In an increasingly complex world the natural human inclination is to oversimplify issues and problems to make them seem more comprehensible and less threatening. This tendency usually generates forms of dogmatism that diminish our ability to think creatively and to develop worthy talents. Fortunately, complexity theory is giving us ways to make sense of intricate, evolving phenomena. This book represents a broad, interdisciplinary application of complexity theory to a wide variety of phenomena in general education, STEM education, learner diversity and special education, social-emotional development, organizational leadership, urban planning, and the history of philosophy. The contributors provide nuanced analyses of the structures and dynamics of complex adaptive systems in these academic and professional fields.
A Critique of Creativity and Complexity
ADVANCES IN CREATIVITY AND GIFTEDNESS
Volume 7

Advances in Creativity and Gifted Education (ADVA) is the first internationally established book series that focuses exclusively on the constructs of creativity and giftedness as pertaining to the psychology, philosophy, pedagogy and ecology of talent development across the milieus of family, school, institutions and society. ADVA strives to synthesize both domain specific and domain general efforts at developing creativity, giftedness and talent. The books in the series are international in scope and include the efforts of researchers, clinicians and practitioners across the globe.

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A Critique of Creativity and Complexity

Deconstructing Clichés

Edited by

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In the last decade the words “creativity” and “complexity” have proliferated in the learning sciences, the organisational sciences, economics, education and the humanities to the extent that they almost sound clichéd. School systems, policy documents, funding bodies and scientific foundations repeatedly invoke creativity and complexity in vision/mission statements and calls for research proposals. This has generated some benefits and problems. One of the benefits is the invigoration of interdisciplinary work pertaining to complex phenomena that cannot be understood from within the borders of insular disciplines. A drawback is the occasional misinterpretation of the nature and nuances of complex, adaptive systems. In this book, researchers and theorists from various disciplines critically examine disciplinary boundaries in relation to the terms “creativity” and “complexity” with the goal of moving beyond clichéd uses of these constructs. The book also includes chapters that apply concepts from complexity theory and creativity in a practical sense.
# TABLE OF CONTENTS

## Section 1: Introduction

Creative Emergence, Order, and Chaos: Grappling with the Complexity of Complexity Theory  
*Don Ambrose*  
3

## Section 2: Complexity in STEM Processes and Structures

Learning: Creation or Re-creation? From Constructivism to the Theory of Didactical Situations  
*Jarmila Novotna & Bernard Sarrazy*  
19

Investigating Mathematical Creativity in Elementary School Through the Lens of Complexity Theory  
*Esther Levenson*  
35

On the Edge of Chaos: Robots in the Classroom  
*Steve V. Coxon*  
53

## Section 3: Interdisciplinary Perspectives on Creative Complexity

The Ubiquity of the Chaos-Order Continuum: Insights From Diverse Academic Disciplines  
*Don Ambrose*  
67

Organisational Leadership for Creativity: Thriving at the Edge  
*Elizabeth Watson*  
87

Complex Regenerative Creativity  
*Marna Hauk*  
97

Pareto Optimum Efficiency Between Chaos and Order When Seeking Consensus in Urban Planning  
*Todd Juhasz*  
123

Subjectivity, Objectivity, and the Edge of Chaos  
*Peter E. Pruim*  
143
# TABLE OF CONTENTS

## Section 4: Creative Educational Frameworks and Initiatives

- **Seeking Chaotic Order: The Classroom as a Complex Adaptive System**  
  *Don Ambrose*  
  159

- **Expansive Notions of Coherence and Complexity in Education**  
  *Bryant Griffith & Kim Skinner*  
  185

- **Complexity, Patterns, and Creativity**  
  *Jeffrey W. Bloom*  
  199

- **A Shakespeare Festival Midwives Complexity**  
  *Kathleen M. Pierce*  
  215

- **The Anthropology of Twice Exceptionality: Is Today’s Disability Yesterday’s (or Tomorrow’s) Evolutionary Advantage? A Case Study with ADD/ADHD**  
  *Jack Trammell*  
  227

- **Mentoring the Pupal: Professional Induction Along the Chaos-Order Continuum**  
  *Kathleen M. Pierce*  
  239

## Section 5: Social-Emotional Dynamics as Complex and Potentially Creative

- **Helping Students Respond Creatively to a Complex World**  
  *Michelle E. Jordan & Reuben R. McDaniel, Jr.*  
  249

- **Toward the Pattern Models of Creativity: Chaos, Complexity, Creativity**  
  *Krystyna C. Laycraft*  
  269

- **Emotions, Complexity, and Intelligence**  
  *Ann Gazzard*  
  291

## Contributors

301

## Subject Index

309
SECTION 1

INTRODUCTION
CREATIVE EMERGENCE, ORDER, AND CHAOS: GRAPPLING WITH THE COMPLEXITY OF COMPLEXITY THEORY

Complexity theory encompasses promising, interdisciplinary attempts to understand the complex dynamics of exquisitely interconnected, dynamically evolving systems. In today's increasingly complex, turbulent world, excessively simplistic, reductive approaches to theory development, research, and practical application increasingly come up short when applied to complex problems. Fortunately, complexity theory can provide helpful correction, overriding the dogmatism that ensues from shortsighted, superficial explanations of nettlesome phenomena. Nevertheless, given its intricacy, attempts to understand and apply complexity theory also can fall prey to dogmatic misconceptions. The chapters in this volume represent insightful attempts to correct some of these misconceptions while finding ways to apply complexity theory to problems and opportunities in transdisciplinary work, general education, STEM education, learner diversity, social-emotional development, organisational leadership, urban planning, and the history of philosophy. More opportunities for creative thought and action in these domains arise from the analyses.

THE DUAL-EDGED SWORD OF SIMPLISTIC REDUCTIONISM

There is growing recognition that reductive treatments of complex phenomena have enabled considerable progress, especially in the natural sciences, while also leading us into dead ends. For example, in a sweeping, interdisciplinary investigation of complexity, leading thinkers from a wide variety of fields recently grappled with the tension between the need to simplify phenomena and the need to recognise and embrace complexity. The editors of the volume explained:

The spectacular progress in particle and atomic physics, for example, comes from neglecting the complexity of materials and focusing on their relatively simple components. Similarly, the amazing advances in cosmology mostly ignore the complications of galactic structure and treat the universe in a simplified, averaged-out, approximation. Such simplified treatments, though they have carried us far, sooner or later confront the stark reality that many everyday phenomena are formidably complex and cannot be captured by traditional reductionist approaches. (Lineweaver, Davies, & Ruse, 2013, p. 3)
These warnings about excessive reductionism came from a group dominated by natural scientists including leading thinkers from astrophysics, biology, evolutionary paleobiology, cosmology, physics, astronomy, mechanical engineering, and the philosophy of science, among others.

Similar cautions arise in other disciplines. For example, economics is extremely influential in our everyday lives because it underpins the workings of our financial system, and of globalised capitalism more generally. However, there have been vigorous criticisms of the rational actor model that dominates standard, neoclassical economic theory (see Ambrose, 2012a, 2012b, 2012c; Konow & Earley, 2007; Madrick, 2011; Marglin, 2008; Schlefer, 2012; Stiglitz, 2010, 2012; Stiglitz, Sen, & Fitoussi, 2010). The excessively sanitised, hyper-reductive model portrays humans as highly rational beings who make solely self-interested decisions based on perfect information sets. The model works well as a driver for empirical and theoretical work in economics but it doesn’t map onto the world very well because the typical human injects considerable irrationality into his or her decision-making, is not entirely self-interested (unless he or she is a psychopath), and rarely has access to anything near a complete set of information for complex decisions. Arguably, the inadequacies of this oversimplified theoretical model contributed strongly to the 2008 economic collapse and to other serious, high-impact economic distortions.

Behaviorism, which dominated psychology in the mid-20th century, represents another example of temporarily productive but excessively sanitised, reductive theory. The behaviourist framework exiled the nettlesome complexities of the mind (anything that could not be measured with precision) to confinement within a metaphorical, cranial black box in attempts to mimic the hypothetico-deductive precision of empiricism in the natural sciences. The paradigm generated progress in psychology but eventually led theorists and researchers into increasingly barren territory. This led to its replacement by cognitive science—an energetic but still flawed new paradigm that was open to more diverse investigative methodologies and more authentic theoretical portrayals of the brain-mind system (see Ambrose, 2003, 2009a).

In yet another example, Bleakley (2010) made the case that effective medical education requires more tolerance of the ambiguity that arises from the complex, dynamic biological and technical systems medical practitioners repeatedly confront in their work, and that this tolerance can arise from capitalising on team learning. The distributed cognition that can arise from ambiguity embracing teamwork can enable medical professionals to diagnose and treat more effectively; however, the dominant model of medical education works against understanding of complex, adaptive medical systems because medicine is ideologically grounded in notions of excessive individualism and the acquisition of discrete knowledge elements.

TRAPPED WITHIN METAPHORS

These rigid theoretical frameworks arise from dogmatic entrapment within one of several root-metaphorical world views. The prominent philosopher Stephen Pepper
CREATIVE EMERGENCE, ORDER, AND CHAOS

(1942) analysed deep-level influences on human thought and action and categorised these influences into world hypotheses, which included mechanism, organicism, contextualism, and formism. As scholars later used these frameworks for analyses of phenomena in various disciplines, the world hypotheses became known as world views (see Ambrose, 1996, 1998a, 1998b, 2000, 2009c, 2012b, 2012d; Cohen & Ambrose, 1993; Dombrowski, Ambrose, Clinton, & Kamphaus, 2007; Gillespie, 1992; Heshusius, 1989; Overton, 1984; Terry, 1995). Each world view is rooted in a metaphor that implicitly shapes thought and action. Each root metaphor structures the development of philosophical, theoretical, methodological, and practical tenets that guide the work of academics and professionals. All of this occurs at very deep, implicit levels and thinkers rarely are aware that their minds are trapped firmly, even dogmatically, within a metaphor.

As with peeling away the layers of an onion, we can peel away layers of implicit conceptual influence to get down to the root metaphor that simultaneously makes us somewhat effective as theorists, methodologists, or practitioners, but also somewhat ineffective because the metaphorical entrapment prevents perception of other options. For example, mid-20th century teachers who excessively used reward and punishment to manipulate their students’ actions may not have realised that they were guided by the advice of psychologists whose thoughts were dominated by behaviourist theory. Many if not most of those psychologists did not realise that the behaviourist theory shaping their work was rooted in the positivist research paradigm. Many philosophers of science who promoted positivism likely did not realise that their philosophical framework was rooted in the mechanistic world view.

This lack of awareness that our thought is rooted down through multiple levels of analysis illustrates one of the strongest reasons for the ubiquity of dogmatism in human thought and action. Dogmatic idea frameworks force us to think more superficially, narrowly, and in more shortsighted ways than we should (for more on dogmatism see Ambrose, 2009b; Ambrose, Sternberg, & Sriraman, 2012; Ambrose & Sternberg, 2012). Table 1 shows the four world views, their root metaphors, the conceptual tenets that emerge from the metaphors, and examples of influences each world view has exerted in academia.

While a very simple system such as a simple machine can be investigated effectively through the lens of a single world view, complex systems nested within complex, multi-layered contexts are far too intricate for us to understand through a single conceptual lens, hence, we see the dogmatic folly of excessive adherence to the rational actor model in neoclassical economic theory, or the behaviourist model of mind in mid-20th century psychology. Pepper (1942) metaphorically illustrated the need for navigation through multiple world views:

Post-rational eclecticism is simply the recognition of equal or nearly equal adequacy of a number of world theories and a recommendation to not fall into the dogmatism of neglecting any one of them. . . . Four good lights cast fewer shadows than one. (p. 342)
Given the increasing recognition of the intricate complexity in complex adaptive systems, these four good lights are needed now more than ever before. Complexity theorists have revealed a wide array of baffling phenomena that show up as patterns in exquisitely complex systems (see Anteneodo & da Luz, 2010; Bleakley, 2010; Boedecker, Obst, Lizier, Mayer, & Asada, 2012; Chen, 2010; Fontdevila, Opazo, & White, 2011; Gershenson, 2012; Kelso, 1995; Lizier, 2012; Mazzocchi, 2012; Miller & Page, 2007; Morowitz, 2004; Schneider & Somers, 2006; Watts, 1999). For example, the innumerable elements of a complex, adaptive system can spontaneously self-organise into intricate, beautiful, and evolutionarily advantageous patterns. The dynamic tension between frustrating chaos and stultifying order can give rise to productive complexity. Also, intriguing behavioural and structural similarities can be seen in very diverse complex systems.

Understanding complex, adaptive systems brings to mind the old Sufi parable of the blind men and the elephant. Similar to the blind men in the fable, an investigator

### Table 1. Root-metaphorical world views as alternative conceptual frameworks for investigation of complex phenomena.

<table>
<thead>
<tr>
<th>World View</th>
<th>Root Metaphor</th>
<th>Basic Tenets (what the world view emphasises)</th>
<th>Examples of Influence in Academia</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mechanism</td>
<td>Machine</td>
<td>Reduction of the whole to its component parts; precision; detail; linear causality; objectivity</td>
<td>Psychologists reducing intelligence to a precisely measurable IQ score</td>
</tr>
<tr>
<td>Organicism</td>
<td>Organism developing through stages toward a particular end</td>
<td>Coherence and totality of systems (the whole transcending its parts); integrative connections; long-term development</td>
<td>Interdisciplinary work (integrating knowledge across disciplines); much theorising about child development</td>
</tr>
<tr>
<td>Contextualism</td>
<td>Ongoing event within its context</td>
<td>Contextual influences; unpredictable emergence of novelty</td>
<td>Cognitive scientists studying the context-embedded mind (contextual influences on thought patterns)</td>
</tr>
<tr>
<td>Formism</td>
<td>Ubiquitous similarity (e.g., Plato’s ideal forms)</td>
<td>Search for patterns of similarity in diverse phenomena</td>
<td>Complexity theorists studying patterns of similarity in the dynamics of complex adaptive systems such as human brains, national economic systems, fractal mathematics</td>
</tr>
</tbody>
</table>

Given the increasing recognition of the intricate complexity in complex adaptive systems, these four good lights are needed now more than ever before. Complexity theorists have revealed a wide array of baffling phenomena that show up as patterns in exquisitely complex systems (see Anteneodo & da Luz, 2010; Bleakley, 2010; Boedecker, Obst, Lizier, Mayer, & Asada, 2012; Chen, 2010; Fontdevila, Opazo, & White, 2011; Gershenson, 2012; Kelso, 1995; Lizier, 2012; Mazzocchi, 2012; Miller & Page, 2007; Morowitz, 2004; Schneider & Somers, 2006; Watts, 1999). For example, the innumerable elements of a complex, adaptive system can spontaneously self-organise into intricate, beautiful, and evolutionarily advantageous patterns. The dynamic tension between frustrating chaos and stultifying order can give rise to productive complexity. Also, intriguing behavioural and structural similarities can be seen in very diverse complex systems.

