

# Critical Mathematics Education: Past, Present and Future

Festschrift for Ole Skovsmose

Helle Alrø, Ole Ravn and Paola Valero (Eds.)



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*Festschrift for Ole Skovsmose*

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## PREFACE

Since the publication of the book *Towards a Philosophy of Critical Mathematics Education* in 1994, the seminal ideas of Ole Skovsmose about mathematics, its role in society and in mathematics education have become a source of inspiration for innumerable mathematics education researchers and teachers around the world, well beyond the protected borders of Denmark and Scandinavia. The issues that Ole Skovsmose had envisioned to be key aspects of mathematics education in a world concerned with democracy and technological development have come to occupy a more central place in the awareness of the broad international community of mathematics education researchers. His contribution to the field has emerged from a unique combination of philosophical thinking about mathematics in relation to society, educational thinking that emphasises the politics and sociology of educational processes in mathematics, and pedagogical thinking that connects the philosophical and educational reflections with issues of relevance for the micro-processes of teaching and learning of mathematics. Thus, Ole Skovsmose's work has influenced and spread alternative and significant theoretical ideas for understanding and interpreting mathematics and mathematics education in contemporary societies.

After his 65<sup>th</sup> birthday in July 2009, Ole Skovsmose retired formally from his Professor position at Aalborg University and has started a new stage in his life as an Emeritus Professor, as a writer and as an artist. As colleagues and friends who have enjoyed thinking with him about mathematics and mathematics education, we have prepared this Festschrift, which is a tribute to Ole's academic life. The book also provides an opportunity to see the state of the ideas of critical mathematics education, their link with other important and related trends in the field, and foremost, their future directions.

Quite a number of researchers in the field of critical mathematics education from five continents contributed to this Festschrift. They were asked to write chapters that, taking their own research as a starting point, would provide an account of the growth of a 'critical mathematics education' approach. Further, they were asked to show how they have engaged with some of the core ideas of critical mathematics education in their own work. The editors have encouraged free, creative writing that opens the possibility of weaving together theoretical and empirical discussions, as well as personal stories of meetings, talks or exchanges with Ole Skovsmose. The result is a collection of papers that represents a diversity of geographical locations where critical mathematics education has been recontextualised to the realities of classrooms and the discourses of mathematics and mathematics education research around the world.

As editors, we want to express our gratitude to the many people who contributed to the completion of the book. First, we want to thank each author. We are proud to present contributions from prominent international researchers within the field of mathematics education. Each one of them has committed their time in producing their papers and reviewing the contributions from other colleagues. Apart from the

## PREFACE

authors we owe a special acknowledgement to several people who have engaged in this festschrift. We would like to thank Alexandre Pais for his assistance with the correspondence with the authors and for taking care of many practical issues during the realisation of the book. As many of the authors are not native speakers of English all the chapters have gone through a language revision. We thank Sheilagh Kelly for her professional contribution to language proofing the book. All the chapters improved significantly with her many suggestions for making texts clearer. We also thank Patricia Perry for her work in setting up the manuscript for publication. Finally, we thank Annette Lorentsen and the Department of Education, Learning and Philosophy at Aalborg University for the generous economic support that helped us to complete the book successfully.

*Helle Alrø, Ole Ravn, and Paola Valero*  
Aalborg, December 2009.

## 1. INTER-VIEWING CRITICAL MATHEMATICS EDUCATION

*Inter-viewing* refers to a dialogical act of viewing together, through conversation, a topic of common interest.<sup>1</sup> The dialogical exploration breaks with the idea that an interview situation consists of questions posed by an interviewer and a series of answers given by an interviewee, as if answers and questions did not format each other and did not allow the interviewer and the interviewee to enter in a relationship of knowing together. In the case of this chapter, our inter-viewing consisted of an e-mail conversation, with a number of exchanges back and forth, between Helle, Ole Ravn and Paola, and Ole Skovsmose.

As editors of this collection, we wanted the readers to have a little insight into Ole Skovsmose's life and work from a different perspective than what academic writings allow. Therefore, we have been keen to get a firsthand impression from Ole himself, and invited him to examine, in conversation, the relationship between some of his main ideas, views and hopes for the future and his life. In our work together with Ole we have heard how many fantastic life stories interwove with his ideas: a childhood in Hjørring, Northern Denmark; the studies in Hjørring Teacher Training College and his first years as a school teacher and a handball trainer; a youth in Copenhagen with military service and studies of philosophy and mathematics; his years as a PhD student under the supervision of Bent Christiansen at the Royal Danish School of Educational Studies; the years of struggle to convince some of his mathematics education colleagues that a 'critical mathematics education' was a sensible and plausible idea to pursue; the division of time between his family and the writing of a PhD thesis; the first international conference he attended (ICME 5 in Adelaide, Australia) where his oral presentation proposal was rejected, but where he heard for the first time Ubiratan D'Ambrosio talk about ethnomathematics; the stay in Cambridge with Alan Bishop and the preparation of *Towards a Philosophy of Critical Mathematics Education*; the beginning of the collaboration with South African scholars and the finger caught in the door of the car, episode that opened the window to a completely new world; the stays and life in Brazil and the learning of Portuguese; the visits to many places in the world and the discovery of new possibilities for thinking about mathematics and education.

In what follows, Ole Skovsmose engages in conversation answering Helle, Ole Ravn and Paola's questions about the life experiences that have influenced his thinking. He also presents his understanding of critical mathematics education as a variety of concerns, and he talks about the challenges that he sees towards the future.

He finishes with a presentation of his meeting with the contributors to this book. From this framework we hope that some of Ole Skovsmose's ideas can be an interesting point of departure for reading the book.

*What are some key points in your life experience that have influenced your thinking?*

An experience of particular importance was my getting familiar with the South African context. In the Introduction of *Travelling through Education* I describe a particular episode. I tell how the very first day I arrived to South Africa in 1993, I came to enter a black township. This experience represents an important development in my perspective on Critical Mathematics Education (CME). I realised how much my thinking about CME was rooted in a German-Scandinavian, if not Danish tradition.

