information in memory and offering encouragement” (Norton & D’Ambrosio, 2008, p. 223). This description
of scaffolding provides a foundation for the learning community that was formed by Prabhu, Czarnocha, Baker and Dias to study the effects of inquiry based learning during problem solving with remedial students in the South Bronx (CIRG CUNY Grant). Scaffolding and structuring of exercises can be seen in the work of Prabhu in the creative learning environment as well as the work on proportional reasoning with rate by Czarnocha, Dias and Baker. In particular scaffolding is important to support the foundational cognitive groundwork for bisociation as the instructor identifies the schemes-matrices that are to be synthesized and the concepts that are to be bisociated.

One goal of social constructivist learning is to support the creative process of students providing meaning to new concepts during problem solving. This goal focuses attention on how students understand and reformulate problem information. That is the process of “decoding” the original problem and the subsequent process in which the individual must “self represents a model” (Singer & Voica, 2013). Koestler (1964) points out that much of the reasoning that mathematicians accomplish during illumination is through visuals and only much later expressed in and with much effort expressed in language. Prabhu employed the Fraction Grid, a self-made visual display of the fractions in order to assist students give meaning to the measurement subconstruct of the fraction. While Professor Dias and Baker employed the two-sided number line as a visual in order to give meaning to proportional reasoning for students transitioning into elementary algebra.

A final component of the Teaching Research Methodology to support and sustain a Creative Learning Environment developed by Prabhu and Czarnocha was a team of collaborative teacher-researchers. The collaboration occurred within the class as instructors worked with colleagues within mathematics and counselors from student development to assist students with affective issues and self-regulated learning. However, it was the collaboration as a team of teacher-researchers that is most remarkable and distinctive. This team of collaborative researchers would be referred to as a community by Lin and Ponte (2008) that is, as a self-grouping through personal interest in educational research. “Communities are regarded as self-selecting, their members negotiating goals and tasks” (Krainer, 2008, p. 5).

The learning community (Unit 5) is not simply instructors lead by an educational researcher. A model we note would leave the instructors passive following the lead of the expert. The top-down model of an active instructor lecturing to passive students has been rejected by social constructivists as not being effective in producing a creative learning environment in the classroom. Following this logic the top down model of a researcher telling teachers how to create and assess pedagogical curriculum we believe needs to be reconsidered. That being said an excellent example of a top-down approach in which the researchers encouraged the teachers to be actively involved is the learning community described by Jaworski (2008). In this community the experts or didactians constantly met with and included the teachers in all phases: curricula and lesson plan development, implementation and assessment and of course refinement of the results.
In our teaching research community the instructors function as both researchers and teachers the creativity and motivation engendered within the community is then brought into the classroom in an effort to infect the students through like minded dialogue with one another. The learning community has much in common with action research (Benke et al., 2008) however the goal is not just to improve student fluency. The teacher researcher operates on two frames of reference simultaneously reflecting upon curricula and pedagogy to influence student affect, engagement and creativity but also reflecting upon the learning theories that underlie learning and creativity within the classroom experience.

The bissociative nature of the TR NYC model provides a well-equipped conceptual framework for understanding and analyzing both affective and cognitive aspects of student’s creativity in the learning process. As such it provides a foundation to motivate student transition from habits of failure to excellence. The community of the TR team of the Bronx supports the transition from habits of failure to success. The members share the successes, joys and disappoints as well as the insights into how theories of learning can be interpreted and translated into craft practice. The importance of a learning community is not only to inspire the instructor to try new approaches, assess and refine her methodology in an effort to inspire student; it also provides a support network when the frustrations that students and instructors feel at low performance and inevitable poor test results lead to discouragement. The axiom that an instructor cannot give up on low performing students first, because everyone else already has, and second, because if you do they will give up on themselves necessitates a community.

REFERENCES


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