Understanding complex, adaptive systems brings to mind the old Sufi parable of the blind men and the elephant. Similar to the blind men in the fable, an investigator
employing the conceptual lens of a single world view might grasp a seemingly crystal-clear glimpse of a portion of the behaviour and evolving structure of a complex adaptive system, such as a creative individual, but could never hope to understand it in its entirety. For example, a mechanistic neuroscientist can clarify the electrochemical communication processes within a neural network within the brain of a creative person but will have great difficulty perceiving the ways in which those neural networks are influenced by subtle changes in other biological subsystems within the body, or by minor shifts in the environmental context that influence the person who owns that brain. A contextual mind theorist would have a better chance to understand environmental influences but would lack the precision and clarity provided by the mechanistic researcher who reveals insights about the electrochemical processes. Moreover, the long-term developmental perspective provided by an organicist developmental psychologist who looks at creativity as an integrative, lifetime process instead of an instantaneous light bulb moment of inspiration (e.g. Gruber, 1989) also is necessary to understand the creative work of the individual in its totality. The more perspectives from diverse disciplines that can be brought together and synthesised, the better, although such synthesising admittedly is a daunting challenge.

While it likely is impossible for a group of theorists and researchers to gain anything near complete understanding of complexity and creativity, an ambitious group can make some progress toward that goal. Our collaborators in this project recognise the intricacies involved in wrestling with the nuances of complex adaptive systems. The composition of our investigative team reflects this recognition. We include theorists, researchers, and professionals from diverse disciplines. Our collective expertise encompasses dimensions of gifted education, creative studies, educational philosophy, mathematics and the sciences, English literature, the history of philosophy, urban planning, and interdisciplinary work. Consequently, the contributors to this volume have shed some illumination on complex creativity by employing Pepper’s (1942) four good lights.

AN OVERVIEW OF THE CONTENTS IN THE VOLUME

The first section of the book applies various constructs from complexity theory to teaching and learning in mathematics and the sciences. In recent years policymakers, citizens, and educators have paid considerable attention to the need for, and enhancement of, STEM expertise. Unfortunately, shortsighted educational reform initiatives preempt the development of complex understanding and higher-order thinking throughout the K-16 curriculum, including in the STEM disciplines (Berliner, 2006, 2012; Ravitch, 2010, 2013). Our contributors in this section suggest some ways to reinvigorate STEM complexity.

Bernard Sarrazy and Jamilla Novotna show how complexity theory can be employed to explore the creative versus reproductive dimensions of mathematics education in their chapter, *Learning: Creation or Re-creation? From Constructivism*
D. AMBROSE

to the Theory of Didactical Situations. They analyse studies of conditions and processes conducive to creative learning within complex contexts. More specifically, they reveal in-depth analyses of developmental dynamics and the ways in which those dynamics can include emergent properties based on complex pedagogical and creative interactions. Essentially, the teaching of mathematics entails the artful establishment of promising conditions for the emergence of creative mathematics understanding. Ultimately, they show that there should be less interest in creation as such than about pedagogical and didactical conditions conducive to its emergence in mathematics learning. The mission of mathematics educators should be to create environmental contexts that enable mathematical creativity.

In her chapter, *Investigating Mathematical Creativity in Elementary School Through the Lens of Complexity Theory*, Esther Levenson used results from empirical observations in classrooms in the city of Tel Aviv, Israel to analyse dynamics of mathematics learning through the lens of complexity theory. Focusing on student interactions with materials, other students, and teachers, Levenson discovered ways in which ideas emerged and were developed. She found that creativity, as it is manifested in the classroom, entails complex, unpredictable, mutual adaptation of all players within the complex adaptive system of the classroom. On the one hand, the teacher and the students are all present in the same lesson and there is a collective experience. On the other hand, different individuals experience instruction in different ways. This chapter outlines the dynamic interaction and interdependence of classroom participants, as well as the tension between pursuing both stability and change. In essence, the author explains how the results of these swirling forces and some principles of complexity (i.e., internal diversity, redundancy, decentralised control) can promote or inhibit mathematical creativity.

Steve Coxon, takes us into an intriguing aspect of science and technology with his chapter titled *On the Edge of Chaos: Robots in the Classroom*. He begins by discussing the role of robots in our world and then turns to the value of robotics as a learning opportunity. Coxon does this by contrasting the processes of robotics with the structure and dynamics of traditional education. He describes the history and nature of educational robotics programs and outlines a variety of current robotics offerings. While making it clear that robots are nowhere near as complex as biological systems, he argues that they allow for enormous cognitive complexity when it comes to students building and using them. He keeps us informed about the research into the effectiveness of robotics as an educational strategy. He also establishes some similarities between large-scale political-democratic dynamics on the edge of chaos and the instructional and learning processes in robotics. Learning is much more dynamic and productive at this edge where there is balance between orderly, authoritarian control and anarchic chaos.

In recognition of the strong, interdisciplinary nature of scholarship addressing complex adaptive systems our next section brings together interdisciplinary
perspectives on creative complexity. Here we include a broad survey of complexity theory in multiple disciplines as well as more specific applications to organisational leadership, environmental sustainability, and urban planning.

Don Ambrose employs a specific construct from complexity theory to generate a very broad-scope exploration in the chapter, The Ubiquity of the Chaos-Order Continuum: Insights from Diverse Academic Disciplines. The interdisciplinary science of complexity is revealing ways in which complex adaptive systems tend to oscillate along a continuum between the extremes of chaos and order. Productive, creative complexity occasionally becomes available when a fine balance emerges from the tension between chaos and order on the continuum. While there is some potential for misinterpretation of this construct, the dynamics of the continuum are applicable to a wide variety of phenomena. This interdisciplinary analysis reveals some ways in which excessive order, excessive chaos, and productive complexity can emerge in human thought and action. Some examples include the tensions between relativism and authoritarianism in identity formation and moral development; laissez-faire market utopianism and centralised regulation in economic systems; relativistic pluralism and universalist monoculture in the culture wars; anarchy and rigid, scientific management in organisational dynamics; incremental wandering and the lure of completeness in the philosophy of science; and the fractured-porous and unified-insular structure and dynamics of academic disciplines. Thematically guided interdisciplinary exploration, dialectical thinking, and the logic of the included middle are proposed as antidotes to entrapment within the counterproductive regions of the chaos-order continuum.

In her chapter, Creative Complexity in Organisational Leadership, Liza Watson discusses creativity and learning in organisations and some ways leadership comes into play in these dynamics. Leadership theories are considered in light of the dynamics of the chaos-order continuum. Watson also contemplates these leadership dynamics while analysing their fit with the industrial age that we are leaving and the knowledge era in which we are currently immersed. In essence, these dynamics revealed by complexity theory can be difficult for individuals and organisations to handle because they can be disruptive even while they provide opportunities for creative organisational progress.

Marna Hauk argues that priorities must change if the world is to shift from degenerative environmental destruction to regenerative sustainability. In her chapter, Complex Regenerative Creativity, Hauk shows that predominant analytic and deterministic methods usually provide knowledge of parts and mechanisms, but they rarely yield adequate answers. Creativity enters the process in the key role of assembling diverse parts, often in unexpected ways. Regenerative design involves both art and science not separately but merging together. The theoretical framework in this chapter employs complexity theory emphasising regenerative creativity as domain-general and transdisciplinary in nature. The framework produces ethical novelty inspired by complex, natural patterns.
Todd Juhasz illustrates the broad applicability of complexity theory in his chapter, *Pareto Optimum Efficiency Between Chaos and Order when Seeking Consensus in Urban Planning*. Based on his experience as an urban planner with transdisciplinary expertise encompassing the biological sciences, architecture, and management, Juhasz establishes a comparison between two case studies of major urban planning projects in two important American cities, one on the East Coast and one on the West Coast. One of these projects suffered from frequent, serious problems and was completed with minimal success. The other project proceeded with fewer problems and led to very successful outcomes. The comparison reveals that successful urban planning and implementation requires artful negotiation to keep the process from disintegrating toward excessive chaos or becoming trapped within excessive order. In contrast, careful, artful urban planning generates a productive balance between chaos and order, which leads to complex yet effective results. Recommendations for the education of students in the urban planning profession are provided.

Rounding out this exploratory, interdisciplinary section, Peter Pruim takes us on a philosophical excursion in his chapter, *Subjectivity, Objectivity, and the Edge of Chaos*. His analysis has two stages. First, he uses the of edge-of-chaos heuristic to classify general epistemological positions. At the extreme of order are the epistemology of the Rationalists and all irrational ideologies where no experience is allowed to count against fundamental principles. At the extreme of chaos are various forms of radical empiricism, including positivism, where reality is identified with experience, which is ever changing and different for every observer and so generalisable theorising is difficult. At the edge of chaos is the sort of empiricism promoted by Quine and Susan Haack, in which the two dogmas of empiricism are replaced by balancing theoretical coherence with observational adequacy. In the second stage of the analysis, Pruim uses this heuristic to describe the history of philosophy of mind: Cartesian dualism, materialist identity theory, materialist functionalism, eliminativism, Wittgenstein and behaviourism, neurophilosophy, and the current scene in cognitive science.

The next section of this book returns us to the nature and nuances of the educational system. It begins with a philosophical analysis of educational purposes and processes. After that, we include more specific insights about the promise of complexity theory in education, from the tension between modernism and postmodernism in diverse forms of expression, to the creativity it reveals in a project blending Shakespearean literature and the performing arts, to the promise of dual exceptionality as a creative advantage, to creative mentorship of new professionals as they make their way into complex work environments.

In their chapter, *Expansive Notions of Coherence and Complexity in Education*, Bryant Griffith and Kim Skinner argue that our culture is embedded within a dynamic tension between coherence and complexity, and that tension generates conceptual chaos. Griffith and Skinner employ complexity theory as a tool for critiques of the excessively mechanistic approaches that dominate education today. They bring into play conceptions of modernism and postmodernism while
looking at ways in which human interactions and context tend to be ignored and marginalised. It is these interactions and contextual influences that enliven education and make it too complex for mechanistic approaches alone to handle. Appreciating and capitalising on epistemological diversity is a theme in the chapter. The authors make room for various forms of cognitive diversity, including domain-specific cognitive frameworks. They also use research findings to illustrate ways in which students can be encouraged to engage in higher-order thinking conducive to complex understandings of text.

Jeffrey Bloom provides a panoramic overview of analyses of complexity in diverse phenomena in his chapter titled *Complexity, Patterns, and Creativity*. Deriving insights from the history of creativity research and from extensive investigations of complex adaptive systems, Bloom uses this analysis as a basis for considering ways in which creativity emerges and complex patterns form. He pays special attention to scientific phenomena, especially the formation and utility of meta-patterns that underpin and sustain the structure and function of complex systems throughout nature. Implications for education arise from the analyses. Especially pertinent are his recommendations for preserving creativity in learning, and for developing a stronger grasp on the pernicious effects of superficial, dogmatic school reform initiatives such as No Child Left Behind and the Common Core standards.

Kathleen Pierce provides an example of socially generated, emerging complexity in her chapter, *A Shakespeare Festival Midwives Complexity*. She explains how preparation for participation in a Shakespeare festival performance creates a community of practice among secondary school students who work along the chaos-order continuum. Procedures employed in the management of the festival seem to provide just the right amount of constraint to nurture complex thinking without inhibiting students’ creativity in interpreting Shakespeare and designing an original 20-minute performance from his plays. The festival day itself provides a series of workshop sessions in theatre arts where students quickly learn new skills, play, and practice in the company of students from other schools. The festival design imposes order and allows for chaos in each of the workshops before complexity emerges in the form of new competencies developed in collaboration with new acquaintances.

Jack Trammell has us think about an issue that straddles the fields of gifted education and special education in his chapter, *The Anthropology of Twice Exceptionality: Is Today’s Disability Yesterday’s, or Tomorrow’s, Evolutionary Advantage? A Case Study with ADD/ADHD*. Some anthropologists and psychologists suggest that the ADD/ADHD arrangement of the prefrontal cortex may have been an evolutionary advantage 20,000 years ago when humans had a greater need to respond rapidly to stimuli in the environment and to consider creative, nonlinear approaches to problem solving. In today’s world, that same brain arrangement is often treated as a disability and the potential giftedness associated with it is overlooked. Trammell briefly examines the historical etiology of ADD/ADHD, considers current neuroanatomical perspectives, and suggests that the degree to which the brain arrangement is considered medically disabling is problematic. He then shows how conceptions of
ADD/ADHD as a disability are being transformed. Finally, he proposes that the concept of twice exceptionality itself actually may be a misinterpretation of a rapidly evolving human brain in which today’s disability can be yesterday’s, or tomorrow’s, special ability.

In the chapter, *Mentoring the Pupal: Professional Induction Along the Chaos-Order Continuum*, Kathleen Pierce employs the chaos-order continuum again, this time to analyse the difficult problems beginning professionals face when making the transition into a complex profession. She shows how beginners in schools and universities often have great problems adjusting and getting up to speed with highly complex professional demands even though those institutions often have established formal mentorship programs. Thinking about the ways in which these experiences oscillate along the chaos-order continuum helps us see how the immense difficulty arising from rapid immersion in highly complex, multilayered processes and contexts establishes chaotic conditions in the beginner’s mind while the excessive order of the bureaucratic procedures typical of induction processes represents excessive order on the continuum. According to Pierce, nuanced mentorship can enable beginning professionals to find a productive balance between these extreme conceptual positions where they can begin to enjoy the fruits of professional complexity.

Our final section provides a look at the social-emotional dimensions of complex creativity. There is increasing recognition that high-level cognition incorporates emotional ingredients, especially when it comes to creative work. These emotional ingredients can be injected through the influence of productive relationships, recognition of the need for cognitive restructuring and integration, and awareness of barriers that can distort the emotional elements of thought.

Michelle Jordan and Reuben McDaniel emphasise the importance of social dynamics in their chapter, *Helping Students Respond Creatively to a Complex World*. They begin by taking aim at the persistent dominance of conceptual frameworks saturated with scientific determinism when it comes to influence over educational philosophy and practice. After addressing that pressing issue they posit knowledge of complex adaptive systems as an alternative framework. They go on to explain how this alternative reveals dynamic complexity in a wide range of phenomena pertaining to education and creativity. They also provide advice about how to help students navigate the contextual intricacies revealed by their analysis. Some of this advice includes developing ways to help young people tolerate and embrace the fundamental uncertainty of complex environments while capitalising on the potential embedded in dynamic relationships. Jordan and McDaniel also provide a wide variety of examples of practical, creative strategies that can be used in classrooms to generate better understanding of system dynamics.