In a lecture that I gave a few days after my arrival I used the expression "education after apartheid". The title of one of Adorno's essays is "Education after Auschwitz". Adorno's point is that it is possible for an education to make a difference. Thus in the first lines of the essay Adorno states that it is an aim of any education to prevent a new Auschwitz from happening again. This can be read as a metaphor, but it can also be taken literally. The claim is that education can play a critical socio-political role that might help to prevent new forms of domination being created with disastrous impact. Adorno's formulation also includes a heavy critique of the education that, during the 1920s and 1930s, did not counteract, but apparently adjusted itself to the Nazi outlook.

In a similar way it made sense for me to state as an educational aim to try to ensure that a new apartheid should not happen again. I find that it makes sense to see education as playing an important socio-political role in the post-apartheid era. At the same time it became important for me to emphasise that there is no recipe to be associated to CME, and, most importantly, that one cannot assume that CME, with its European roots, can be transposed to a South African context, or to any other context for that matter. A critical approach is always in need of being reworked and changed. Thus CME becomes a most uncertain approach.

*What comes to your mind when thinking about your experiences in a Danish context and the formation of your ideas in CME?*

No particular event comes to mind. My way into *critical* mathematics education was a slow movement. My way into education and mathematics education, however, was straight. From when I was very young I wanted to become a teacher, and during the period 1963–1967 I was educated at Hjørring Teacher Training College. At that time I certainly did not think or hear anything about critical education. I liked teaching very much, and at that time the teaching practice, as part of the teacher training, was substantial: half a year as there was a general lack of teachers.

In 1967–1968 I joined the civil service at Hillerød, close to Copenhagen. I had decided to try to do further studies, and one evening per week I went to Copenhagen, in my grey uniform, to join a course in philosophy. Once I saw a pile

of chairs and tables blocking the entrance of a neighbouring lecture hall. I wondered what that could mean. That pile was my first meeting with the 1968 student revolution. Later in 1968 I started studying mathematics at Copenhagen University, but at the Department of Mathematics there was not a single trace of any 1968-movement. In 1971 I started studying philosophy, and I became familiar with Critical Theory and critical education in general as part of my philosophy studies. It was during this time that my imagination was first captured with speculations about the possibility of formulating a critical mathematics education.

During my university studies I taught different evening courses, and during the period 1972–1977 I was teaching at Copenhagen Teacher Training College. There, one of my colleagues was translating Paulo Freire’s work into Danish. Working at a teacher training college gave me much inspiration. I finished my university studies in 1975, and started to prepare a proposal for a PhD study. My idea was to combine constructivism with a critical approach. In 1977 my proposal, after several reformulations, was accepted at The Royal Danish School of Educational Studies<sup>2</sup>.

I do not feel that there is a particular experience, which somehow opened my way into critical mathematics education. It was not any direct route and one of the consequences of this gradual development is that there has never been a moment where I have assumed a particular already formulated theoretical position. Sure I have been inspired from a variety of sources, but I have never used pre-established formulations as ‘justification’ for my position.

*If one can say that there is a ‘critical mathematics education trend’, how do you see its roots or past, the present and the future?*

It might in fact be difficult to claim that a critical mathematics education *trend* exists. Thus, it is not easy to point out any defined CME research program. There are no well-defined educational practices, which are claimed to represent CME. There is no political agenda that represents such a trend. CME does not really work as a label, which is assumed by any larger group of educators. Nevertheless, I think it makes sense to talk about CME, and look at its past, present and future.

During the 1970’s critical education was often formulated with references to Critical Theory. However CE and CME developed with some tensions. If one assumes Habermas’ perspective, scientific domains are constituted through different knowledge-guiding interests. Thus, natural sciences, including mathematics, are based on a technical interest; the humanities on a hermeneutic interest; while the social sciences can be constituted through an interest in emancipation. The assumption of CE is that this interest should guide educational activities and research, and that it constitutes a proper CE.

This interpretation, however, makes any CME appear suspicious: How could an education within a domain guided by a technical interest come to serve any emancipatory interest? How could any instrumental reasoning be associated

with CE? Furthermore, interdisciplinarity was assumed important for any CE, so what to think of critical *mathematics* education? Much of the early writings in CE simply ignored any CME approach. CME had to develop through its own theoretical means, although still much inspired by other critical movements.

In the Scandinavian countries the work by Stieg Mellin-Olsen played an important role. In Germany many contributed to the formulation of CME. I can refer to Peter Damerow, Christine Keitel and Dieter Volk. If one wants to get an impression of what could be called the first European wave of CME, I suggest that one browses through Dieter Volk's book *Kritische Stichwörter zum Mathematikunterricht* from 1979. It presents a variety of topics that were addressed at that time.

In the USA, from the beginning of the 1980's, an important contribution to CME was presented through the work of Marilyn Frankenstein and Arthur Powell. They suggested the term "criticalmathematics" in one word. In this way they wanted to emphasise that with the concept was about a particular form of mathematics. This idea was not broadly recognised, and in fact I think their work includes a problematic assumption about the specificity of CME. However, Frankenstein and Powell's work was highly inspirational, and they developed CME with strong references to the work of Paulo Freire, and in relation to the particular context of the USA and the many class and race conflict there.

I see CME as represented by many different approaches in mathematics education, and certainly by many approaches that do not use the label of CME. For instance, the whole movement of mathematics education for social justice I see as an example of CME. The approach suggested by Renuka Vithal talking about "a pedagogy of conflict and dialogue" can also be an example. Different examples are found in many discussions of mathematical modelling. Much work in ethnomathematics shares the interest and concern of CME. And today several people in Brazil contribute explicitly to the development of CME.

Such different approaches represent concerns with respect to mathematics education that to me indicates a critical position. This brings me to consider the future.

*Would you please touch upon a few key points of development for the future of critical mathematics education? In which direction would you like to see the development of critical mathematics education happen?*

There are many issues to discuss related to the future, but let me first say something very general about the past.

