Leaders in Mathematics Education: Experience and Vision

Alexander Karp (Ed.)
Teachers College, Columbia University

This book consists of interviews with the most important mathematics educators of our time. These interviews were originally published in the International Journal for the History of Mathematics Education and are now being offered to a wider readership for the first time, collected in a single volume. Among the individuals interviewed are scholars from Brazil, France, Germany, Russia, the United Kingdom, and the United States who have made a significant impact on the development of mathematics education in their countries and internationally. The interviews cover their biographies, including their memories of their own studies in mathematics and their intellectual formation, their experience as researchers and teachers, and their visions of the history and future development of mathematics education.

The book will be of interest to anyone involved in research in mathematics education, and anyone interested in the history of mathematics education.
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Edited by

Alexander Karp
Teachers College, Columbia University

With contributions by

David Lindsay Roberts
Prince George’s Community College
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This book consists of interviews published in the International Journal for the History of Mathematics Education. When this journal first appeared in 2006, it was the only international scholarly periodical devoted to this field, and it remains so to this day. Interviews with major figures in mathematics education began to be conducted and published in the journal in 2007, becoming one of its regular features. Gradually, a certain stable set of questions developed, which were posed and discussed in the course of virtually every interview, though naturally discussion was usually not restricted to them alone. In this book, thanks to the kind permission of the journal's publisher, the Consortium for Mathematics and Its Applications (COMAP), it has been possible to reproduce virtually without changes all of the interviews published thus far. We expect that they may be of interest to a wide readership, including those readers whose interests are relatively far from the history of mathematics education. The journal plans to continue publishing interviews, and of course the list of people who may be considered leaders in mathematics education is not exhausted by those represented in the interviews in this book.

The present volume offers its readers interviews with eleven different individuals—individuals with different lives, different interests, and different countries of residence—Brazil, France, Germany, Russia, the United Kingdom, and the United States. Each of these people speaks about his or her life and work, and in addition each interview is prefaced by a brief note about the interviewee, so there is no need to give any other general information about the people interviewed here. Nonetheless, it will be useful to say a word about the history of mathematics education, including those periods of it which are touched on in the interviews, as well as about the interview genre as a means for studying history and in its other functions.

THE HISTORY OF MATHEMATICS EDUCATION AS A SCIENTIFIC DISCIPLINE

The beginning of research in mathematics education may be variously dated, depending on how one chooses to understand what it consists in, and how one determines the point at which it should be defined as a separate field of research. In any event, it is clear that the first doctoral dissertations in mathematics education, defended in the United States, were devoted specifically to history (Jackson, 1906; Stamper, 1906). Subsequently, however, the empirical-analytic tradition, which
relied on statistical methods, came to dominate the field; its absolute hegemony was broken only in the 1980s (Kilpatrick, 1992). Historical studies in different countries had undoubtedly been conducted earlier as well, but in the context of an increasing awareness of the limitations of the previously dominant methodology, interest in them began to rise. Important steps in this development were represented by the publication of a two-volume work on the subject by Stanic and Kilpatrick (2003), the formation of a special topic study group devoted to the history of mathematics education at the International Congress of Mathematics Education (beginning in 2004), the appearance of the International Journal for the History of Mathematics Education, and the appearance of special conferences devoted to the history of mathematics education (Bjarnadóttir, Furinghetti, & Schubring, 2009; Bjarnadóttir, Furinghetti, Matos, & Schubring, 2012; Bjarnadóttir, Furinghetti, Prytz, & Schubring, in press). As experience with the publication of the Handbook on the History of Mathematics Education (Karp, Schubring, 2014) shows, however, very many aspects of this history still remain virtually unresearched.

The methodology of such research itself, including the understanding of what its object consists of, still remains in its formative stages (Karp, 2014). The historian of mathematics education usually asks how the teaching of mathematics changed and how it happened that a society or government implemented (supported, permitted, allowed, tolerated—many shades may be distinguished here) specifically one or another form of instruction. The specific characteristics of the given field consequently lie in the fact that, while its methodology is in essence a classically historical one, based first and foremost on the analysis of surviving texts (in the broad sense of this word), the texts themselves are usually mathematical or devoted to the teaching of mathematics. For example, a new textbook was introduced, distributed, and promoted in a specific country (or region), and the historian must be capable of identifying what exactly its novelty consisted in and understanding the reasons for its success or lack of success within a framework of no longer purely educational categories, but more general social ones as well.