In her chapter, *Toward the Pattern Models Of Creativity: Chaos, Complexity, Creativity*, Krystyna Laycraft provides a new approach to the study of creativity in adolescents and young adults engaged in complex, creative endeavors by combining the idea of self-organisation with theories of emotions. Employing
qualitative research methods she found some differences in the creative work of young people, but also discovered common phases such as differentiation/chaos, integration/complexity, and dissipative structures/creativity (products of creativity in the forms of new movements, new writings, and new paintings). Creativity of the young people under study was intertwined with strong emotions of interest, joy, and acceptance. These dynamics encouraged global, open, and exploratory modes of attention, stimulated thinking, and enriched imagination. All of this deepened emotions, leading to more curiosity, enthusiasm, delight, passion, resourcefulness, and love. Creative individuals became more sensitive, more open, and receptive to their internal and external worlds. They seemed to become more resourceful, imaginative, empathic, and spiritual.

Ann Gazzard concludes this section, and the volume, with her chapter, *Emotions, Complexity, and Intelligence*. She shows how the edge of chaos hypothesis from complexity theory can elucidate our understanding of emotional intelligence, in particular its foundation in the early childhood years. She draws insights from psychology, neuroscience, and other fields to shed light on the complex dynamics of emotional development and barriers that suppress or distort that development. Based on syntheses of these insights, she concludes with recommendations for enhancing and strengthening emotional intelligence in young children.

While our motley coalition of investigators from multiple academic and professional fields has employed analytic insights from Pepper’s (1942) four good lights, we certainly have not covered all of the conceptual territory relevant to the nature of complex adaptive systems. That territory simply is far too expansive to grasp in a single project and much more can be done in future investigations. Others have developed important insights about the creativity-complexity theory nexus (e.g., Richards, 2001, 2010; Schuldberg, 1999; Sterling, 1992) and we hope that our project augments their work. Our primary purpose has been to expand awareness of the promise and intricacies of the meeting place between complexity theory and creative effort. We encourage future development of theory and research along these lines.

**REFERENCES**


SECTION 2

COMPLEXITY IN STEM PROCESSES AND STRUCTURES
JARMILA NOVOTNA & BERNARD SARRAZY

LEARNING: CREATION OR RE-CREATION? FROM CONSTRUCTIVISM TO THE THEORY OF DIDACTICAL SITUATIONS

The mere fact that the result of original work in the mathematical field is called sometimes a creation or invention, sometimes a construction or discovery, shows all the multiformity of mathematical experience.


It is indispensable that every teacher, every day, begins his/her class as if the knowledge that he/she proposes to students were discoveries for the first time in the world and as if this meeting was decisive for ... the future of mankind.¹

G. Brousseau (1986)

INTRODUCTION

The idea of the child “creator” is historically associated with its activity and with “construction” of connaissances and savoirs. These ideas developed within pedagogical streams, namely in active pedagogy, and their boom was brought about with the emergence of constructivism especially with Piaget. In fact, Piaget’s theory triggered a fundamental breakdown in the conceptions of learning seen as adaptation to the environment and of knowledge seen as a dynamic process of adaptation between the subject’s schemes and the object of the knowledge. Despite the fact that constructivism significantly influenced teaching and learning, the theory did not enable pinpointing of the conditions under which a situation becomes a didactical situation with didactic properties; an error is constructive for new knowledge only through the regulations that enable its avoidance. Application of this theory in teaching and learning (Aebli, 1966) led to reinforcement of the idea that the child is the “creator.” Constructivism also led to the belief in the inevitability of the development of logical-mathematical reasoning and therefore of the student’s mathematical mind independent of teaching/learning situations as such. But descriptions of cognitive processes in Piaget do not enable us to study and define the didactical conditions that enable us to carry out these processes. This became the object of the first theorisation of didactical phenomena at the end of the 1960’s (Brousseau, 1997).

The properties of these situations can be derived neither exclusively from the student (the epistemic subject) nor exclusively from mechanisms of their pedagogical

D. Ambrose et al. (Eds.), A Critique of Creativity and Complexity, 19–33.
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limitations. Nor can they be derived only from the examination of knowledge as such, but properties of these situations emerge from the complex interplay of retroactions that show up in the situations with which the student interacts. These retroactions must be at the same time sufficiently transparent but also sufficiently “rich” for the student if conditions in which the students learn mathematics are to be created.

This chapter is an attempt to show that we should be less interested in creation as such than about pedagogical and didactical conditions (belonging to mathematics) of its emergence. One of the fundamental paradoxical dimensions of creation that we will try to elaborate is that what students have to learn is precisely what the teacher cannot teach them, and so it therefore corresponds to what students have to “create.” In fact, mathematics that already has been discovered is “dead” mathematics, and it is brought to life through its use by researchers but also by teachers as the crucial teacher’s role is based on creating the conditions where students may learn mathematics by applying their knowledge in new contexts. With these ends in mind, teachers create situations that show their students the use and interesting aspects of mathematics that they have been taught; but the teachers cannot place themselves in their students’ position in order to teach them (Similarly, one cannot walk or speak or sleep for a one-year-old toddler even though, of course, one does his/her best to help the child to learn.). This is what mathematics educators call the didactical contract (Brousseau, 1997; Novotná & Hošpesová, 2009; Sarrazy, 1995).

In other words, to teach mathematics is to create conditions for re-emergence of mathematics. Creation of these conditions forms the core of the teacher’s work (Bureš & Hrabáková, 2008).

The aim of this chapter is to deconstruct the mystic dimension of this creation in order to grant the teaching of mathematics and the learning of mathematics in their places in both areas: didactical and pedagogical. Didactical suggests learning ready-made mathematics, and pedagogical implies maintaining the sense of creation by the student as a condition and initiator of appropriation of a mathematical activity.

CREATION AND EDUCATION

Considered here in the domain of mathematics, the process of creation as emergence of something new of an intellectual and aesthetic value is fascinating, and it surprises and impresses us through the disruption that it reveals and fills us with enthusiasm with its aestheticism.

Mathematics, rightly viewed, possess not only truth, but supreme beauty – a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show. The true spirit of delight, the exaltation, the sense of being more than man, which is the touchstone of the highest excellence, is to be found in mathematics as surely as in poetry. (Russell, 1917, p. 60)
LEARNING: CREATION OR RE-CREATION?

Creation seems to have one divine dimension because it appears as manifestation of reality it reveals and that eludes our immediate comprehension. Let us say that we would like to believe as we like to believe the magic of magicians or the fiction in a movie, but the spectacle is more fascinating than the knowledge of how it happens that spoils the delight. Is there anybody who has never been fascinated by the ingenuity of our own children and also of our students? Everything seems to be happening but something has eluded our comprehension; and this is just the mystery that fascinates us and that we do not try too hard to penetrate if we do not want to break the spell; at best when we speak about it, we refer to gift or talent. Creation (as manifestation of talent) therefore seems to be in opposition to education and more specifically to teaching. This conception of creation denies education. But the fact is, as we will demonstrate in this chapter, that there is no creation without education.

CREATION AT THE CROSSROADS OF PARADOXES

Etymologically, creation is a process consisting of “extracting something from emptiness,” giving rise to a being or a thing *ex nihilo*. For a long time this concept was attached purely to the religious domain: the God as the creator of the world, seven days of creation, etc. It was as late as the end of the 18th Century that the word started to be used also for “the assemblage of beings and created things”; in other words, something that can be produced even by human activity. However, the idea of uniqueness is still attached to this concept. On the one hand, there is the uniqueness of what is produced (creation as an object) but also the uniqueness of the creator. It is very common in the history of humankind that creations are called by the names of their creators: Pythagoras’ Theorem, Fermat’s Last Theorem, the Poincaré Conjecture, Einstein’s Theory of Relativity, and Bolzano’s Paradoxes of the Infinite.

But we are facing a problem here: how can we tell what an original is, first, unique product? This can only be said in reference to the collective, already existing productions. Therefore, there is no creation without a community as community recognises its value in production and decides whether something is worth keeping and valuable for the life of the humankind in practical, aesthetic, scientific, and technological utility. This is one of first paradoxes brought about by creation: Uniqueness and uncommonness of creation cannot exist without the group that recognises these qualities; creativity, which is undoubtedly individual, is basically a collective phenomenon. Without a community, there is no creator.

The second paradox is an immediate consequence of the first paradox: how to recognise what is new? Paraphrasing Menon’s dialogue we could say: If we recognise some production as something new, we must have some prior knowledge of it because otherwise we would not have been able to recognise it at all. If we have no prior knowledge, we cannot recognise it as such. The histories of sciences, arts, literature, and other disciplines have witnessed a considerable number of works that were acknowledged years, even centuries, after their production. For example, the numerous works of Bolzano (1781–1848) were recognised as late as 1920.
Education is at the heart of overcoming this paradox of what we are dealing with. The fact is there is no creation without collective memory of production. And education is nothing but the transmission of works. Therefore, there is no creation without education. This is the second paradox: There cannot be creation without transmission of what has already been produced, and at the same time creation cannot be the product of transmission because the only thing that can be transmitted is that which exists.

Here we identify a phenomenon that teachers are more than familiar with: New knowledge can only be born in relation to the former knowledge (which it often destroys). This is undoubtedly a source of inequality; creation can only be born from plentitude and not from ignorance or naivety that is sometimes claimed in connection with young children.

The third paradox is a direct consequence of the second one. If a society, a group, or a family can transmit only what already exists, then each creation requires a transgression because it means to make the non-existent exist and to make the hidden visible. But psychoanalysis shows that the hidden has always kept its relationships with savoir and desire; the subject cannot desire what he/she does not have, what is concealed from him/her. Here is one of the prime movers of creation as looking for savoir: a savoir to see. Consequently, what is hidden is always burdened by prohibitions as with the forbidden fruit of the tree of knowledge in the Garden of Eden from the Bible’s Book of Genesis or children’s particular attraction to the forbidden or the hidden. Freud (1962) calls this space “the scopic drive” where the “desire to see” is born. Thus creation is always transgression, transgression from the rules in order to see differently, transgression from the hidden in order to penetrate what is masked. But transgression – and therefore creation – cannot be the product of an explicit command or docile execution of the order to disobey. Thomas Kuhn (1962) speaks in this respect of “scientific revolutions,” “revolution” in the sense of considering the things in a different way from a different point of view as in the case of Andrew Wiles who studied Fermat’s Last Theorem from Galois’ and Taniyama-Shimura’s works (Singh, 2002). This is the third paradox: To create is to let oneself be, to make the decision to become an author and to accept the position of “the person who founds and constructs.” In Latin auctor referred to the God, a God creator. Referring supposes that the subject is autonomous and free but as we have already suggested, freedom is possible only in the framework of a culture that is necessarily collective.

This third paradox has its philosophic expression in Kant’s (1991) famous formula characterizing the Enlightenment spirit to: “Argue as much as you will, and about what you will, but obey!” (p. 50). An ice-hockey player, a painter, a musician, and a mathematician are free to choose how they play but cannot define the game in which they take part. The freedom of players is possible only if they play by the rules that define the conditions of the possible and play inside the respective communities hockey players, painters, musicians, and mathematicians. The space of creation lies in the space in between the individual freedom and the collective restrictions on
how to play, between the structural dimension of the game and the way of playing it. But let us state again that creation is far from restricted only to novelty. It must be of interest and of value to the institution or the community in which it appears. So the creation is always an encounter between a culture at a given moment and the singular desire of a subject nourished by this culture.

The phenomenon is typical for teaching and learning as teachers do (often silently) the sorting out of what must be remembered and what can be forgotten. This dimension of creation is therefore valuable for the teacher. The dimension of marginal novelty and of possible discovery of what Giroux (2008) calls atypical conducts are characterised by: marginal character, non-adapted to restrictions, and specific to the stakes of the mathematical situation. For that matter, that is why atypical conducts are relevant for the area of didactics. They cannot be mistaken for “inefficient or deviant conduct” or for “behaviour that has not been adjusted to the assigned problem.” As Giroux explains, atypical conducts “confirm the role of the antagonist dimension of the milieu (situation)” and “are thus consequences of appropriation of the real stake of the situation.” For instance, mathematical creation requires a game, like the space of limited freedom that is parallel to the role of hinges; if the hinges are too tight it is impossible to open the door, too loose, the door cannot be closed.

To summarise, we can say that creation is an anthropological phenomenon, closely linked to education. Creation enables birth of novelty that is at the given moment considered as useful for the culture and which will be recognised as such by an institution furnished with a collective memory. It is located on the crossroads of the following three paradoxes:

- Paradox of the collective subject that poses the question of relationships between the individual and the society;
- Memory paradox that characterises the dialectics of the ancient or of the “already known” and the new;
- Paradox of authority: to create the means to let oneself discover something that does not yet exist; this permission puts in dialectical tension docility and transgression. Without rules and norms creation is not possible: a human, like a student, is at the same time autonomous and heteronomous. Freedom is possible only through voluntary acceptance of a set of restrictions that define the space of its creation.

These three paradoxes of creation are also in the background of any educational activity and, as we will see, they show more clearly in mathematics education. Let us start by considering the following extract from an episode from a mathematical lesson with 9-year-old students.

Elodie tells her teacher that some pages in her dictionary are missing: “Look, it jumps from page 122 to page 211!” The teacher takes the opportunity to ask: “How many pages are missing in Elodie’s dictionary?” “Easy!” some students
say. Some students tap on their calculators … then many of them suggest “89,”
to which the teacher says: “No!” says the teacher. Refusing to validate these
new answers, the teacher says: “You must not only find the right result but
also prove that it is correct!” Thirty minutes pass, students offer many results
but none of them is valid. Suddenly, Lou announces proudly: “Here it is! I
found it, and I can prove it!” Lou comes to the blackboard with her dictionary
and says: “I counted it!” Her dictionary is open on page 123 and she starts
counting every page, turning the pages one by one. The others protest: “This
is cheating!” The teacher’s reaction is: “I forbade no method. All methods are
allowed when it comes to savoir!”

It is true that Lou did not invent counting as Antonín Dvořák did not invent the notes
with which he composed his requiem. What Lou created is the use of counting here
and now in order to establish with certainty the solution to the posed problem. The
solution itself was not included in the knowledge of counting. It is largely due to
the character of novelty of her procedure that makes other students treat her as a
“cheater.”

MATHEMATICAL EDUCATION – CREATION OR REPRODUCTION?

To teach is to pass down to the younger generations what the previous generations
have produced with the aim to prepare children for the society in which they will
live their lives. Because children are unfinished, dependent, and ignorant, they have
to be educated. The educator, albeit the parent or the teacher or anybody else, shows
examples, explains, justifies, forbids, and says what can be done and what cannot be
done, but will never succeed unless the child joins in the educational project.