The notion of *critique* changes with time. Critique as an epistemological category can be associated with Descartes' universal doubt. Descartes has been called the first modern philosopher, as he tried to bring philosophy beyond the reach of dogmatism. His universal doubt was a preparation for an unlimited application of rationality. Kant established critique as an overall preparation of basic categories for all rational activity, including any scientific enterprise. Marx took critique in a different direction. He tried not only to criticise economic theories, but also to criticise the very economic reality. In this sense, he turned

critique into both an epistemic and a political activity. However, different as they are —Descartes, Kant and Marx— they share the conviction that it is possible to establish critique on some kind of solid foundation.

Critical Theory demonstrates a first attempt at a solid foundation. Thus critique becomes established as a multifaceted activity. It could address very many issues: social, economic, and political. It could address literature, music, or art. It could address ways of living, cultural priorities, gender issues, daily-life issues, and education as well. Furthermore, critical theory provided different forms of critical activity, thus I am very impressed by Walter Benjamin's anarchistic patchwork approach, which he demonstrated in the Arcade project. Still, to a certain extent, Critical Theory maintains the assumption that there are some grand notions, like empowerment, that direct a critical activity. In other words, such a notion of critique pre-supposes that it can get some kind of foundation.

*And the future? What about the future?*

In my opinion the future development of critical mathematics education, of critical education and any critical activity for that matter, has to do with taking a step beyond the assumption that critique can be a guided activity. My point is that critique, instead, has to face uncertainty. In fact I find that critique can be seen as an expression of uncertainty. The challenge for a critical activity for the future is to face that there are no guidelines, no foundation, to be found for any critical activity.

One could relate this challenge to some of the points formulated by Derrida, who developed much of his philosophical insight during the same period as Habermas. Derrida was, like Benjamin, inspired by surrealism leading to a kind of methodological anarchism. One of Derrida's many steps —taken in all different directions— brought him beyond dualism in epistemic matters. This allowed him a more free style formulation of theoretical discussions, including his deconstruction of dualities, also the one between good and evil. In this way he anticipated that critique would loose orientation.

Foucault, directly inspired by Nietzsche, finds that knowledge and power not only interact but also are integrated. Thus, Nietzsche has presented knowledge as a human tool for survival. There is not any epistemic significance of knowledge that distinguishes it from other human secretions like saliva or perspiration. Foucault could go very long in emptying knowledge for any epistemic content, leaving power to dominate the terrain. But Foucault does not slide into absolute relativism. Thus he makes a distinction between “powerless truth” and “truthless power”, a distinction Nietzsche seems ready to dissolve.

In fact, if this distinction becomes dissolved, any critical activity will loose its meaning. Like Foucault, I operate with this distinction. I do not assume any absolute relativism. Anyway, the notion of critique has moved very far from any possible guidelines. To formulate and exemplify what this could mean is a real challenge for the future. This becomes a challenge for critical education in general, as well as for critical mathematics education. This is the reason why I try to link any critical activity to a profound uncertainty.

*Well, this was very general, but what does this mean for mathematics education?*

Mathematics education is a social, political, economic and cultural phenomenon. It is taking place in the world around us. It appears to have important functions. At least it is difficult to imagine society operating without the whole apparatus of mathematics education playing a significant role.

But it appears extremely difficult to identify what are the particular functions of mathematics education. And it is difficult as well to identify what could be the functions of alternative forms of mathematics education. There are a variety of suggestions for how to see the functions of mathematics education. Once, I suggested that one could relate these functions to the dominance of exercises with one and only one correct answer. Working with such exercises can hardly be claimed to support a deeper understanding of mathematical thinking. But maybe it develops what I have referred to as a prescription readiness. Such a readiness might appear useful from a business perspective, when trying to optimise production procedures. Well, naturally mathematics education (elaborated in a different way) could come to serve other functions. Thus a quite different discourse is developed around the notion of 'social justice'.

All such claims might be true in one context or in another, and from one perspective or from another. In this sense mathematics education is an open ended process that could come to function in many different ways depending, for instance, on the students' foreground. My point is that mathematics education does not have any essence, but could take all kinds of formats. And these formats have to be addressed with reference to the particular context—economic, political, social, and cultural—where mathematics education is taking place. One cannot expect CME to take any prototypical format.

Thus, one should not think of any form of curriculum for critical mathematics education. Such an education cannot be specified through certain approaches with general applicability. There are no universal guidelines that ensure any critical approach. What one could hope to do is to provide some educational possibilities. Thus, a landscape of investigation only represents a space of possibilities. This space might be different from the space provided by sequences of exercises. But, still, possibilities can be acted out in all possible directions. This also applies to those possibilities that have been developed with a critical approach in mind.

*It appears, then, that you have emptied critical mathematics education from both direction and content?*

To some extent this is true. At least I think that a critical activity needs to be separated from any assumption about the existence of critical guidelines and directions for what to do and where to go. Uncertainty becomes an integral part of a critical activity.

Grand notions like 'conscientização'—crucial for Freire's formulation of his education as a 'cultural action for freedom'—and 'empowerment'—which has played an important role in the educational discourses inspired by Habermas and

Critical Theory— have guided critical education. Recent educational movements have taken up the grandiose notion of social justice. For me, such notions bring us into a paradox.

On the one hand, we cannot assume that such notions are connected to any well-defined meaning. They might be metaphors for clusters of unconnected ideas. On the other hand, one is in need of some such notions in order to formulate any form of critical concerns. A critical activity cannot *only* represent uncertainty. It also represents concerns, and the formulation of concerns immediately brings us to the formulation of visions, aspirations, and hopes. So for me a critical approach relates to uncertainties, concerns and imaginations. And, as a consequence, a critical approach becomes paradoxical.