Meanwhile, contrary to widespread opinion, the processes taking place are by no means simple. Indeed, it is frequently assumed that students have been taught the same thing in school mathematics for at least the last two hundred years, and that mathematics has always remained pure, in the sense that neither politics nor the problems of the surrounding world in general have exerted any special influence on anything in mathematics classes, since quadratic equations are solved in the same manner by everyone. If people do add anything more to this simplistic picture, then it is merely that teachers ought to be paid more, which will supposedly immediately make them better teachers, and that schools need order—although both of these considerations already go beyond the specific boundaries of mathematics education itself. In reality, on the one hand, curricula do change in significant ways; and on the other hand, avoiding the influence of society while living in society really does turn out to be impossible.
Researchers have recognized (Schoenfeld, 1985) that even when they have to do with such a pure and mathematical activity as mathematical problem solving, they must take into consideration the beliefs of the problem solvers. In solving educational problems, including problems in mathematics education, beliefs turn out to be even more important—and this is already one (though hardly the only) way in which the influence of society penetrates into textbooks that might seem to exist far from the problems that agitate society.

The historian in such cases is often concerned less with analyzing the situation that existed in a particular year or at a particular moment, than with understanding ongoing processes. This in itself immediately calls into question the brief references to “culture” that one sometimes encounters when some specific characteristics are discussed, “culture” being understood in this situation as something given and unchanging (as when people simply assume that in some countries every teacher pays special attention to reasoning and proving, or knows how to provide good examples for the rules being studied—why?—well, “that's just culture!”). The historian asks: How did this culture develop? Why did it not develop elsewhere? Does it change over time, and if so, how? Seeking answers to these and other questions, the historian must analyze complex and unobvious interconnections, which can be understood only by comparing such sources as textbooks, curricula, or the transcripts of strictly educational discussions, with a far wider range of sources—letters, diaries, newspapers, the speeches of politicians (by no means about mathematics), and much else that often seems to have no relation to mathematics education at all.

Schubring (1987) compared the methodology that must be employed by a historian of mathematics education with that which has been used since the eighteenth century in studying Ancient Greek poetry—for a better understanding of which, it turned out to be necessary to study Greek politics and even Greek economics. It turns out to be necessary to reconstruct the context in which various changes in mathematics education took place. Appreciating the difficulty of recreating such a context, Schubring (1987) proposed that researchers select a “unit” for such studies that would reveal “the relevant dimensions” in interaction. As one such possible “unit,” he named the life of the mathematics educator (textbook author).

The lives of important figures in mathematics education are precisely the focus of this volume, which, one would like to think, can thus help to shed light on the “context” of the changes which these individuals witnessed or the changes to which they contributed (both the former and the latter have been significant). It will be useful, therefore, to attempt, however briefly, to describe what exactly took place in mathematics education during the period discussed in this book.

THREE QUARTERS OF A CENTURY IN MATHEMATICS EDUCATION: ACHIEVEMENTS AND CHALLENGES

The people interviewed in this book have lived and live in different countries, in which life is different, and consequently the developments taking place in education,
including mathematics education, are by no means identical. One thing, however, can be asserted with confidence about practically all countries: many more people are now being taught mathematics than in the past.

We can point to at least two reasons for this. The first is that the world has become mathematical. Already at the beginning of the twentieth century, the Russian poet Alexander Blok wrote about steel machines in which “integrals breathe.” Today, integrals or discrete mathematics “breathe” not only in machines, but in almost any quotidian operation. Some understanding of mathematics has consequently become indispensable far more broadly than in the past. The other reason lies in the fact that an explicit division of society into gentlemen—who study, among other things, mathematics—and everyone else has become impossible. As a descendant of commoners, Martin Eden—the protagonist of a novel published at the beginning of the twentieth century (London, 1909)—is immediately made to know his place by the members of the higher spheres whom he encounters: Eden has never heard of trigonometry. After the Second World War, society strove to avoid such an opposition (or at least claimed to do so), and in fact it did now need to worry about attracting an increasingly growing number of well-educated workers.

Thus arose the problem of creating a “Mathematics for All,” which of course we can hardly consider solved, but which has exerted an enormous influence on the transformation of mathematics in schools. Since it is obviously impossible for us to analyze all sides of this transformation here, we will simply mention the fact that the very problem of “Mathematics for All” stimulated discussions about who these “all” were, in other words, both about the psychological styles and specific characteristics of the learning of mathematical material, and about the historical and cultural-practical characteristics of students and this mathematical material (one might say, ethnomathematics). At the same time concerns about the fact that the school course in mathematics was losing much of its substantive content as a result of being adapted to education on a mass scale began to be expressed with greater frequency and urgency. “The people don’t need it!” was a formula widely used in the Soviet Union, often invoked by party leaders to motivate the closing of theaters and the refusal to publish books that they considered undesirable. Likewise, in the teaching of mathematics, in many countries there developed a tendency to assume that the people do not need anything overly complicated, and that consequently, no one needs it (although “the people” never authorized anyone to make such an assumption). It became necessary to fight specifically for a substantive mathematics for all.