In fact, what the teacher expects is neither mere docility nor respect to what
was said, but the teacher does not expect mere imitation of what was said or done
nor mere memorisation of the rules that have been taught. The teacher expects
special use of what has been taught in new situations that the students have not
yet met. It is exactly in this respect that learning may be considered as a sort of
what Baker and Hacker (1986) call normative creation, creation because the subject
explores new spaces, solves new problems; also normative because the way it is
done must conform to the rules that define the space of his/her action. Similarly, a
football player may create his way of playing but within the framework of the given
rules of the game that—at the same time—restrict his activity as well as define his
freedom. The difference between a good and a bad student is not in their knowledge
of algorithms but in the way they use them. After all, the teachers are not wrong
when they say to a student: “You know the lesson but you have not understood a
thing!” Learning, of course, is storing in memory but comes to surface mainly in
the form of students’ own production in the limits given by the rules as they have
been taught.
The idea of the child as creator developed rather late. However, it was definitely here in the 18th Century with the Lumières and their concept of social contract. For many years it was one of the fundamentals of active pedagogy. Let us recollect at this point what J.J. Rousseau wrote in *Emile*:

Let him know nothing because you have told him, but because he has learnt it for himself. Let him not be taught science, let him invent it. If ever you substitute in his mind authority for reason, he will cease to reason; he will be a mere plaything of other people’s opinion. (Rousseau, 1991, p. 564)

One could assume that Rousseau had anticipated even the essence of didactics that developed in the end of 1960s with the theory of didactical situations (Brousseau, 1997). This theory outlines a method that enables the student to understand and “discover science” independently, on his/her own:

Keep the child dependent only on things. [...] Never offer to his indiscrete will anything but physical obstacles or punishments that arise from the actions themselves, [...] Experience or lack of strength alone ought take the place of law for him. (Rousseau, 1991, p. 238)

This method is correct as learning enables us to change the students’ points of view. Although we can teach this change, we cannot force it upon the students. It is similar to suffering; everybody knows what suffering is, but suffering is an individual experience and does not come to surface even though others might feel and sense that another person is suffering. Moreover it is not possible for anybody to suffer in place of the other; one can try to help, try to soothe, but one cannot suffer in another’s place.

The idea of the child creator is often replaced by new pedagogies that tend to be rooted in the romantic movement reinforced by Piaget’s constructivism. Piaget claims that everything that one teaches a child prevents the child from inventing it (Piaget in Bringuier, 1980). Piaget was right; children show extraordinary plasticity in their behaviour. Children seem always ready for challenges, come up with new problems, and think up new forms of adaptation to situations. Moreover, children never cease creating more and more complex problems. But what Piaget did not explain—as it was not in his project—are the conditions in which students can meet problems that allow them to learn exact mathematical knowledge. If these conditions are ignored, creation is isolated from its educational and cultural dimensions. We end up with the previously mentioned ideology of talent. The theory of didactical situations developed in response to this blind spot in Piagetian theory. What properties must situations have if they are to enable students to learn what cannot be taught directly? What constitutes the core of mathematical activity where learners search, make conjectures, confront convictions, justify, insist, persuade, prove, and do mathematics?
Example of a Situation

This is a situation of the study of linear applications: Enlargement of a puzzle (extracted from Guy Brousseau (1997, p. 177).

Instructions: Here are some puzzles. You are going to make some similar ones, larger than the models, according to the following rule: the segment that measures 4 cm on the model will measure 7 cm on your reproduction. I shall give a puzzle to each group of four or five students, but every student will do at least one piece or a group of two will do two. When you have finished, you must be able to reconstruct figures that are exactly the same as the model.

Development: After a brief planning phase in each group, the students separate. The teacher has put an enlarged representation of the complete puzzle on the chalkboard. Almost all students think that the thing to do is to add 3 cm to every dimension. […] The result, obviously, I that the pieces are not compatible.

This situation clearly shows that students can learn by interacting with the situation; the situation enables (among other things) students to invalidate the classical model of addition: If $4 \rightarrow 7 (4 + 3)$, then $5 \rightarrow 8 (5 + 3)$.

On the condition that each student gets one puzzle with the goal of illustrating the use of proportionality, will all students learn the same? Definitely yes. The students will have learned the same functions and same mathematics, but in this situation they will learn something extra—a way of doing mathematics. Let us recollect at this point, for example, that thanks to various manipulations by cutting and comparing that Galileo managed to work out the formula for calculation (approximate) of the surface of a cycloid.

This example demonstrates that if a teacher teaches a rule, the student learns simultaneously a way of doing mathematics without being explicitly taught the rule—very similarly to the “practical sense” in the sense of Bourdieu (1984). This way of doing mathematics contributes to mathematical education in that it determines, not mechanically, the use of teaching rules. It is obvious that we must study the conditions of the students’ mathematical activity. The tension between the two aspects—the rule and its use—is often the centre of attention to those looking for improvement of teaching and learning mathematics. The relationship between the mathematical rule and its use is generally approached from one or the other of the following opposing poles.
LEARNING: CREATION OR RE-CREATION?

- Sometimes the stress is put on teaching the concepts and on exercises and problems designed to reinvest and drill knowledge already taught.
- Sometimes the focus is on mathematical culture, on the possibilities of a student’s creation, and on the construction of knowledge.

This opposition is misleading because even though it marks one of the borders between “active” and “classical” pedagogy, it makes the teacher ask many questions. Should teachers make their students’ “heads quite full” or “heads well done” in Montaigne’s words? We should hope for both. This is one of the contemporary impasses of teaching and learning mathematics. One of the possible reasons for this impasse is the fact that this debate is counterproductive. Because knowledge of algorithms does not mean better knowledge of arithmetic, just as knowledge of the rules of chess allows us to “play chess” but does not prevent us from making mistakes when playing or losing the game. The idea is simple but not trivial.

CONTRIBUTION TO THE STUDY OF CONDITIONS OF CREATION:
RESPONSIVENESS TO DIDACTICAL CONTRACT

Methodology

Asked to solve a set of arithmetical problems in four different situations (see below), 155 students aged 9 were presented the following problem that we call “pseudo-multiplicative”:

A snail is at the bottom of a well. He decides to leave the well. We know that it will take him 6 days to get out of the well. How long will it take three snails to get out of the well following the same path?

This type of problem brings a less ordinary use of multiplication because it requires the student to: (1) use multiplication knowledge to demonstrate that his/her solving process cannot make use of this operation and (2) produce an answer without calculating (less usual use in school).

All four situations are situations of evaluation, yet they differed from each other in their level of authority. For instance, depending on the situation, the person posing the problems might have been another student, the teacher, or the researcher. Each situation also differed in the individual or collective stakes connected to verification.

- Situation 1: The researcher asks the students to solve a series of problems (among which the target problems are included). The students are informed that the test will not be marked.
- Situation 2: This test is presented to the students as a competition between classes, in which each class chooses which of the posed problems will be assigned to the other classes. The test is divided into two phases: 1) the presentation of the rules of the “competition”; each student poses a problem and solves it and consequently submits it to the researcher; 2) the test itself: the researcher poses
his/her own problems. Here, the nature of the activity is not individual as in the other situations but collective.

• Situation 3: Each teacher carries out evaluation that he or she would normally carry out in the end of the term. This evaluation includes the target problem. Here, the nature of the activity is clear to the students: the activity is individual and marked by their teacher.

• Situation 4 (“warning situation”): The aim of this situation is to verify whether the students are able to correct a “defective” problem assignment. That is why the students were informed of the presence of both non-calculable and calculable (classical) problems in the given set of problems.

The results clearly show that the decisions concerning the target problem (absence of an answer, answer calculated “3 x 6,” or the correct answer “6 days”) can be accounted for by the type of the situation rather than by the students’ school level. When the investment is collective and the degree of authority is less developed, the students take the freedom to produce an answer without calculating. What we now have to explain are the differences in student solutions manifested within the same situation.

**Didactical Environment and Responsiveness to Didactical Contract**

The responsiveness to didactical contract is a concept that we designed for labeling the students’ decisions in relation to the situations where they had to make non-ordinary decisions and let themselves do something that they were not used to doing. The following is an example of responsiveness to didactical contract:

A few days before this episode, the teacher taught his 9-year-old students the following algorithm for calculating the difference between two numbers.

![Algorithm Image]

In the evaluation prepared by the teacher, the researcher included the following exercise: How would you carry out the following calculations?

a. 875 - 379 = ______________________

b. 964 - 853 = ______________________

c. 999 - 111 = ______________________

Sixteen out of 19 students applied the algorithm taught by their teacher in all three exercises including the following third one.

999 - 111 = 1008 - 120 = 1088 - 200 = 888.

It can be presumed that students ask themselves what the teacher expects of them. Should they show that they master the taught algorithm or should they use the classical procedure (that would be much simpler) for answering the question c? It is this implicit attitude that we call “responsiveness to didactical contract.”

28
LEARNING: CREATION OR RE-CREATION?

Of course we do not intend here to measure the students’ creative abilities; we want to contribute to evaluation of the flexibility of their knowledge according to the situations that form the major components of the creativity—their ability to let themselves do something non-ordinary.

Analysis of Effects of Didactical Environments on Creativity

We studied the effects of two strongly contrasted models of teaching on the phenomena of responsiveness to didactical contract. The following were some observations:

- The Magisterial Model bases work on repetition, and the basic teaching scheme could be described “show-retain-apply.” The teacher hopes that students will be able to generalise and will apply the piece of knowledge elsewhere. Teaching strives to hand over procedures and algorithms necessary for solving problem-types that are often rooted in social life. Typical for such teaching are weak openings and low varieties of situations. During lessons, teachers quickly teach a solving model and then assign their students increasingly more complex exercises that are consequently checked collectively on the blackboard. These classes are weakly interactive.

- The Activating Model corresponds to what could be called “active pedagogy” and often refers to Piagetian constructivism. The problem is considered as the privileged tool for “making sense” to knowledge. Teaching is characterised by strong variability in the organisation and management of situations. Teaching regularly uses group work. Assigned problems are in most cases complex and open. These classes are strongly interactive.

Let us now turn our attention to the effects of the culture of classes on the phenomena of creation: How can these ways of teaching influence students’ approach and attitude to “novelty”? Do they or do they not allow a non-ordinary solution?

Results and Comments

In case of the “snail” problem, it was observed that 48% of students from the “activating” model produced answers without calculating. In contrast, 17% of students from the “magisterial” model produced answers without calculating \[\chi^2 = 6.08; p < .04\]. These differences are valid for the same school level independent of the situation of their production. These models show to be suitable for explanation of the phenomena of responsiveness to didactical contract. In other words, the more chance the students have to confront the rules with weakly repetitive situations, which is the case in the activating model, the more they let themselves use them in new situations. Reciprocally, the more repetitive the teaching is, which is the case of the magisterial model, the less the students let themselves deviate from the use of rules.
Can we assume that one model of teaching should be given priority over the other one? This would definitely be wrong. If we assign the same students problems of high difficulty in weakly decontextualised situations with the same or very similar context to the one previously described, results concerning the influence of teaching styles may be radically different.

Conditions of the Experiment

The problems used for the experiment correspond to the fourth additive structure of Vergnaud’s typology (1982). This structure is specific as it uses only positive and negative transformations (win or lose) without any indication of the initial numerical state. An example of this type of problems follows:

Dominika plays two rounds of marbles. She plays one round. In the second round she loses 4 marbles. After the two rounds she has won 6 marbles. What happened in the first round?

The experimental plan is classical: 22 problems of various difficulty were assigned to students in a pre-test. This was followed by 2 lessons in the interval of one week from one lesson to another; in the end the same problems were assigned to students again. An ‘index of progression’ \( I_p \) was defined for each student.

Results and Comments

When evaluating the effects of the teaching, two aspects were considered.

- The first consideration is called \textit{efficacy} and corresponds to the measure of effective performances recorded in the post-test when controlling variables likely to have influences the observed results (the students’ school level).
- The second consideration is called \textit{equity} and measures differential efficacy for a given group of students, taking into account their initial level (the pre-test results).

The magisterial model looks to be more fair with students making significantly more progress than students from the activating model \( f_1 = 3.73; p < .05 \) and also more efficient since student performances are significantly better \( f_1 = 5.10; p < .01 \). These effects are evident in particular for weak students \( (\text{efficacy: } f = 20.26; p < .01 - \text{equity: } f = 20.26; p < .01) \).

Should these last results make us re-evaluate our previous conclusion, and do results confirm that the magisterial model is more beneficial than the activating one this time? On the one hand, all research is only a window opened to the universe of practices whose temporalities are not analogous. Nothing here allows us to conclude that if the number of lessons was greater – in our research we were limited to two lessons on the involved teachers’ request – the performances would necessarily be the same. The speed of learning is probably slower in the activating model, but the
time provided for the teacher also has very significant effects on structuration and management of didactical organisation (Chopin, 2011).

CONCLUSION

The results lead to the fundamental questions about how and where to direct education. Should the goal of education be the “head quite full” or the “head well done”? Should we look for good teaching of algorithms or should we allow students to be creative and use these algorithms in new situations? This makes us ask what kind of women and men the school should produce. Obviously, if the magisterial and activating models always appear hand in hand, they are in a paradoxical relationship. Here we again come across the paradoxes of creation discussed in the first part of the text.

The theory of didactical situation was born from theorisation and scientific study of the conditions that allow us to overcome this paradox; if its recognition in the scientific community is undoubtable, its dissemination and use in teacher training remains strongly limited, which Marchive (2008) points out in his recent study. Is this something to be regretted? Definitely yes, because teacher training seems to be an efficient tool that will enable teachers to avoid this impasse. It is crucial that teachers believe in student creativity, but this pedagogical belief leaves teachers often insufficiently prepared if they are to construct conditions for mathematical creation. Pedagogical willingness as such or humanist spirit are powerless tools when teachers come face to face with students’ ignorance and lack of comprehension.

It would be desirable to develop significantly teachers’ didactical culture, but it would be wrong to think that it could replace pedagogical knowledge. It would be a serious mistake because teachers as well as students need both certainty and illusion.

If an educator contributes to clarification of the conditions under which creation of savoirs new to the student is possible (that do not depend on the student but on the mathematical culture itself), it is the pedagogue and nobody else who is responsible for preparation of socio-affective conditions that will enable his/her students to take part in an activity. This activity must be an adventure for students that only they can experience and that nobody else can do it in their place—the adventure of grasping the whole world in one day and in turn engaging themselves in the adventure of mastering the subject matter. How can one imagine that they would be able to produce the new unless they have had the chance to experience it actively? This is our noble mission: to create conditions enabling this mathematical creativity.