*Could you tell us a bit about the relationship with the authors of the chapters in the book?*

Let me try to put some comments in a form of inverse chronological order. I have met Alexandre Pais in meetings and conferences in Portugal, Brazil and Denmark. He brings powerful ideas to any discussion, and I am sure that he is going to contribute to the further development of CME. Sikunder Ali Baber came from Pakistan to Aalborg to complete his PhD studies. He brought my attention to the importance as well as the difficulty of proposing a CME within a Muslim context. Diana Stentoft became PhD student at the same time as Sikunder. She helped me to see the notion of foreground and identity in their complex dialectical relationship.

I have known Gelsa Knijnik for a long time, and I appreciate very much her critical approach to ethnomathematics. It is only recently, however, that I had the opportunity to visit her in Porto Alegre. Here I met Daiane Bocasanta. She was defending her Master thesis, which brings not only new issues but also a new literary style to mathematics education.

Núria Planas welcomed me during my stay in Barcelona, and we discussed issues of poverty and exclusion. This brought new dimensions to my notion of exclusion and students' foreground. I have known the work of Marta Civil for a long period, and I am always impressed by the way she combines the specificity of a study with a broad outlook.

In the Mathematics Education and Society conferences, which for me represented a continuation of the Political Dimensions of Mathematics Education conferences in the early 1990s, I have met many people who have given important contributions to the social turn in mathematics education. One is Keiko Yasukawa, with whom I have explored many details of mathematics in action.

In 1993 I visited Patras, where I was invited to present ideas of CME at the annual meeting of mathematics teachers in Greece. One person from the audience asked me a serious and complicated question, which I tried to answer the best I could. It was Anna Chronaki. Since then we have met many times around the world.

I was introduced to Renuka Vithal in the 6th International Congress on Mathematics Education in Quebec in 1992. This became the start of my involvement in the South African project. Soon after John Volmink (the South African director

of the project) Renuka and I managed to meet and set up the research proposal. John had to pass Heathrow Airport on transit, and so did Renuka although on a different route. So I took the plane to Heathrow, we met for a couple of hours in Terminal 2, and then I returned directly to Denmark. This became my absolute shortest trip to London, but it became extremely important and fruitful as we in fact did set up the project, which was to run for the next 10 years.

The Nordisk Matematikundervisning Seminar (NOMUS) conferences emerged from the collaboration between mathematics educators in Bergen and Aalborg. However, soon this collaboration got a broader platform and a special NOMUS spirit of collaboration and support was developed among participants. Marit Johnsen-Høines made a unique contribution to this development. I have learnt much from Marit about the importance of seeing language as a way of being.

I became attracted to the work of Paul Ernest, who demonstrated a strong interest in philosophic issues. We experienced a particular cooperation through one of the groups of mathematics students I once supervised at Aalborg University. Paul had sent me the preliminary manuscript of his book about social constructivism, and the group wanted to work with precisely this issue. The group was very enthusiastic communicating with Paul. They learnt a lot —and actually Paul acknowledged their comments—, and so did I.

In 1989 I visited Berlin twice, once before the wall fell, and once just a week after. At that time I was briefly introduced to Eva Jablonka, but it is first of all through her writings that I know Eva. I appreciate very much her analytical sharpness, and I find her contributions to the critical perspective on mathematics education through the focus on mathematical modelling to be very important.

In 1984 I joined in my first international conference; it was the International Congress in Mathematics Education in Adelaide. It was here Ubiratan D'Ambrosio in his very powerful plenary talk introduced the ethnomathematical perspective. However, it was only later at the ICME in 1988 in Budapest that I met Ubiratan personally. One evening we had a long conversation, where we talked about mathematics, philosophy, culture, Denmark and Brazil. Since then we have met many, many times.

The two people I have known for the longest period are Lena Lindenskov and Palle Rasmussen. Lena was a student at Roskilde University and involved in the initial attempts of formulating a critical approach to mathematics. I met Palle when I, in 1982, started working at Aalborg University, but I have known his name long before that. He belonged to that committed group of university students who, in the years after 1968, worked hard on introducing and developing CE in Denmark.

Let me now return to the editors of the present book, with whom I have had important co-writings. Ole Ravn became my PhD student in 1999. He opened a new space for reflections through his PhD study, and this space became very inspirational for our present co-writing on the philosophy of mathematics. I met Paola for the first time in 1996 in Sevilla. She asked if she could come to Denmark and become my PhD student. I tried to point out all the difficulties such a move would involve. However, she insisted, and in the most excellent way she proved that this was the right thing to do. We have had a very important co-operation. I met

## INTER-VIEWING CRITICAL MATEMATICS EDUCATION

Helle in 1988 when I, for a two-year period, moved from the Department of Mathematics to the Department of Communication at Aalborg University. Helle emphasised the importance of seeing learning in terms of communication, and this became crucial for our shared writing about dialogue and learning in mathematics education.

### NOTES

- <sup>1</sup> Our use of this term is inspired in the work of interview methodology by Steinar Kvale, particularly in his book *Inter-views: An Introduction to Qualitative Research Interviewing*, published in 1996 by Sage Publications.
- <sup>2</sup> The Royal Danish School of Educational Studies was an institution of higher education and professional development for teachers in Denmark. In year 2000 it was transformed into the Danish University of Education. Nowadays it has become a faculty of Aarhus University.

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## 2. CRITICAL DIALOGUE IN MATHEMATICS EDUCATION

The qualities of communication in mathematics education influence the qualities of learning mathematics. This hypothesis is investigated in *Dialogue and learning in mathematics education: Intention, reflection, critique* (Alrø & Skovsmose, 2002). From that study the authors claim that qualities of a dialogue such as inquiry, equality and risk-taking can be seen as equivalent to qualities of critical teaching and learning.

This chapter develops this claim in relation to teacher education. Our emphasis is on professional development and empowerment of student teachers in teacher education. What could it mean to teach and learn mathematics that aims at developing critical democratic competencies in this context —for the student teachers and for the pupils? What is the role of communication in this critical process of teaching and learning mathematics? A key concept we are going to develop is the notion of *innovation competence*, which means the competence of being able to challenge already existing practices and critically to think, plan and act in alternative ways. How can such a competence be stimulated in a learning conversation that focuses on possibilities rather than on limits? This question can be related not only to learning mathematics, but also to the professional development of teachers in general.