Meanwhile, along with the growing number of students being taught mathematics, changes were taking place in the understanding of what mathematics was. The 1960s-70s witnessed a second wave of international reforms (if what took place prior to the First World War, under the guidance or influence of Felix Klein, is considered the first wave). “Mathématiques modernes,” “New Math,” the “School Mathematics Project,” “Kolmogorov’s reform”—the reforms that took place in different countries had different names, and their representatives often emphasized
(justly) the differences between them. Nonetheless, the directions of the reforms were similar, and this in itself refutes any simplistic notion of these developments as the outcome of some local intrigue. As is true of many other reforms, a tradition has developed of seeing these reforms in mathematics education in a negative light—for some unknown reason, it is claimed, a lot of incomprehensible abstractions were introduced into the school curriculum, but, fortunately, people thought better of it in time, and eventually got rid of them all. It is not difficult to see that, although some things were undoubtedly gotten rid of, a great deal nonetheless remains in place to this day, and in any case, the impact of the reforms on the whole mathematics education community was enormous. Many of those interviewed in this book were active participants in the reforms that took place at the time, and some among them were also leading developers and authors of later curricula and textbooks.

It must also be pointed out that, both while the reforms were being implemented and during later periods, mathematics education increasingly became subjected to government involvement or influenced by nationwide forces. Indeed, schools could not help but feel the government's influence already during much earlier periods—thus, readers will find in this book reminiscences about German schools under the Nazis or Soviet schools under Stalin. But after World War II, nationwide policies gradually began to play an increasingly greater role in Western Europe and the United States as well—among other factors, grants to support the development of various projects, including scientific projects, acquired unprecedented importance. In the 1980s, in the United States, the Standards movement began to develop, and soon increasing attention began to be paid to assessment and accreditation. In general, mathematics education policies, in which many of the individuals interviewed in this book were involved, became considerably more significant than they had been previously.

At the same time, the discussion of the problems of mathematics education crossed borders: internationalization became one more crucial characteristic of the ongoing developments. Important steps in this direction had already been taken at the beginning of the twentieth century, when the International Commission on Mathematical Instruction (ICMI) was established, but after the Second World War, the ICMI was not simply reborn, but substantially expanded its role. Even more importantly, internationalization now was not limited, so to speak, to one-time meetings or even joint publications: contrasts and comparisons with international parallels became a norm and scholarly publications themselves are now usually addressed to an international audience. Those interviewed in this book are major figures in specifically international mathematics education.

The close connections that formed between mathematics educators in different countries would likely have been impossible without the development of technology. Since our aim here is to name the most significant changes in mathematics education, we cannot avoid saying a word about the influence of the technological revolution. Technology (in the broad sense of the word) has always exerted an influence on the development of education (Kidwell, Ackerberg-Hastings, & Roberts, 2008); nonetheless, only in the last half-century has it became so fast-paced and
all-encompassing. Schools are subject to enormous, and sometimes direct, pressures from the manufacturers of computers and software providers, but a still greater influence comes from the surrounding world, in which more and more improved mechanisms keep appearing, not to make use of which in education seems inconceivable. Technology influences both how people teach and what they teach. New possibilities open up, and some old techniques and procedures die out. The developments taking place are by no means simple, and while being drawn in by the breakneck pace of technological innovation, it is important not to lose (and desirable to augment) the “eternal” values of a subject and its teaching. Among those interviewed in this book are individuals who spent many years pondering how this might be done, and they share their experience and observations here.

The last point that must be mentioned here is not the least important. Whatever date one chooses to designate as the beginning of research in mathematics education, it is clear that the field became a mature discipline, capable to a certain extent of attracting attention on a mass scale, only several decades ago, and not least thanks to the efforts of the people interviewed in this book. The point is not only that now incomparably more scientific journals devoted to mathematics education are published than, say, fifty years ago, or that more conferences are organized, or that a vastly greater number of people earn doctoral degrees in mathematics education—it is natural to see all of this as a reaction to the ubiquitous pressures to raise teachers to a higher scientific level, which frequently compel people to write and publish who would have otherwise never had any interest in doing research. More important is the fact that research in mathematics education has acquired (or is in the process of acquiring) its own varied subject matter, its own methodology, its own values, and its own classics, among which are many of the works of the people interviewed in this book.

THE INTERVIEW AS A MEANS OF UNDERSTANDING AND AS A MEANS OF COLLECTING DATA

Many stories have survived about clever journalists or critics who, without troubling to think too much about books, asked their authors questions about what exactly they wanted to say in their writings. Leo Tolstoy (1984) once replied that, if he “wanted to say in words everything that he intended to express in a novel, he… would have to write the very novel he had written… all over again” (p. 784). Novels are indeed written in words that do not articulate ideas directly, but depict “images, actions, and situations” through which the author expresses what he or she wishes to say.