ACKNOWLEDGEMENT

The research was partially supported by the project GAČR P407/12/1939.
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NOTES

1 Translation into English: J. Novotná. French original: « Il est indispensable que tout enseignant, chaque jour, commence sa classe comme si les connaissances qu'il propose à ses élèves étaient découvertes pour la première fois au monde et comme si cette rencontre était décisive pour... l'avenir de l'humanité.»

2 In the chapter two types of knowledge are distinguished: connaissances and savoirs. Briefly we can say that “isolated parts are acquired as savoirs connected by connaissances” (Brousseau, Sarrazy, 2002). For a detailed description, see e.g. (Brousseau, 1997).

3 Example: Taniyama-Shimura’s conjectures came back to “life” in connection with the success of Andrew Wiles with the last Fermat’s theorem.

4 G. Brousseau was the first who was awarded Felix Klein’s medal from ICMI in July 2004 at ICME congress in Copenhagen.

REFERENCES


This chapter explores the practical application of complexity theory to the investigation of mathematical creativity in elementary school classrooms. While previous studies have investigated ways of assessing students’ potential for mathematical creativity (Lee, Hwang, & Seo, 2003; Mann, 2009) and specific ways of promoting mathematical creativity (e.g. Levav-Waynberg & Leiken, 2012), this study investigates mathematical creativity as it emerges in classrooms as part of the regular lesson. Three classrooms were observed where the teachers were not explicitly implementing a program aimed at promoting creativity. Yet, when the lessons were reviewed, it became apparent that they included episodes where manifestations of mathematical creativity were evident.

Analysing exactly which of the classroom participants were being creative, who was responsible for what, and what led to the creative endeavor in these classrooms proved difficult. In a previous article (Levenson, 2011), I employed theories related to collective creativity (e.g. Hargadon & Bechky, 2006; Paulus, Larey, & Dzindolet, 2000) and theories related to collective mathematical understanding (Martin, Towers, & Pirie; 2006) to investigate notions such as collective fluency and collective flexibility in the classroom. In this chapter, I look at the same data using the lens of complexity theory.

Looking at the same situation from different perspectives is not new (e.g. Even & Schwarz, 2003) and can afford the researcher a richer and more detailed view of the data (Bikner-Ahsbahs & Prediger, 2006). In this case, complexity theory takes into account the dynamic interaction and interdependence of classroom participants focusing on and making sense of how knowledge, and in this case creative thinking, emerges at the group level. In addition, as Hurford (2010) claimed, “it may well be that the most important affordance of systems-theoretical approaches to learning is in the language of complexity itself, because the language helps all stakeholders to fabricate their own internal models of dynamical learning systems” (p. 583). This chapter is the result of applying this approach, along with its attending language, to investigating the emergence of mathematical creativity in elementary school classrooms.
MATHEMATICAL CREATIVITY IN THE CLASSROOM

As the focus of this chapter is on creativity among young students, it is not concerned with the creativity of a few eminent persons who have made a significant and lasting contribution to society (sometimes known as Big-C creativity). Instead, this study is concerned with everyday creativity (little-c creativity) as it is manifested in the classroom. It focuses on students’ “novel and personally meaningful interpretation of experiences, actions, and events” (Kaufman & Beghetto, 2009, p. 3). This view is in line with Runco’s (1996) view of creativity as “manifested in the intentions and motivation to transform the objective world into original interpretations, coupled with the ability to decide when this is useful and when it is not” (p. 4).

Focusing on mathematical creativity, Liljedahl and Sriraman (2006) differentiated between professional and school-level mathematical creativity. Professional mathematical creativity relates to work that significantly extends the body of knowledge and opens up new directions for other mathematicians. School-level mathematical creativity includes unusual and/or insightful solutions to a given problem or viewing an old problem from a new angle, raising new questions and possibilities. This study adopts the view that mathematical creativity is “an orientation or disposition toward mathematical activity that can be fostered broadly in the general school population” (Silver, 1997, p. 75). As such, the product of mathematical creativity in the classroom may be original ideas that are personally meaningful to the students and appropriate for the mathematical activity being considered.

One of the hallmarks of creativity in general and mathematical creativity specifically is divergent thinking. Divergent thinking is often measured in terms of the fluency, flexibility, and originality of ideas produced. Silver (1997) related fluency to “the number of ideas generated in response to a prompt” (p. 76). Flexibility, according to Silver (1997) refers to “apparent shifts in approaches taken when generating responses to a prompt” (p. 76). Leikin (2009) evaluated flexibility by assessing if different solutions employ strategies based on different representations (e.g., algebraic and graphical representations), properties, or branches of mathematics. Flexibility may also be thought of as the opposite of fixation. In problem solving, fixation is related to mental rigidity (Haylock, 1997). Overcoming fixation and breaking away from stereotypes are signs of flexible thinking. Haylock further differentiated between content-universe fixation and algorithmic fixation. Overcoming the first type of fixation requires the thinker to consider a wider set of possibilities than at first is obvious and to extend the range of elements appropriate for application. The second type of fixation relates to when an individual adheres to an initially successful algorithm even when it is no longer appropriate.

Novelty and originality are also related to mathematical creativity. According to the systems model of creativity, when an individual employs the rules and practices of a domain to produce a novel variation within the domain content, then that individual is being creative (Sriraman, 2008). In the classroom, this aspect of creativity may
manifest itself when a student examines many solutions to a problem, methods or answers, and then generates another that is different (Silver, 1997). In this case, a novel solution infers novelty to the student or to the classroom participants. Originality may also be measured by the level of insight or conventionality with respect to the learning history of the students (Leiken, 2009). The terms originality and novelty may seem synonymous; and indeed, some researchers have used them interchangeably. However, each of these terms stresses different elements. Novel may refer to “new” while original may refer to “one of a kind” or “different from the norm.” While it seems likely that a “one of a kind” idea will also be “new” and vice versa, it is sometimes the case that an idea, especially one raised in the classroom, may be new to a student; but if other students have the same idea, it may not be original.

One of our goals as mathematics educators is to promote mathematical creativity among our students. Toward this end, several studies have focused on the teacher and pedagogical aspects of teaching for creativity (Bolden, Harries, & Newton, 2010; Jeffrey & Craft, 2004; Sawyer, 2004) while other studies have focused on the types of tasks that can promote mathematical creativity (Kwon, Park, & Park, 2006; Silver, 1997). However, creativity is not always something that can be planned for or predicted. More likely, it can be occasioned. Especially in the classroom, where students interact with each other, the teacher, the content, and the environment, it may be said that the creativity that emerges is the result of different agents coming together to complement each other thus opening up possibilities that might not have existed or been acted upon at the individual level. Put simply, viewing the classroom as a complex system affords us the possibility of examining how creativity may emerge. The next section briefly reviews theories related to complex systems and how they relate to education.

THE CLASSROOM AS A COMPLEX ADAPTIVE SYSTEM

The classroom may be referred to as a Complex Adaptive System “in which many players are all adapting to each other and where the emerging future is very hard to predict” (Axelrod & Cohen, 1999, p. xi). This is in contrast to a simple system, which has a limited number of components, few interactions, and is decomposable (Casti, 1994). If the connections between the components are severed in a simple system, the system will more or less function as it did before. The classroom may be viewed as a complex system because the components of the system—the teacher and the individual students—may have different goals and driving forces, yet each individual is highly connected with the other; the decisions and actions of one may affect the decisions and actions of others (Hurford, 2010). In addition, the knowledge and insights that are shared and creativity that emerges can hardly be predicted at the onset of the lesson.

Emergence is a central theme of complexity theory. It implies that “given a significant degree of complexity in a particular environment, or critical mass, new properties and behaviours emerge that are not contained in the essence of the constituent elements,
or able to be predicted from a knowledge of initial conditions” (Mason, 2008, p. 2). In other words, a slight change in initial conditions, or a slight change in the behaviour of one agent, may shift the direction of the larger system. The system maintains itself by adapting to new situations and learning from emergent interactions.

Davis and Simmt (2003), in their review of the parallels between complexity science and theories of knowing, suggest five features or conditions that must be met in order for systems to arise and maintain their ability to adapt and learn: (1) internal diversity, (2) redundancy, (3) decentralised control, (4) organised randomness, and (5) neighbour interactions. Internal diversity refers to the different ways members of the community contribute to finding solutions to a given problem. In a sense, it represents the possible responses to emergent circumstances and thus its quality ensures the survival of the system. On the other hand, if members of the system are to communicate, they must share some similarities such as background, language, and purpose. Redundancy refers to the similarities that enable the system to cope with stress and allow for different members to compensate for others’ failings. The complements of internal diversity and redundancy are also related to studies that have investigated group creativity. For example, a situation where diverse individuals come together to solve a problem (such as in the work place) may begin with divergent thinking, but eventually, ideas must converge in order to solve the problem at hand. On the one hand, the different backgrounds and knowledge bases of a diverse group may contribute different perspectives for consideration. On the other hand, diversity may be so wide as to hinder individuals as they strive to understand different ideas and come up with an agreed-upon solution (Kurtzberg & Amabile, 2001).

When viewing the classroom as a system, decentralised control means allowing students to share in the decisions about what is and what is not acceptable. Organised randomness refers to a “structural condition that helps to determine the balance between redundancy and diversity among agents” (Davis & Simmt, 2003, p. 154). In a complex system there are boundaries and constraints, but the possibilities within these boundaries are rich and numerous. In the mathematics classroom this could mean providing enough of a framework so that students can focus on a certain problem while simultaneously allowing sufficient freedom for students to offer various and flexible responses. The last condition, neighbour interactions, refers to mathematical ideas or insights that interact with each other. In the following sections, I illustrate how mathematical creativity can be seen to emerge by viewing the classroom as a complex system.

CLASSROOM EPISODES

Setting

In this section, three episodes are reviewed. Each episode depicts segments of a mathematics classroom where the teacher was observed teaching a “regular”
INVESTIGATING MATHEMATICAL CREATIVITY IN ELEMENTARY SCHOOL

...lesson. That is, the lessons were not planned specifically for observation nor did they have specific aims other than to review or teach what was considered part of the scheduled content sequence. The teachers taught according to the mandatory mathematics curriculum using state approved textbooks. They taught in local public schools located in the same middle-income suburb of Tel Aviv, a major city in Israel. During the school year, each class was observed approximately ten times. During classroom observations, the focus was on students’ interactions with materials, other students, and teachers and the ways in which “ideas are picked up, worked with, and developed by the group” (Martin, Towers, & Pirie, 2006, p. 152). All lessons were video recorded and transcribed by the researcher who also took field notes during the observations. For each episode, some preliminary background of the classroom is offered, followed by a transcription with minimal comments. After the transcription, elements of mathematical creativity evident in the transcription are reviewed. The emergence of this creativity is then analyzed through the lens of complexity theory by referring to some of the features of complex systems such as internal diversity and redundancy.

Episode 1: Internal Diversity, Redundancy, and Occasioning Mathematical Creativity

This episode was taken from a sixth grade class consisting of 28 students, 16 girls and 12 boys. The teacher, Hailey (not her real name), had 14 years experience teaching mathematics in the elementary school system, mostly teaching fifth and sixth grade classes. The lesson depicted below took place in the middle of the school year, where the main topic of the lesson was multiplication of decimal fractions. The class had already been introduced to this topic and had already practiced the procedure for multiplying decimal fractions during previous lessons. Hailey put the following problem on the board: \( \_ \times \_ = 0.18 \), and asked the class, “What could the missing numbers possibly be?” Many children raised their hands and the teacher commented, “There are many possibilities.” She then called on one at a time:

Gil: 0.9 times 0.2.
Teacher: Another way. There are many ways.
Lolly: 0.6 times 0.3.
Teacher: More.
Tammy: 0.90 times 0.20.
Teacher: Would you agree with me that 0.2 and 0.9 is the same [as 0.90 and 0.20]? I want different.
Miri: I’m not sure. 9 times 0.02.
Teacher: Nice. Can someone explain what she did?

(The teacher and students then review the rules for multiplying decimal fractions.)

At this point, note that although Gil and Lolly gave different answers both may be considered similar in that they consisted of two numbers with one digit after
the decimal point. Tammy broke the mould by using numbers with two digits after the decimal point. From a mathematical point of view, as noted by the teacher, Tammy’s answer is the same as Gil’s. On the other hand, from a student’s point of view, 0.9 may be very different from 0.90. In addition, as will be shown later on, different representation of the same number may afford different possibilities and thus representing 0.9 as 0.90 may not only be acceptable but even preferable.

Tom:    What about 0.18 times 0.1?
Tad:    No.
Teacher:   Why not?
Mark:    [The answer would be] 0.018 because there would be three digits after the decimal point.
Teacher:    Ah. Ok. Thank you. We want a number with two digits after the decimal point.
Gad:    0.18 times 1.
Ben:    And 1 times 0.18.
Teacher:    You’re using the commutative property of multiplication. But, it’s really the same as Gad’s answer.
Toby:    18 times 0.1?
Many students:    18 times 0.01.
Teacher:    Let’s move to another problem. (The teacher writes on the board the following problem: __ × __ = 0.012.)

Regarding mathematical creativity, the task presented by Hailey may be called a multiple-solution task that invites divergent thinking (Leiken, 2009). All together, the class produced five different correct solutions. Perhaps, if more time was available, the class could have produced more solutions. Regarding flexibility, the second solution, 0.6×0.3 followed more or less the same strategy as the first solution 0.9×0.2. The last three solutions, 9×0.02, 0.18×1, and 18×0.01, differ from the first two solutions but may be considered similar to each other. Each example consists of one factor that is a whole number and a second factor that is a decimal fraction with two digits after the decimal point. In addition, there was one attempt to find a solution that included a factor with one digit after the decimal point and a second factor with two digits after the decimal point.

Viewing the episode through the lens of complexity theory, what stands out is the balance between internal diversity and redundancy that allows the students to not only raise different suggestions but to evaluate each other’s suggestions as well. Diversity was specifically promoted by the teacher who encouraged the students to find different solutions. Diversity may also be found in the various solutions, acceptable and unacceptable, that arose during the episode. Diversity may also be seen in the different ways in which students contributed. Some students hesitantly offered solutions implicitly seeking confirmation. Others boldly stated their solution. Still others took the role of evaluators. Yet, the students were also able to compensate for each other’s deficiencies. There were three instances where one student put forth an incorrect or unacceptable solution and others, building on the idea, corrected the
situation. When the teacher does not accept Tammy’s solution of 0.90 times 0.20, Miri is able to build on Tammy’s idea of changing the place of the decimal point and come up with an acceptable solution. When Tom changes both the factors and the place of the decimal point, he comes up with an incorrect solution. Gad, using the same factors, comes up with a correct solution. When Toby suggests the incorrect solution of 18 times 0.01, many students chime in with the correct solution without the teacher intervening. In other words, the dynamic interactions among participants allow the system to right itself.