### AN EVALUATIVE APPROACH TO MATHEMATICS EDUCATION

Ole Skovsmose is wandering back and forth on stage looking distant and thoughtful —a characteristic manner of his when lecturing. As the audience we experience a *déjà vu*. We feel like we have experienced the situation before, and automatically we anticipate what is going to happen next. Thus, we expect to be invited into one of Ole's current critical discussions on mathematics education. Even more we expect to be invited into critical co-reflection on ideas that he is on his way to develop. We even know that our comments might be referred to in his further research.

Ole claims that classroom communication has some characteristics that we normally may not recognise because we are used to this way of talking —in classrooms. We automatically adapt ourselves to the predominant culture of the classroom. We automatically adapt ourselves to a discourse arguing that assessment and evaluations are made for the sake of the pupils' learning, for the sake of their positive development.

Ole pretends to be a pupil who is exposed to the teachers' evaluative voice.<sup>1</sup> He picks up utterances from the teacher that we all recognise as quite usual and perhaps trivial because we know the classroom. The teacher's voice appears strongly, though: "This is not good enough!" "You should know ..." "You fail..." "You have to do more exercises in order to pass." Pupil: "But I did my best..." Teacher: "Well, it is not good enough!"

Without further comments, Ole turns his speech: "*What if...*" he says holding his right hand under his cheek, "...*what if...* I would enter another culture that has some communicative characteristics in common with classrooms ...*what if...* I would enter a doctor's surgery for consultation? I am an adult, and the doctor examines me as his patient."

Ole presents a metaphorical example in which he lets the doctor adopt a teacher's voice. The doctor assesses Ole as his patient and expresses his judgement that the patient fails: "Your blood pressure is far too high. You cannot enter this consultation. You will have to achieve better in order to be accepted as a patient in my surgery. You have to do your tasks over (and over) again until you become well enough to enter a new consultation and have a new test. Sorry you fail. Standards must be maintained."<sup>2</sup>

This metaphorical story helps us observe the teacher's voice from the outside. Ole identifies some communicative aspects represented in the teacher's evaluative voice. He has chosen an example that challenges us to consider and reconsider: "Does one really have to be healthy in order to enter a doctor's consultation? Does one really have to be good at mathematics in order to get help from the mathematics teacher? What is the difference between these helping relationships?" Both teachers and doctors can articulate some utterances and they might be interpreted as fruitful (or not) in both contexts. However, many utterances would hardly be accepted as fruitful in any other contexts than schools. Ole's story inspired us to further investigations of teaching and learning discourses.

Much mathematics education has a predominant evaluating approach. What is right and wrong, what could have been done differently (i.e., better). In the classroom this can be observed in a very common pattern of communication that could be characterised as Guess What the Teacher Thinks.<sup>3</sup> The teacher asks a question, the student answers, and the teacher evaluates the answer.

Teacher: How much is  $17 + 17$ ?

Student: 34.

Teacher: Very good.

The teacher knows the answers to his questions beforehand, and the students are supposed to guess what the teacher is aiming at. In previous studies we have seen that the student participation in this kind of classroom communication is often limited to 'minimal response'.<sup>4</sup> This means that students show little activity and responsibility for their answers; they do not engage much in learning.

An evaluative approach is also predominant in teacher education practice (Johnsen-Høines & Lode, 2006). An evaluating perspective on what has been done can be very useful, e.g., in learning from mistakes, but it might also be an obstacle to empowerment of student teachers and renewal of educational practices. An evaluative perspective can restrict investigative approaches within mathematics education as well as within mathematics. Further, it might be difficult to develop an innovation competence and a critical educational thinking, if one has mainly participated in evaluating conversations pointing to what is wrong and what could have been done better.

#### LEARNING CONVERSATIONS IN MATHEMATICS PRACTICE (LCMP)

The story in the doctors' consultation makes a point of departure for and gives further support to our elaboration on the communicative practice that develops as part of the LCMP-project (Johnsen-Høines, 2009a). In the LCMP-project, the student teachers are challenged to investigate mathematical issues such as calculus, statistics, geometry, and linear algebra. They read about, practise and discuss investigative dialogical approaches to mathematics education, which imply ownership, empowerment and critique.<sup>5</sup> They are in charge of running seminars where they discuss articles and experiences from practise, and they write their theses regarding the interplay between theory and practice.<sup>6</sup> Such educational approaches aim at supporting empowerment of the students for their future positions as mathematics teachers.

The LCMP project is connected to a school development project, 'Real-life Education', that implies education and industry partnership. Learning mathematics is situated partly at workplaces partly at school. Student teachers participate in the developmental work as part of their practice teaching. This practice makes a basis for establishing a learning community between pupils, student teachers, teacher trainers and researchers.

A main focus of the LCMP project and thereby of this community is to inquire about communication in the contexts of teaching and learning mathematics. The project aims to provide greater insight into ways of studying learning as it takes place *in* the activities and *between* the activities; and how learning processes can be understood as ongoing and referential learning processes. Thus, we do not regard learning only as situated in different practices. Rather we study learning processes by inquiring how the participants make and use references for further learning in and between different contexts. Communication is seen as an analytical tool, in order to get insight in such processes, and as a didactical tool in order to support such referential and ongoing learning processes. We try to develop subject based reflective dialogues.

#### SUBJECT BASED REFLECTIVE DIALOGUE

The notion *subject-based reflective dialogue* can challenge the evaluative approach to teaching and learning mathematics in teacher education and in school practices. A subject-based reflective dialogue is an educative approach that aims to explore

how the situation might generate discussions for further development; hence, it has a future-oriented perspective. Instead of evaluating the qualities of a certain course, the subject-based reflective dialogue departs from activities that have taken place in order to discuss potential didactical implications and possibilities of the activity. A subject-based reflective dialogue is generated in the practice teaching situation, released from the evaluative aspects and developed as a subject-based interest; as a foundation for subject-based reflections. This challenging approach implies an investigative and dialogical approach and aims at developing critical competencies through educational dialogue (Johnsen-Høines, 2009b). For the student teachers as well as for the teacher trainers and researchers this implies that they become able to enter unfamiliar communication practices as regards teacher training. They will have to learn how to enter a future-oriented educational dialogue.