Another feature of a complex system is decentralised control. In this episode, although the teacher introduced the task, she invited the students to “agree” with her that 0.9 times 0.2 is the same solution as 0.90 times 0.20. More poignantly, we see the students beginning to evaluate each other’s solutions while the teacher merely asks them to explain their disagreement. Organised randomness was produced in one sense from the task itself bound by the need to find factors that multiply to 0.18 and yet with many possible solutions. To this, the teacher added additional bounds, pointing out that 0.2 and 0.20 are essentially the same and that using the commutative property of multiplication does not lead to different solutions. As opposed to the teacher’s invitation to the students to agree with her that 0.9 is the same as 0.90, in the case of the commutative property, she declares the status of this solution without attempting to view the solution from the student’s point of view. In other words, the teacher set certain constraints but still left room for variation. Finally, when considering neighbour interactions, two major ideas may be considered “neighbours” in that they are tossed around and interact with each other producing a variety of solutions: the number of digits after the decimal point and the factors of eighteen. To summarise, the collective fluency and collective flexibility exhibited in these episodes (Levenson, 2011) can be seen to emerge from the classroom behaving as a complex system.

**Episode 2a: Balancing Stability and Change in a Fifth Grade Classroom**

This episode took place in a fifth grade classroom taught by Nina (not her real name). Nina had eight years experience teaching fifth and sixth grades. There were 28 students in the class, 12 girls and 16 boys. The students had previously been introduced to decimal fractions, had learned to convert back and forth between decimal fractions and simple fractions, and they had recently learned to add and subtract decimal fractions. The main topic of the current lesson was reviewing addition and subtraction of decimal fractions. The following problem, taken from the classroom textbook, was given as a homework assignment by the teacher and was reviewed in class at the request of one of the students.

Complete the following sequence:

\[
\begin{array}{c}
50 \\
100 100 100 100
\end{array}
\]
The following discussion ensues:

Teacher: After 30/100, mmm hundredths, and again, mmm hundredths, and again. They want a sequence. What is a sequence?
Uri: It continues with jumps.
Teacher: Equal jumps. The jumps must be equal. What types of jumps are there? (A few students say out loud different numbers: 25 and 25/100.)
Teacher: That’s the size of the jump. You mean to add 25/100.
Uri: Can the jumps be in multiplication?
Teacher: Wonderful. That’s exactly what I mean. If I jump by adding 25/100 then the next will be 50/100. Now, you mentioned another type of jump. We didn’t learn that yet…it’s part of next year’s syllabus. But, there are also multiplication jumps. Who said that going from 5/100 to 30/100 means that I added 25 [hundredths]? I can also multiply…
Sam: By 6.
Sarah: 180.
Teacher: 180 hundredths.

The first part of this exchange focuses on clarifying what is meant by jumps (they have to be equal), types of jumps (adding or multiplying), and sizes of jumps (25/100). It is worth noting that Uri thought of multiplication jumps despite the fact that multiplication of simple fractions and decimal fractions had not yet been introduced. Furthermore, although it is technically part of next year’s curriculum, the teacher does not dismiss this idea. Finally, two more children contribute to the idea by carrying out the actual multiplication.

Uri: That’s what I did at first. But I thought it was a mistake.
Teacher: Is that allowed?
Tina: I thought that it would be a mistake.
Nat: But then you get big numbers.
Teacher: So, you can use a calculator.
Nat: Then you can also divide.
Teacher: You can divide, but not here (referring to the jump from 5/100 to 30/100).
Nat: You can multiply by 6 and then divide by 3.
Teacher: Ok. That’s also a type of sequence. Multiply by 6, divide by 3, and then again multiply by 6 and divide by 3.
Tina: But, that’s not good. You need equal numbers.
Teacher: This is a different type, but it is certainly acceptable. Let’s try it.
Tina: But, it won’t come out. You need equal numbers.
Teacher: Let’s just say that when the textbook requests a sequence, they generally don’t mean this type. They usually mean jumps that are the same each time. But, this is definitely a sequence.
Tina: But, they are not all equal.
Dan: You can also have more than two types of jumps. Multiply, divide, and add.
This episode displays several instances of mathematical creativity. Most notable are the original ideas of Uri and Nat. Uri suggests using multiplication, despite the fact that this operation has not been introduced to the class in conjunction with fractions. Nat suggests continuing the sequence with unequal jumps, an original idea in light of the teacher’s comment that jumps must be equal. Finally, Dan suggests employing three mathematical operations at once in the same sequence. Perhaps, employing multiplication and division may be allowed because they are essentially inverse operations. But to consider addition in the same sequence as multiplication and division is indeed original. In short, the mathematical creativity displayed in this episode by the students is an illustration of what Liljedahl and Sriraman (2006) termed school-level mathematical creativity. It includes unusual solutions to a given problem that essentially show how students can view a familiar problem (such as continuing a sequence) from a new angle, raising new questions and possibilities (such as having unequal jumps).

Regarding flexibility and its counterpart fixation, two students claim that they thought of using multiplication but dismissed the possibility. Nat, who claims that he would end up with “big numbers,” shows signs of content fixation. Recall that according to Haylock (1997), overcoming content fixation requires the thinker to consider a wider set of possibilities than at first is obvious and to extend the range of elements appropriate for application. In this case, the content fixation may have been brought on by previous textbook examples that refrained from using “big” numbers. Keeping the door open for additional possibilities, the teacher is quick to negate this excuse. Going from a standard review exercise to exploring the nature of mathematical sequences, the teacher also exhibits flexibility in her willingness to change directions. All together the class came up with four different solutions to the problem.

When viewing this episode through the lens of complexity theory, what first comes to mind is the tension between pursuing stability and change (O’Day, 2002). On the one hand, the teacher begins with the familiar practice of reviewing homework. Knowing that the aim of the homework is to practice adding fractions, the teacher advocates using equal jumps to finish the sequence. So far, we have stability. Using multiplication jumps, Uri introduces a change. This results in another change, the production of big numbers. Uri’s reluctance to use multiplication may be attributed to content fixation or to his desire for stability. We thus begin to see a connection between creativity and complexity theory. When the teacher and students are ready to give up stability, they move forward with original ideas. Throughout the episode, Tina is the voice of stability as she repeatedly calls for equal jumps. The teacher attempts to pursue both stability and change by acknowledging that equal jumps may be the norm and possibly what the textbook author implied, but unequal jumps are acceptable. Finally, the system adapts to the original idea of unequal jumps and accepts sequences that employ two and even three different operations.

Referring back to the features of a complex system mentioned by Davis and Simmt (2003), internal diversity can be found in the different roles played by the
participants. Some students raise ideas (Uri, Nat, and Dan), some act upon the ideas of others by carrying out the arithmetic (Sam and Sarah), while others raise objections (Tina). Yet the students and teacher were able to interact and build upon each other’s ideas. Uri suggests using multiplication jumps. Nat expands upon Uri’s idea by suggesting the use of both multiplication and division jumps in the same sequence. It seems like he is responding to the problem of producing big numbers. If we employ division jumps along with multiplication jumps, then the sequence will not increase so rapidly. Dan continued Nat’s premise of using unequal jumps and suggests employing three mathematical operations at once in the same sequence. Would Dan have come up with this idea if Nat had not previously suggested using unequal jumps? Would Nat have suggested using unequal jumps if Uri had not brought up the possibility of multiplication jumps? Of course, we cannot answer these questions. However, these questions illustrate the interaction between the participants as well as the interaction among ideas that Davis and Simmt (2003) call neighbour interactions.

Another feature of the complex system is decentralised control. Is there one person at the helm steering the others? While at first it seems that the teacher is setting the rules; she quickly allows the students to change the course of action. She also asks the other students if multiplication jumps are allowed, deferring her judgment until the students have thought about it themselves. Finally, while the activity is bound by the need to continue a mathematical sequence, there is sufficient room for several possibilities to arise. In other words, there is organised randomness. To summarise, the original ideas that emerge in this lesson can be seen to emerge from balancing change with stability and are supported by the classroom acting as a complex system.

**Episode 2b: Adapting to the Emergence of Creativity**

This episode is a direct continuation of the previous episode. After the above discussion, the teacher goes back to reviewing decimal expansions, addition and subtractions of decimal fractions, and reviews another homework problem involving a sequence. She then presents another sequence from the classroom textbook, but one that was not given for homework.

Teacher: Here’s one that’s more difficult. Let’s look at another problem. Build a sequence that has in it the numbers 0.2 and 1.1.

(Four children raise their hands.)

Teacher: (Four children raise their hands.)

Dan: Add 0.9.

Teacher: You’re saying to place them next to each other and then the difference is 0.9. Then what would be the next number?

Judy: 2.

Teacher: And then?

Mark: 2.9.

Teacher: But you can make a different sequence. Who says that the two numbers
have to be next to each other? It doesn’t say that they have to be one next to the other.

Tali: You can do jumps of 0.3.

Teacher: Jumps of 0.3. Let’s see. What would come next? (The teacher writes on the board 0.2, 0.5, 0.8, 1.1) Are there more [ways to complete the sequence]?

(The bell rings signalling the end of the period, but the teacher continues.)

Teacher: Sit a minute. One more second.

Dan: You can put the sequence in backwards order and do subtraction.

Teacher: Ok. You can start with 1.1. (The teacher places this sequence on the board.)

Unlike the previous task where the first two numbers in the sequence were given, in this task, two numbers are given but are not placed in any specific order. In essence, this task as well as the previous task, are examples of what Sullivan, Warren, and White (2000) termed a content-specific open-ended task. Both tasks have a starting point but an open goal. While students have to build a sequence, the type of sequence, the types of jumps, and the length of the sequence are not given. The second task has a greater degree of openness in that the starting point is also undetermined. These tasks foster activities such as investigating, creating, and communicating and often require creative thinking. In the above segment, the teacher takes advantage of the situation in order to promote flexibility. In other words, she seems to be less interested in promoting fluency and more interested in trying to encourage the students to think of various ways of placing the numbers. Moving in an entirely different direction from the ones suggested by the students, she raises another suggestion:

Teacher: I have another idea. You can expand the numbers. (The teacher writes on the board 0.20, leaves a lot of space, and then writes 1.10)

Tomer: 0.9

Shay: Nine and a half.

Teacher: 0.90 so the expansion is by 10 and then I can do jumps of 0.45. Is that allowed?

Tomer: Yes.

First, looking at the different solutions to this problem, we may count four solutions where each solution stems from a very different way of combining the numbers into a sequence. The last solution was presented by the teacher. In this last segment, a shift takes place, not only in the mathematics involved but in the prime initiator of creativity. While in the first part of Episode 2a and in the first part of Episode 2b, various students suggest different solutions. In this last segment, it is the teacher who exhibits creative thinking by expanding the given decimals from tenths to hundredths. Recall that the teacher in the first episode claims that 0.9 is the same as 0.90. In this episode, the teacher specifically expands 0.9 to 0.90 in order to open up the way for many more possibilities and solutions. In other words, she adapts the representation of the number to suit her goals. It is the teacher who displays
flexibility, joining the collective effort to come up with various ways of placing the two numbers in a sequence.

That the teacher joins in in the effort to find an original solution is significant also in terms of viewing the classroom as a complex system. Recall that one of the hallmarks of a complex system is its ability to adapt and respond to the environment. Throughout Episodes 2a and 2b, we see this process unfolding by following the interactions among the students and the teacher as well as among the students and themselves. Slowly but surely we see the teacher becoming more and more involved and drawn into the creative process—first by monitoring novel ideas, then by accepting them, and finally becoming totally immersed in the creative activity. We see how the teacher chooses an additional open-ended task more challenging than the previous one and offering additional opportunities for flexible thinking. Then, the teacher insists on continuing the lesson even after the bell rings to signal the end of the period. Finally, she herself comes up with an original solution to the task. The teacher no longer stands apart but has integrated into the classroom system. The system has adapted and responded to the environment. To summarise, the creativity displayed in this episode can be seen to emerge from the adaptations and internal adjustments that evolved during the episode.

Episode 3: How Can Insight Displayed by One Individual be Viewed through the Lens of Complexity Theory?

In this episode, I deviate from the path. Previously, I presented episodes in which it was evident that many participants were involved in the creative process. This episode focuses on one individual and raises several questions regarding how creativity may or may not emerge in a classroom for which it is unclear if a complex system is in place.

The same teacher, Hailey, from the first episode, also taught a fifth grade classroom, which included 32 students, 15 girls and 17 boys. In this episode, Hailey was introducing for the first time subtraction of mixed numerals. The following example is written on the board:

\[
\begin{array}{c}
3 \frac{1}{2} \\
-1 \frac{1}{2}
\end{array}
\]

Teacher: Let’s think together. We’ll solve this problem in column form. What do I have to do?

(Many students raise their hands.)
Ian: One half is three-sixths. Two-thirds is four-sixths.
Teacher: Now, wait a minute. What did Ian do? Raise your hands. What did Ian want to do?
Penny: Expansion.
Teacher: He wants to do expansion so both fractions will have what?
Abe: So, they’ll have the same six so that you can solve it. And there is a half which is three out of the whole – uh – six. And two-thirds is 4…
Teacher: So, what is he doing?
Harold: A common denominator.
Teacher: So, what should I write here?
Harold: Three out of six.
Teacher: You lost something on the way.
Miri: The half.
Abe: The three wholes.
Teacher: So, it’s three and three-sixths.

The teacher, together with her students, brings the second fraction also to a common denominator and rewrites the problem. The following discussion then ensues:

Teacher: From which side should we begin to solve this problem? From the fractions or the whole numbers?
(Students debate from where to begin.)
Miri: From the fractions.
Teacher: We start to solve this problem from the smallest place value. What does this remind you of? Which other problems do we start from the smallest place value?
Lev: Subtraction and addition (of multi-digit numbers) written in column form.
Teacher: Correct.

The teacher then poses the problem of taking away 4/6 from 3/6 and together with the students they exchange the 3 and 3/6 to 2 and 9/6 and proceed to subtract. To summarise the procedure, the teacher once again reminds the class that subtracting fractions is essentially the same as subtracting whole numbers claiming, “You broke up the whole. Just like we solve subtraction (of multi-digit numbers)… say 33 take away 14… you take from the tens digit and add to the ones digit.”

Up until this point, the episode seems devoid of creativity. In addition, it is difficult to view the class as a complex system. The teacher seems to be in control. She has a goal and is working towards that goal. After doing out loud another similar example, a student raises her hand:

Amy: I have a question, but it’s not exactly related.
Teacher: That’s OK. It doesn’t matter [if it’s not related].
Amy: Let’s say I had the problem 32 take away 34. I would have to do an exchange. You can’t do two minus four.
(The teacher writes the example in column form on the board.)
Amy: So can you do the exchange?
(The teacher writes on the board.)