In Alrø and Skovsmose (2002) a dialogue is interpreted in terms of inquiry co-operation including qualities of inquiry, risk-taking and equality. First, a dialogue is seen as a conversation of *inquiry*, which aims at developing new insights and learning. In this sense a dialogue implies an open and curious attitude towards the subject and towards the understandings of both self and others. It implies a willingness to question things and to examine what is new and different but also what is considered knowledge already acquired.

A dialogue takes place in the space between what is already known and what one might come to know. A dialogue has no answers to questions beforehand. Answers may emerge through a collaborative process of inquiry with the purpose of getting to know. This unpredictability means that a dialogue also includes *risk-taking*. It is not possible to foresee which kind of challenges and experience will occur during the inquiry. A dialogue can bring about new fruitful insights, but it might also bring about unpleasant challenges (Skovsmose & Saljö, 2008). The purpose of a dialogue is that one will come to experience things in new and different ways that open up the possibility of change.

Further, a dialogue is based on *equality*. However, this does not necessarily mean symmetry or agreement. Diversity in knowledge, experience and interest can be seen as a challenge in dialogues. Thus, it is a dialogic competence to be able to contain diversity and to develop new ideas from diversity. Equality in dialogue means equality in the interpersonal relationship including mutual acceptance and respect. Entering a dialogue cannot be forced upon anybody, and there is no competition in the sense of winning and losing.

While studying dialogue in mathematics education Alrø and Skovsmose have identified a series of dialogic acts namely getting in contact, locating, identifying, advocating, thinking aloud, reformulating, challenging and evaluating. These acts of communication between teacher and students and among students in the classroom can occur in different clusters and combination, and they are seen as indicators of dialogic teaching and learning taking place (p. 128 f.).

As noted in the introduction the inquiring qualities of dialogue, (making an inquiry, risk-taking and equality), correspond with the qualities of learning mathematics that refer to critical mathematics education as described by Skovsmose (1994). “Thus, a critical theory of learning comes to contain dialogue as a

defining concept.” (Alrø & Skovsmose, 2002, p. 135). Critical mathematics education aims at mathemacy. Mathemacy encompasses not only mathematical knowledge and competencies to apply mathematics in everyday and professional situations, but also judgement and critique of mathematical ideas and what is going to be learned. The reflective component is crucial for the competence of mathemacy (Skovsmose, 2005). Skovsmose (1994) talks about ‘Mündigkeit’<sup>7</sup> as an educational goal and defining element of mathemacy. This political goal of mathematics education correlates with Mellin–Olsen’s (1989, p. 26) discussions about the functionality of knowledge, about being able to use mathematics critically and independently for developing society and one’s own life. The use of knowledge is to be seen as political, i.e., mathematical knowledge can be used in order to influence society.

According to this framework mathematics education has an important political dimension that requires development of certain competencies on the part of the upcoming generation of teachers. It is what we refer to as *empowerment* and *innovation competence* of the student teachers. Thus, we see it as an important educational goal in teacher education that the student teachers learn mathematics, theories about teaching and learning mathematics, teaching methods and the content of the curricula. One aspect is to learn about these issues, the next step would be to practice teaching accordingly. However, an innovation competence would also demand that they learn how to reflect on and challenge existing practices of mathematics education and to offer alternative ways of critical thinking, planning and acting. Again, this is a discussion about the functionality of knowledge. We find it relevant to regard Mündigkeit or empowerment as an educational goal and defining element of innovation competence. A subject-based reflective dialogue should be seen as a way to gain this competence.

The LCMP project has a focus on developing subject-based reflective dialogues and empowerment of the students in order for them to gain innovation competence. In what follows we want to discuss what this could mean in a specific situation of the initiative ‘Real-life Education’. We refer to a first meeting between representatives from industry, student teachers, teacher trainers and researchers as an empirical case. The purpose of this meeting is to plan the participation of a group of 9th grade pupils in the project. We track the conversation as it develops and try to identify dialogic elements of critical mathematics education.

#### INNOVATION COMPETENCE: A CASE STUDY

We attend the very first meeting involving a school and a company. We are a total of 14 participants all together: manager, student teachers, teacher trainers, and researchers.<sup>8</sup> The meeting is opened by the company manager with a presentation of localities, personnel and production. The company supplies the oil industry with a variety of off-shore supplies, from huge mechanical components for installations, chemicals, radio active substances to more trivial support for daily household. The presentation gives information about the company and the overview works as a frame for seeking educational approaches as well. The partnership is in its infancy,

and most of the participants have not experienced this kind of co-operation previously. Nor had the company communicated with schools prior to this meeting. We recognise that we cannot distinguish any 'experts' in this room.

The conversation develops between the manager and a teacher trainer participant searching for various options and limitations regarding the involvement of the pupils in the project. The managers' proposals for pupil activities are formulated like "we should look for something concrete and palpable for the pupils and students to enter"; "For instance they could measure some pipes."<sup>9</sup> The teacher trainer confirms the manager's ideas by adding: "Yes, we will have to think a little mathematics into it, but it cannot be too simple". These cautious formulations could be due to the fact that the participants do not know each other and want to make a start for establishing a fruitful relationship. They are getting in contact. Perhaps they do not want to provoke each other by giving the impression of higher expectations than necessary. However, these comments from responsible adults do not reveal any faith in the mathematical competencies and creativeness of 15 year olds. They move further on by discussing how to organise the co-operation and focus on different sorts of limits due to security concerns. It appears that the teachers seem to accept that the pupils will visit the factory for one day only during the autumn. We recognise some kind of anxiety among the student teachers. They turn towards each other and towards the researchers; with no sound their faces and eyes seem to question the situation: "Just one visit cannot be what we have been envisioned?" The teacher trainers confirm that they do not think that it is possible to realise more than one visit. It would be difficult for the company to handle the group of pupils, and it would be difficult for the school to organise.

The student teachers have not explicitly participated in this encounter, but then suddenly Torunn<sup>10</sup> enters the conversation in a curious tone of voice: "What if we could manage... not to take all the children to the company at the same time... but organise for groups to go. They could investigate one topic each... and then come back and tell the fellow pupils what they have found out? They would probably not have a total overview, but at least they could be sort of experts on some issues. Other groups could investigate other activities and connect to different issues? We are many adults involved." Torunn has offered an alternative approach to the pupils' participation in the project. They could each focus on different issues. This suggestion is formulated tentatively as a hypothetical question '*What if we could...*' She is trying to locate and identify pedagogical possibilities. The tutor teacher seems to appreciate the idea, but instead of making an inquiry into it she focuses on the practical limitations that she sees: "That sounds like a good idea, but how would we handle it? The different groups would work on different aspects of mathematics and on different parts of the curriculum. How could we, as teachers, obtain an overview and handle the variations?" Despite this resistance, Torunn still wants to advocate her idea concerning a broader content that should be worked on, and she continues in an inquiring mode regarding the reservations formulated by the teacher: "What if the whole class could continue working on the ideas that each expert group brings back? Couldn't these issues be seen as common content? Perhaps they would even create more engagement in the learning activities, because

they are going to learn from peers and not from teachers?" Torunn stresses the importance of engagement in the learning situation, an issue that has not been mentioned earlier. She suggests a new model for teaching and learning, which indicates an *innovation competence*. The manager and the teacher trainers seem to think of learning mathematics in a traditional way. The question for them, using the factory as a context, seems to be what kind of mathematical exercises can be made that are not too difficult for the pupils to solve. Torunn, however, is able to create a vision of possibilities within the experimental situation that challenge already existing ways of working with mathematics education in the classroom. This challenge includes new ways of planning and new ways of organizing for learning. This challenge puts new demands on pupils' activities and allows for them not only to learn mathematics, but also to learn about and to critically reflect upon how mathematics is used for certain purposes in a real life context.

Another student teacher named Ketil seems to elaborate on this idea when he says: "As I understand it, you store huge amounts of chemicals at the base. I understand that some of them are dangerous, and that you even handle nuclear substances. My question is: Can this be utilised? Can we find a way that the pupils become interested; can they do some investigations, be inspired, and learn more about chemicals, chemical reactions and handling of chemicals?" The manager does not know but she is willing to contact the engineer who is responsible for chemicals. "We have to be careful. We have strong rules due to security", she repeats. Ketil moves further: "What if the pupils could make some trivial experiments outside?... Or what if they could visit the laboratory?... We have to present something that they will become curious about. I suppose this could be possible for some of the pupils, for one or two groups?" Ketil refers to Torunn's idea. "Your focus should be on mathematics, shouldn't it?" the teacher trainer asks. "Yes, but in this field mathematics is implied too, and some of us studied science last year". Ketil responds, "I would like to facilitate the students' subject oriented curiosity".

The student teacher, Ketil, suggests a critical approach for pupils' activity in the company. They could "make experiments", "visit the laboratory" and "become curious". In other words, they might create handling of chemicals as a landscape of investigation (Skovsmose, 2001). The manager naturally makes some reservations about rules and security, but she does not totally reject the idea. Obviously, the teacher trainer does not clearly connect such kinds of activities to learning mathematics, but Ketil still advocates his idea: "Mathematics is implied, too". Ketil's suggestion not only shows an interest for the pupils' to learn mathematics and how mathematics is applied. He also envisions how the pupils might engage in critical activities of how mathematics is used within the field of handling chemicals.

The student teachers' suggestions are supported by one of the researchers who adds: "What if the student teachers and the pupils could participate in an inquiring dialogue with factory professionals in order to examine things that have not yet been investigated sufficiently? I know that this is not easily done. But that would be the ultimate situation." Ida, a third student teacher, turns around and addresses

the manager: "Could it be... that the pupils could present some results from their work? Here, at the company? Is it possible that they could present results that might be interesting for you?" People are nodding; everybody supports this idea. The manager says that it would be nice to organise for presentation and for instance also for exhibition. "But", Ida adds, "I got the idea on the basis of asking: 'What is in it for you?' Could the company benefit from this co-operation?" The manager admits that she has not thought about such possibilities. "This is really new for us, and we have to develop the practice and discuss such things on our way. I see that you have an important issue, but I must admit that to me it is quite an unfamiliar way of thinking." "Perhaps this would have some impact on the processes, to what extent the pupils involve themselves and take ownership," Ida adds. Ida's suggestion is a radical one in many ways. It changes the perspective from 'the company is available for studying mathematics in real life' to 'pupils might be able to produce insight that could be of interest for the company'. Maybe the company could benefit from the co-operation and not just the other way around. Further, such a perspective might influence the students' involvement and ownership of their work. And ownership is a very important issue in learning processes, as well as in learning mathematics (see for instance Mellin-Olsen, 1987). We interpret Ida's contribution as an example of innovation competence that she is able to bring to the fore in pursuing an inquiring, risk-taking, equal examination of the possibilities of 'Real-life Education' with this company.

A new partnership is going to be established and the participants make their positions. In the meeting presented above we can identify different perspectives on this project. The factory has agreed to the partnership, and takes a first step to inform the other partners about the work place. This establishes a scene in which searching for educational possibilities can be done. The manager stresses that she has no competency within education, and that the company has no experience in participating in this kind of co-operation. She is presenting the company as an available context for 'Real-life Education' rather than taking the stand as a collaborator. The information about the industrial activities is well structured and her tone of voice is friendly, honest and reliable. When discussing suggestions put forward by the students, she is friendly listening, she often refers to restrictions and rules, but she is willing to find out what ideas might be possible to realize. (The reserved attitude of hers might for example be related to discussions at the workplace about security and what this partnership could mean for the daily work.) Thus, we think we can identify an *informative and reserved manager voice*.