Amy: So, 12 take away four is eight. And then two take away three.
Don: Is minus.
Teacher: Minus one. But this -1 is in the tens digit. So it's minus ten plus eight.
Which is?

In the beginning, this lesson seems unrelated to creativity. There is no problem posed or task for the students to implement that might occasion creativity. Seemingly out of the blue, Amy poses a question. She inquires about extending the procedure of subtraction to a new domain, that of negative numbers, a domain not yet introduced in fifth grade. In essence, Amy is employing the rules and practices of a domain (subtraction that requires regrouping) to produce a novel (for her and for the class) variation within the domain content. According to Sriraman (2008), this is one aspect of being creative. It may also be said that Amy raises new questions and possibilities related to the old problem of subtracting, another aspect of creativity. Finally, the time it took for Amy to raise the possibility of extending subtraction with regrouping to negative numbers reminds us of the incubation period that may be considered part of the creative process. In fact, Sriraman (2008) points out that in general, a mathematician’s creative process follows the four-stage Gestalt model of preparation, incubation, illumination, and verification. In the above lesson, we see a microcosm of these stages. The teacher prepares the stage by analysing in detail the process of subtracting mixed numerals and relating this to subtraction of multi-digit numbers learned previously. During this time, Amy is quiet. We do not hear her voice, although, we can infer by what comes later, that she was listening and processing what was going on. Following this quiet period, Amy comes up with a question, which in her opinion, is not quite related to the classroom topic. While we cannot know for sure when exactly she came up with her idea, it reminds us of the incubation period followed by the illumination. Next, the teacher helps her verify that indeed her idea is on target.

While different aspects of mathematical creativity were noted in this episode, it is less obvious with this lesson that the class is behaving as a complex system. It is difficult to detect the diversity. The teacher seems to be in control. Where is the organised randomness?

Yet complexity theory is not only about viewing the classroom as a complex system. It is about viewing learning as an emergent process and viewing knowledge as distributed. Recall that a simple system is one that may be deconstructed into its components and where the outcome may be predicted. Amy’s insightful question could not have been predicted nor can it be explained without looking at the context in which it took place.

To begin with, the teacher does not merely dictate a procedure. She involves the students at each step. According to Morrison (2008), “In complexity theory, learning becomes a joint voyage of exploration, not simply of recycling given knowledge ... The teacher is vital, intervening judiciously to scaffold and create the conditions for
learning-through-self-organization and the child’s emergent knowledge…” (p. 23). Noticeably, the teacher connects the new procedure to previously learned procedures and emphasises the mathematical principles that are the basis for these procedures. In other words, what is emphasised in this lesson is what Davis and Simmt (2003) termed neighbour interactions, mathematical ideas or insights that interact with each other. Thus, instead of focusing on the interaction between participants, in this episode the focus is on the dynamic interaction and interdependence of ideas. Diversity may be seen in the ideas – subtraction of mixed numerals, subtraction of multi-digit numbers, and subtraction resulting in negative numbers – yet they are similar enough (i.e. redundant) to enable interactions. Finally, while the teacher had a goal and aimed to navigate the lesson and the students’ learning, she was open to seemingly unrelated questions. Amy’s insightful extension of the mathematical principle behind subtraction with regrouping to a new domain of numbers can be seen to emerge as the boundaries of the lesson are stretched. As such, we may surmise that while there were boundaries, there was also an openness to bend these boundaries and that Amy, and most probably other students as well, were aware of this possibility. In other words, there was organised randomness. To summarise, in this episode it might seem that one individual displays creative thinking. But by viewing the episode through the lens of complexity theory, the individual’s creativity can be seen to emerge from the dynamic interaction of ideas in an environment that supports adaptations.

RELATING COMPLEXITY THEORY TO EMERGENT CREATIVITY

In the previous section, complexity theory was used to analyse emergent creativity. Taking a closer look, each episode focused on different aspects of creativity. In addition, although complexity theory in general was used to analyse each episode, different features of this theory are emphasised in different episodes. Is it possible that when certain features of a complex system are emphasised, different aspects of creativity will emerge?

In the first episode, the teacher encouraged fluency by presenting a multiple-solution task and explicitly stating that the task had many solutions. Through the lens of creativity, we saw an emphasis on diversity, both in the solutions and in the different ways students participated in the lesson. While of course a certain amount of redundancy was necessary for the interactions to be effective, it is possible that an emphasis on internal diversity promotes the fluency of ideas—one aspect of mathematical creativity. In both the first and second half of the second episode, the aspects of creativity that are most notable are flexible thinking, which leads to novel solutions. In the first half of the episode, the lens of complexity theory focuses on how the system balances between stability and change. In the second half of the episode, the focus was on the system’s adaptation to the environment. Essentially, the system’s ability to adapt is a result of the balance between stability and change. Thus, it is possible that change and adaptations lead to novel ideas. In the first two
episodes, creativity is evident in a number of classroom participants. In the last episode, creativity is evident with one individual for whom the process of creativity seems to follow the path of preparation, incubation, illumination, and validation. During this episode, the lens of complexity theory focuses on the interactions between ideas mediated by the teacher in conjunction with the students. Perhaps paying meticulous attention to mathematical properties along with focusing on the interaction between these properties lays the groundwork for insightful questions. Perhaps the relative stability promoted by the teacher within the complex system of the classroom allows for individuals to take their time in adapting new knowledge to existing knowledge thus allowing creativity to emerge from the process.

Of course, it is too simplistic to claim that any one single feature of a complex system is responsible for one particular aspect of creativity. In fact, the very idea of decomposing the system into its parts negates one of the basic principles of complexity theory—that the whole is greater than the sum of its parts. However, recognizing the classroom as a complex system does not mean we ignore the features of the system. Instead, as exemplified in the episodes, we can use principles of complexity such as internal diversity, redundancy, and decentralised control to organise the classroom in a way that supports the emergence of different aspects of mathematical creativity.

The contribution of complexity theory to the study of creativity in the mathematics classroom is not only about viewing the classroom as a complex system, it is about viewing knowledge and creativity as emergent processes and viewing knowledge, and possibly also creativity, as distributed. Teachers should be aware that providing tasks that may occasion creativity is but one step towards promoting mathematical creativity. Creativity cannot be made to happen. However, setting up a classroom environment that encourages diversity, supports interactions among participants and ideas, and allows for a certain amount of instability, may well support the emergence of creativity even when it is not planned for.

REFERENCES

INVESTIGATING MATHEMATICAL CREATIVITY IN ELEMENTARY SCHOOL


STEVE V. COXON

ON THE EDGE OF CHAOS: ROBOTS IN THE CLASSROOM

Robot is a term first coined in 1920 by Czech playwright Karel Capek from the Czech word for forced labor, *robota* (James & Leon, n.d.). Although once only a playwright’s fancy, today’s robots labor around the world and beyond, and they are playing increasingly important roles in society. Robots are autonomous machines that respond with motor movements to input from the external world through sensors in accordance with computer programs. From autonomous submarines working to stop a deep sea oil spill disaster, to robotics in industry, surgery, homes, military operations, space exploration, and rescue, robots do work for humans that is monotonous, tiring, dangerous, or beyond human capacity. Even the most complex computer programs are vastly simpler and less chaotic than human thinking and the most complex machines are likewise simple in comparison to living systems. Robots can be seen as increasing order in the world and as very simple in their functioning relative to humans. One may place robots in the far lower right hand corner of the Chaos-Order Continuum (Ambrose, 2009). It is instead student thinking and interactions that I argue in this chapter can be pushed to the Edge of Chaos in constructivist robotics-based educational programs. While robots are themselves arguably simple, students working in groups to design, build, and program robots to solve difficult problems requires complex thinking and interaction quite different from within a traditional classroom setting.

To understand the complexity involved for student thinking and interactions in robotics educational programs as they are explained below within a constructivist context, it is important here to draw a contrast with the traditional classroom setting. Robotics educational programs usually fall into two categories: academic competitions outside of the school day as a series of open-ended and challenging tasks and classroom use of robotics similar to problem-based learning. Both activities are constructivist in nature as children work to construct meaning by working with others with great autonomy, physical materials, and tasks that require higher order thinking processes. Such educational programs lead to much more complex thinking and interactions for students than the traditional classroom in which a teacher and text are seen as the sole sources of information, work is individualised and simplistic in nature with close-ended, single-answer problems, and children are dominated by the teacher to restrict their movements, discussions, and habits. Robotics programs have been fundamentally different from their inception.
HISTORY AND PROGRAMS

As with other models of classrooms that present an alternative to the traditional characterisation above, such as constructivist classroom models, robotics educational programs have existed longer than many today would expect. The Soviet Union’s 1957 launch of Sputnik had such a large impact on American science, technology, engineering, and math (STEM) education (Flanagan, 1979; Super & Bachrach, 1957; Wai, Lubinski, & Benbow, 2009), that it could arguably be considered the impetus for robotics education in the U.S., despite the fact that the first programmable robot had only just been developed (Robots.com, n.d.). Robotics use in the classroom was not immediate, but began just over 20 years after Sputnik. While robots were just becoming common to industry in the 1970s (Robots.com, n.d.), LEGO Logo, which connects LEGO bricks and motors to the then-popular LOGO programming language, was introduced for children in the early-1980s (Fox, 2007; Logo Foundation, 2000). In contrast with traditional modes of instruction, working with LEGO LOGO involved student application of understanding computer programming as opposed to the mere consumption of information. To wit, LOGO inventor Seymour Papert was a protégé of Jean Piaget (MIT Media Lab, 2007).

Still, there is no evidence that the use of robotics in schools was widespread until the advent of LEGO MINDSTORMS and the corresponding FIRST LEGO League (FLL) competition that began in the late 1990s. Notably, the LEGO product line takes its name from Papert’s (1980) book Mindstorms: Children, Computers and Powerful Ideas. Today, many kits exist for engaging students in robotics including with K’NEX, LEGO MINDSTORMS, the T-Bot mechanical arm, and Tetrix, which allow students to build sturdy robots with aircraft-grade aluminum. There are several robotics competitions available to K-12 students at international, national, and regional levels including FIRST Robotics Competition (FRC), Junior FIRST LEGO League (Jr.FLL), FLL, Fire Fighting Robot Contest, VEX Robotics, and Carnegie Mellon Mobot Races (Coxon, 2009; Robots.net, n.d.; Tallent-Runnels & Candler-Lotven, 2008). Curriculum units for classroom instruction are available for several of the kits both from some of the companies that produce the kits and from third parties (e.g., Coxon, 2010; Toye & Williams, n.d.).

LEGO MINDSTORMS Kits

The most common robotics kit for both competition and classroom use are the MINDSTORMS sets sold by LEGO Group, now in the third generation (EV3). While other kits exist, most are either pre-built robots that children may program or are not actually robots in that they operate by remote control (and therefore not autonomously based on input from the external environment via sensors). The EV3 is an update to the NXT and RCX kits that have been sold since the 1990s. The EV3 includes 541 pieces that, as with other LEGO sets, can be combined in a very large number of ways. While robots are simple in comparison to living systems,
there is a great deal of complexity here in terms of allowing practically unlimited possibility for student building. Danish mathematicians have demonstrated that six 2 x 4 LEGO bricks of the same color can be combined in nearly one billion different ways (Eilers, Abrahamsen, & Durhuus, 2005). The possibilities in combining 541 pieces, most of which are more complex themselves than the 2 x 4 bricks in the above calculation are, for all practical purposes, limitless. Increasing complexity further, the kit includes three to four motors and five sensors: color, gyro, ultrasonic, and two-touch. The sensors allow for various inputs from the external world to lead to motor operations based on computer programs written by children. While instructions for several robots are provided with the kit and hundreds are available online or in books, the number of combinations of pieces is limited only by the user and the problem at hand that robot is engineered by children to solve.

**FIRST LEGO League**

Competitions are one of the two primary delivery models for the use of robotics kits that I focus on in terms of complexity. The largest competition and the focus of this chapter is the FLL for ages 9-14 utilising the LEGO MINDSTORMS kit. The competitions have become widely available and are growing fast. FIRST programs now have a total reach of more than 300,000 K-12 students in more than 70 countries after beginning in a single high school gym with a few dozen students in 1992 (US FIRST, 2013). FLL is an academic competition in which students build robots to manipulate LEGO objects based on a real-world science theme. For example, in a recent FLL competition, Power Puzzle, the theme was energy production and use. Participants were required to build and program a robot that could add a solar panel to a house and replace a pick-up truck in its driveway with a fuel cell car—all made from LEGO bricks. The competitions have multiple facets. Not only do robots compete, but participants also compete for awards in teamwork, robot design, and a presentation on a public service research project that they conduct before the competition. Each facet of the competition adds greatly to the overall complexity of student thinking and interactions within the program.

**Robotics in the Classroom**

Increasingly, robotics have been incorporated into science education programs including the school day and in after-school programs other than competitions. Designing, building, and programming robotics is seen as an introduction and inspiration to engage children and adolescents in STEM fields. These fields account for the majority of America’s economic growth and tremendous improvement of the human condition (National Academy of Sciences, 2005). Problem-based learning (PBL), such as can be conducted with robotics, is potentially engaging for children including the gifted (Allen, 1996). In PBL, students are given a multi-faceted problem and materials. They are then tasked with solving aspects of the problem.
through research, experimentation, or engineering (as with LEGO robotics). Engagement in science learning through real-world problem-solving potentially leads to college majors in STEM fields (Wai, Lubinski, & Benbow, 2009). Meta-analysis has revealed that, with a powerful effect size of 1.48, enhanced context strategies have the highest effect size on achievement of all researched forms of science education (Schroeder, Scott, Tolson, Huang, & Lee, 2007). According to Schroeder et al. (2007), enhanced context strategies include real-world learning and problem-based learning. Robotics is especially appropriate for aiding with both of these enhanced contexts in the classroom. The same meta-analysis found that instructional technology use has a moderate overall effect size of .48 on science achievement. Robotics falls into this category as well. While a smaller effect size, it is still a meaningful one. Some research already exists specifically on the effects of robotics use in classrooms, which will be explored here.

Research on Robotics in the Classroom

As is common to educational technology generally, the research on participant outcomes lags behind usage. The use of robots in education had rarely been researched in K-12 classrooms prior to 2010, but is now growing rapidly. In a search of ERIC database using the keyword “robot,” only 35 articles were published between 2000 and 2010. However, between 2010 and 2012, 89 new articles appeared. Only a relative few of these involved K-12 environments with many others used in laboratory settings, medicine, and at the university level. To understand the complexity involved, a selection of those studies that involve K-12 classrooms is presented here.