The teacher trainer's contribution can be seen in correspondence with this. However, she is searching for possibilities by referring to the curriculum. What mathematics should be learnt, and how could mathematics be applied to workplace activities. She also refers to the curriculum of the students: "Your focus should be on mathematics, shouldn't it?" We can interpret the teacher trainer's voice as an *evaluative voice*. If the utterance was interpreted authoritarian in this context the conversation could have been limited and the activities restricted. However, by Ketil's response, an investigation into the function of mathematics in the interdisciplinary field is suggested and initiated.

The students' (and the researcher's) contributions can be seen in opposition to the *informative*, *reserved* and *evaluative* approaches. The students are silent listeners in the first part of the meeting. As researchers we observe a change when 'one visit only during the autumn' is articulated. Their quiet, polite, listening, mode changes when Torunn formulates a hypothetical what-if-question implying a suggestion for all of us to consider. She is addressing the problem: 'one visit only during the autumn' by challenging the participants to investigate other possibilities. Although this is met by reserved, evaluative comments, Torunn still articulates her inquiring approach. Ketil and Ida also address and partly confront the informative, reserved and evaluative approaches that were established initially. They are met by evaluative or moderating comments, but they return by advocating and elaborating on their ideas. Even though the students' contributions can be seen as three individual actions (they elaborate on three different issues) a referential line becomes obvious. Each contribution represents an attempt to investigate possibilities for establishing alternative learning environments. They all refer to issues like learning from each other, curiosity, engagement, empowerment and reflections of mathematics in use. We can identify their approach as an *innovative voice* with a predisposition for a *subject-based reflective dialogue*.

The development of the conversation can be described as interplay between conflicting approaches. The innovative approach was sharpened through the process as a challenge to the informative, reserved and evaluative approaches, but still all of these different approaches were present in the room. Maybe the *informative*, *reserved* and *evaluative* approaches articulated a need for challenging. Maybe they provoked the students. Maybe it contrasted an approach that the students had developed before they entered the meeting. Maybe these approaches challenged their expectations and their need to articulate themselves. Maybe the fact that the researchers and student teacher educators were present had a supportive function of the students' approach; also by the common references they had from previous discussions at the university college. Maybe the incident can be understood as an ongoing and referential process situated in different practices.

To creatively question existing practices and critically search for, suggest, discuss, try out and evaluate new ideas is a vital characteristic of innovative competence. The critical and reflective movements between conflicting perspectives become fundamental. The approaches identified above can be seen as conflicting, in opposition to one another. And they might be present at the same time, in the same context. During the meeting in the factory different approaches were observed and described as articulated by different (groups of) participants. However, it is important to have in mind that different approaches might be (and often are) articulated by the same person. The conflict represented by different approaches is to be seen as interpersonal as well as intrapersonal. Thus, conflicting approaches can exist *in* and *between* people; in and between contexts. Conflicting approaches mean challenging approaches, but their presence does not necessarily mean destruction or oppression. Conflict can be constructive if participants do not

remain locked into particular perspectives, but are willing to investigate alternatives. We consider that to cope constructively with conflicting perspectives is an important part of an innovation competence.

The student teachers at the meeting showed such an attitude in their contributions. They managed to go into a dialogue in the sense of making an inquiry, taking a risk and maintaining equality although being the less experienced among the participants. Through their inquiring attitude they managed to maintain equality. The inquiring attitude could be heard in their tone of voice and seen in their use of dialogic acts such as: locating, identifying, advocating and challenging. They took the potential risk of presenting a challenging perspective, not as an ultimatum, but as a possible new way of planning for the pupils to learn mathematics that stimulated everybody in the room to reflect on alternatives.

They did not engage in personal controversies, they kept the didactical focus in their invitation to mutual investigation of hypothetical possibilities. This field of investigation is the field of imagining and not knowing (yet). It is the field of learning and change. The students showed an interest in and a willingness to go into this field of potential insecurity. In this way such a mode of inquiry includes risk-taking. It takes guts to stay in the field of conflict, unpredictability, and uncertainty.

Thus, the dialogic qualities of inquiry, risk-taking and maintaining equality are also defining qualities of innovation competence. Innovation competence in mathematics education can be seen as a defining element of Ole Skovsmose's notion of mathemacy and one that we advocate should form part of teacher education that aims for mathemacy.

## NOTES

<sup>1</sup> The notion 'voice' refers to Johnsen-Høines' (2004) elaboration on the theoretical framework of M. M. Bakhtin, where voices are identified as personal voices but also as utterances that carry the 'history of traditions'.

<sup>2</sup> Ole's story is cited from our memory.

<sup>3</sup> See Young (1992, p. 106 f.) and Alrø & Skovsmose (2002).

<sup>4</sup> See Lemke (1990) and Alrø (1995, 1996).

<sup>5</sup> We refer to Mellin-Olsen's (1989) concepts related to production of culture, empowering and control of knowledge; Skovsmose's (1994, 2001 and 2005) perspectives on critical mathematics education and landscapes of investigation; dialogical perspectives on learning as developed by Alrø & Skovsmose (2002) and by Lode & Johnsen-Høines (2007); and to the development of learning community in mathematics as developed by Jaworski, Fuglestad, Bjuland, Breiteig, Goodchild & Grevholm (2007) as support for the learning community we are trying to realise.

<sup>6</sup> The study of mathematics at the university college is one full year of study (60 ects), they deliver two theses (each of them are 15 ects). The students have 30 ects of mathematics as compulsory background and will have 90 ects when they have finished the study described here.

<sup>7</sup> Mündigkeit (German) corresponds with empowerment and 'myndighet/myndighed' and 'myndiggjøring/myndiggjørelse' (Norwegian/Danish).

<sup>8</sup> The two of us are the present researchers.

<sup>9</sup> The quotations in this section are extracts from our field notes during the meeting. Subsequently, the student teachers have confirmed and commented on our description.

<sup>10</sup> Torunn, Ketil and Ida are pseudonyms for the present student teachers.

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