Of the K-12 studies available, the greatest number involve the FIRST competitions, the largest and most widely distributed robotics programs, followed by those that use the same MINDSTORMS sets in the classroom. Perhaps the largest study to date was conducted with participants involved in the FRC, a high school level competition, by the Center for Youth and Communities (CYC) at Brandeis University. Melchior, Cohen, Cutter, and Leavitt (2005) conducted a follow-up survey of participants, coaches, and parents, finding that, in comparison to their peers, FRC participants were 35% more likely to attend college, twice as likely to major in a STEM field, nine times as likely to have an internship during their college freshman year, and even twice as likely to perform community service. Of course, it is unclear if those differences were created by FRC participation or if they are simply indicative of students interested in such competitions. In a similar study from CYC of the FLL, an elementary and middle school level competition, Melchior, Cutter, and Cohen (2004) conducted a survey of FLL participants, coaches, and parents. Of those surveyed, 94% or more believed that FLL participants had increases in such areas as programming skills, understanding of how science and technology can solve real-world problems, problem-solving skills, and leadership skills.

Some studies of FLL suggest that student learning in robotics competitions may generalise to other contexts. In a qualitative study, Petre and Price (2004) observed
several robotics competitions in the Seattle area, including FLL, and interviewed participants and coaches. Key themes that emerged included students’ desires to complete the tasks, the open-endedness of competition, and the social context. Based on these interviews, the researchers suggest that robotics works effectively to increase understanding of programming and engineering principles, and that this learning was generalisable to other programming and engineering situations.

Geeter, Golder, and Nordin (2002) were attempting to increase the number of FLL teams in Iowa through a university-based program. Based on their observations during this time, they reported that middle school students competing in FLL gained a better understanding of engineering; improved creative thinking, critical thinking, and problem-solving skills; and increased self-confidence levels, interest, and involvement in science and math. The authors suggested that these skills would help students regardless of chosen career path, but do not report on their methods for making these assertions aside from observation during their program.

Robotics has been less well studied in the classroom. The studies that do exist continue to explore robotics education largely through qualitative means. Korchnov and Verner (2010) conducted a qualitative study of student teachers and their pupils involved in a robotics curriculum. They found that self-confidence, learning effort, and coping with learning pressures improved along other variables.

An even smaller amount of research has been quantitative. Williams, Ma, Prejean, Ford, and Lai (2007) found that middle school students using LEGO RCX (the predecessor to the NXT) in the classroom improved in physics content knowledge using a pre- and post-assessment. The researchers also looked for student gains in scientific inquiry skills but did not find significant changes. Their study suffered from a small sample with heavy attrition.

Verner (2004) conducted one of the most rigorous studies using Robocell, a robotic arm that can move through five joints. He looked specifically at middle and high school students’ (n=128) gains in spatial abilities on 12 spatial tasks. Over the course of treatment with Robocell curriculum, students improved from an average of 46.5% correct on the pre-assessment to 62.4%.

In a similar investigation, Coxon (2012) conducted a controlled intervention study simulating the FLL competition with 75 9-14 year-old gifted students and found significant and meaningful gains on a measure of spatial ability used as a pre- and post-assessment, especially for males (cohen’s d = 0.87) and groups traditionally underrepresented in gifted programs. The use of robotics may also increase aspects of creativity. In a study of Jr. FLL participants from high poverty schools, Kim & Coxon (2013) found a significant increase in divergent thinking.

**ROBOTICS, CONSTRUCTIVISM, AND THE EDGE OF CHAOS**

In *The Ubiquity of the Chaos-Order Continuum: Insights from Diverse Academic Disciplines* in this volume, Ambrose makes the argument that ideal democratic dynamics occur at the edge of chaos on the Chaos-Order Continuum. With a balance
of both individual freedom and prudent regulation, democratic systems are nurtured. This occurs on the model between anarchy on the extreme chaos end of the spectrum and both left- and right-wing authoritarian governments on the extreme order side of the model.

I argue that this operates the same in schools. The earlier characterisation of the traditional classroom is on the extreme order side of the model: the teacher and text are the sole sources of authority, students work as individuals to answer single-answer, close-ended questions, and student behaviour is tightly controlled as students spend most of their time silently seated at desks to receive information passively. In such classrooms, it is unlikely that students’ creative, critical, or other such complex talents will be developed fully. As with government systems in collapse, anarchy may sometimes prevail for the new, underprepared, or substitute teacher. There again, it is unlikely that such complex student talents will be developed. In the middle, approaching the edge of chaos, the ideal classroom for the nurture of talents within robotics programs can exist with maximal opportunity for working with peers with great autonomy and physical, highly-manipulative materials toward solving complex problems (often called missions in robotics competitions). Whether during the school day or in competition outside of school hours, these programs fit within the constructivist paradigm.

Brooks and Brooks (1993) provide a useful, 12-point framework of constructivist classrooms that is paraphrased as the numbered list below. Such classrooms are ideal for reaching an exquisite balance between order and chaos, including with robotics.

1. Encourage autonomy and initiative. Robotics programs not only allow for student autonomy, they encourage it. Survey research of participants in the FLL program reveals that motivation and student initiative are high (Melchoir et al., 2004). However, teachers who focus on order and domination in the classroom will tend to repeat the example traditional classroom contrast provided. This may be due to a fear of losing control or of chaos. It may also be due to a fear of ignorance. It is impossible to be fully aware of every robot design or even of every programming solution. This stands in contrast to the traditional model in which the teacher is expected to know every answer, albeit to the limited set of content and limited set of answers. If teachers are able to move to the side and provide guidance and coaching as needed, but not provide solutions or allow for failures, they are more likely to find their classrooms on the edge of chaos with student autonomy and initiative at their apex.

2. Use manipulative, interactive, and physical materials. In contrast to the traditional model, robotics programs as constructivist educational models put students in direct and continuous contact with physical materials. While the act of programming alone is an abstract activity, the physical nature of robotics makes such sets ideal tools for the classroom. As the pathways through which students interact with the materials are both without limits of possibility and with an ultimate purpose in a problem to be solved, an exquisite balance between
ON THE EDGE OF CHAOS

3. Use cognitive terminology. Cognitive terminology, such as “create” and “evaluate” as well as other academic language is made tangible for children through the use of robotics. Students must both create and evaluate to produce robots that may successfully complete tasks, and these terms can be further engrained through the use of engineering design loops. Strategies for problem solving become useful and through repeated use may become habits of mind for even young children. Such work and terminology interlace and likely become normal facets of participants’ vocabulary as they work on the edge of chaos.

4. Student needs drive lessons, strategies, and content. In the traditional classroom example, content, lessons, and teaching strategies are decided ahead of time by policy makers far removed and with no teaching experience or significant understanding of educational systems, research, or philosophy. Lessons are relatively fixed and unalterable and they are unrelated to individual student learning in the classroom. In the constructivist robotics program, lessons on programming or engineering principles may be taught as needed: by the teacher, a guest expert, or students who have discovered or sought needed knowledge through their own initiative. Strategies are shifted based on constant assessment by student teams or their coaches on reaching goals toward having the robot solve challenges. The teacher is not necessarily the expert on all aspects and student learning may quickly surpass the teacher’s knowledge. This is desired and arguably the ultimate goal of education. If the teacher was always to be the sole font of knowledge, with students never to surpass this level, the best student would be limited to something just shy of the teacher. Instead, the constructivist robotics classroom is a launching point for student learning: an aerie, not a cage. With this understanding on behalf of the teacher, much more is possible and the focus is reversed from a concern over what students do not know to what expertise is accruing.

5. Teachers discover students’ understandings of concepts before sharing their own. Following directly from number 4, teachers do not make assumptions about student conceptual development beforehand, but come to understand each student’s understanding before sharing their own. It may even be unnecessary for the teacher to share as students may either already have an understanding of the concept at hand or develop one through experience with the materials and process. Moreover, when students do not have a well-developed understanding of a concept or hold a misconception, it is likely better for the teacher to guide the students to better understanding through questioning and continued engagement in the robotics activities. For example, a common student misconception is that when a robot is not functioning as desired, it is acting “weird” or “crazy” and that its “behaviour” is based on luck and not within student control. A teacher might guide the student reasoning by questioning physical or programming issues that may have led to the problems, such as a weak battery, loose wheel, or change in conditions that affect a sensor’s input (e.g., lighting with a light
sensor). In this way, the teacher serves as a coach in contrast to the traditional example of a teacher serving as a one-way output of knowledge. In the latter example, the student would merely be wrong and the teacher would provide the answer, leading to more order, but less conceptual development.

6. Encourage students to engage in discussion. Students in robotics programs usually work in pairs or small groups and must work together in order to accomplish their goals. Likewise, a teacher acting as a coach, as in the previous example, must necessarily play a two-way role both listening and using what is heard to question. In both places, the communication is open-ended and not fixed, allowing for a vacillation between order and chaos. A classroom focused on order alone, where the teacher provides ready solutions to the problems and assumes that student thinking is limited and simplistic, is unlikely to generate innovative ideas and solutions to solve the problems set out for the robot. Instead, the traditional, order-focused teacher asks questions with single answers to which students can be either right or wrong.

7. Encourage student inquiry. While the previous examples involve teacher questioning, students are likewise encouraged to question in the constructivist robotics program. They should question their teammates, experts, and the teacher. What are alternative approaches to solve this challenge? Is there a more efficient way to program this? How will other tasks or missions be affected if I redesign my robot for this new challenge? Questions might be more metacognitive as well, and can be encouraged by the teacher. For example, how is this process like what engineers face in the real world? Students from classrooms more closely associated with the traditional example might need encouragement. Such questions in that model may be discouraged or at least downplayed, if they even come up at all in such a mentally limiting circumstance.

8. Seek elaboration from students. Stemming from inquiry is the need to evoke elaboration. In particular, students whose earlier education was within the traditional system may not generate elaborative answers. Such classrooms do not encourage them or model the complexity otherwise inherent in engaged learning. In general, American children are becoming less creative, with the strongest loss being their ability to elaborate upon their ideas (Kim, 2011). Kim and Coxon (2013) offer three likely possibilities for this loss: nationalised minimum competency standards and the associated single-answer tests, video game play and video game addiction, and television. The evidence of excessive television watching’s negative impact on children both in terms of cognitive and health aspects is especially strong. American children ages 2–12 watch around 30 hours of television per week on average, varying a bit by age (McDonough, 2009). The television is on during meals in about two-thirds of American households and on for every waking hour in more than a third (Rideout, Foehr, & Roberts, 2010). At 30 hours per week, an average American child watches about 25,000 hours of television between the ages of 2 and 18, or about three out of 17 years between infancy and adulthood. This is greatly more than the
approximately two years of total time in school over the course of the 13 years
between kindergarten and 12th grade. The effects of this appear almost entirely
negative. Television watchers begin reading later and read less well, are less
active and more likely to suffer from ill-health, have decreased attention spans—
arguably a key facet in the ability to elaborate and engage in fewer creative
activities (American Academy of Pediatrics, 2001; Anderson et al., 2001; Lillard
& Peterson, 2011; Rideout, Vandewater, & Wartella, 2003; Tomopoulos et al.,
2010; Vandewater, Bickham & Lee, 2006). Students from households that allow
this may require more prodding from the teacher to elaborate on their ideas, to
experiment, and to begin to think for themselves.

9. Engage students in experiences that might engender contradictions. Drawing
on the earlier discussions on conceptual understanding, students with
misconceptions and under-developed conceptions, such as the example of
why a robot may not be functioning as anticipated, need these experiences that
contradict their original idea or hypothesis. In the robotics classroom, there is a
constant testing of ideas and experiencing failure. One can build, program, and
test through repeated iterations without finding success with the very tangible,
physical robot. This is in contrast to the traditional classroom in which there is a
single set of correct answers and the basis is abstract rather than physical. Even
in the best traditional classrooms where students might be allowed to conduct an
experiment after reading about a principle in their science text, the experiment
is not student-developed, but instead a series of steps to be followed. If followed
correctly, the student arrives at a known answer. If the experiment ends without
the expected result, it is considered a summative failure instead of a potential
discovery or first iteration of a continuing process. In this manner, the student
is acting as a technician and not a scientist. With robotics, when students are
allowed to engage in problem solving without an enforced set of instructions,
they will likely fail to achieve their goals many times before finding success,
and likely have to continue to refine their robot through many more iterations to
increase its reliability.

10. Allow wait time after posing questions. Provide time for students to construct
meaning. The wait time suggestion is certainly a valid one within robotics, fitting
well within the sections on questioning, inquiry, and discussion all discussed
above. Time to work is an essential factor in complex learning beyond just wait
time after questions. Time is often ignored in traditional classroom settings
where the clock is a cruel overseer. Students need many hours to participate in
robotics programs, often 40 or more to complete a PBL unit or to prepare for a
competition (Coxon, 2009), and this time is better segmented in two to four hour
blocks. Within the 50 minute time blocks often allotted to children in schools,
cooperative projects in which students must engineer and program a robot to
perform complex tasks on open-ended problems where success might be found
only after dozens of iterations do not fit well. Wait time in questioning allows for
student thought, much more necessary in a complex constructivist environment
than in a single-answer traditional classroom where the “best” students are the fastest to blurt out memorised answers. Students need a wealth of time to construct understanding.

11. Nurture students’ natural curiosity. With obvious delight, babies experiment with gravity through repeated iterations of food dropping and watching items hit the floor. Toddlers find endless joy in sand and water play, and kindergarteners bubble with excitement when exploring a creek. Schools can do much to nurture curiosity, but seem just as able—and more likely—to hamper it. The constructivist classroom on the edge of chaos, including with robotics programs, seems a particularly apt way to continue to nurture curiosity. As students have autonomy and time with materials, in discussion, and in working with peers, it seems more likely that curiosity will be maintained or extended. This is in stark contrast with the traditional classroom where there is little to be curious about given its focus on what is known rather than on what is still to be created or discovered and on the way the world works in the past and present and not on how it might be. There is little to be explored within the traditional context, limited to the abstract, the knowledge of the teacher and the finite text.

CONCLUSION

The edge of chaos is a fine line between highly controlled educational programs on one extreme and anarchistic programs on the other. The edge serves as a place where students may generate creative ideas and elaborate upon them, increase autonomy, motivation, and curiosity, and move toward higher, more complex levels in their development. A constructivist educational program is an ideal setting for this to occur, and robotics programs offer considerable possibility within such settings to move student thinking and interactions into deeper complexity and toward edge of chaos. Students in such situations likely improve upon skills useful across domains, particularly in the STEM fields, including problem solving, spatial ability, and aspects of creativity. Such situations mimic well the most challenging professional fields, including the sciences and engineering, far more than do the traditional educational programs. Such constructivist programs should be our goal if we are to prepare students to think complexly and generate solutions to their future world’s emerging problems.

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