Matters stand differently with scientific writing. Of course, the differences between a scientific article and an interview (even one that contains references and certain other attributes of a scientific paper) are obvious; nonetheless, briefly summing up what has been done is standard scientific practice, and it is all the more valuable when such a brief summation is given by the authors themselves. Not all the people interviewed spoke in equal detail about their writings, and—to repeat the obvious again—the reading of the scholarly works themselves cannot be replaced by any
brief paraphrase; even so, however, the reading of the interviews in this book can in our view help to understand the work of the interviewed individuals, and this makes the interviews valuable even for those who have no interest in history. Context, the importance of which for the historian was pointed out above, is likewise important for researchers of mathematics education whose interests may lie far from history, since it helps to achieve a better understanding of the problems that confronted the authors of works that became classics, and the role of the results they obtained.

Here, however, we must inevitably touch on the question of how precise an interview can in principle be, and we must say a word about the interview in general as a source of information. Having barely broached this subject, we find ourselves in the thick of the most diverse judgments and discussions, since interviews have long ago become a commonplace source for the collection of data for scholars in various fields, and in history there has emerged a whole branch, oral history, which is based on interviews and the analysis of interviews. Morrisey (1983) not without irony wrote that “Although oral history has professionalized itself since Allan Nevins issued his call in his 1938 work, Gateway to History, no oral historian can completely fit the job description with all its many requirements” (p. XX). Alas, neither can the interviewers hope that their questions are always absolutely ideal, nor are the interviewees insured against, say, memory lapses. The aim of the present book—the broad publication of a collection of interviews, not a thorough historical analysis of them—is more archival in nature than historical in the full sense of the word. As far as interviews are concerned in general, they should be treated no differently from any other sources when they are analyzed and interpreted: they must be juxtaposed with each other, contrasted, and so on. Nonetheless, a particular observation must be made about the distinctive characteristics not of interviews in general, but specifically the ones published here.

Very often, the objective of oral history is seen to lie in making heard and preserving the voice of the silent majority—and this is indeed an extremely important goal, but it was not the goal that was pursued when the interviews published here were conducted. This book contains interviews with people who published a great deal and many times expressed their views about the development of mathematics education. Their perceptions of surrounding life are most naturally understood not only in the context of the reminiscences of other people about the same period, but also in the context of their own published works. Such a contextualization helps to achieve a better understanding not only of their own works (as we have argued above), but also to a certain extent of the time and events that they talk about.

In conclusion, a word about how the interviews were conducted. Nine interviews were conducted by Alexander Karp specifically for the International Journal for the History of Mathematics Education. These interviews were intended for publication in the journal, so the distinction between a transcript and an actual audio recording, analyzed ad nauseam in the scientific literature, has no bearing on this case, as transcripts were sent to the interviewees for editing; nor is this altered by the fact that some interviewees preferred to limit themselves to minimal corrections, preserving
the conversational style, while others saw fit to edit the text more substantially. The interview with Heinrich Baursfeld was conducted over email.

Two interviews (with Jeremy Kilpatrick and with Isaak Wirszup) were conducted by David Lindsay Roberts several years before the journal was founded, and their full text is stored in the Archives of American Mathematics, Center for American History, University of Texas at Austin. The interviews prepared by David Lindsay Roberts for the journal consisted of excerpts from these interviews, with additional materials in the interview with Kilpatrick drawn from the correspondence between Kilpatrick and Roberts.

Each interview may be read on its own, independently of the others; therefore, the notes for each interview appear separately, and some of them are repeated.

Finally, I would like once more to express my gratitude to the interviewees for all the time and work that they have devoted to the texts published in the book, to David Lindsay Roberts for his contributions, to the chief editor of the International Journal for the History of Mathematics Education Gert Schubring, who read each interview prior to its publication, for useful discussions, to the COMAP Executive Director Sol Garfunkel for publishing the Journal and his kind permission to reproduce the Journal publication of the interviews, to the Director of the Program in Mathematics at Teachers College, Columbia University Bruce Vogeli for his support of the Journal and many important recommendations, and to my colleagues in the Program in Mathematics Henry Pollak, N. Philip Smith and Erica N. Walker for all their help with this book.

REFERENCES


**AFFILIATION**

*Alexander Karp*
*Teachers College, Columbia University,*
*New York*
The following interview with Michèle Artigue was conducted in October 2013. Michèle Artigue obtained her degrees in mathematics from the École Normale Supérieure in Paris and Université Paris 7. Her subsequent teaching and scholarly work were likewise connected with Université Paris 7, where she taught and in 1985-1988 was the director of IREM. Later, she was a professor and a chair of the mathematics department at the University Institute for Teacher Education in Reims, and yet later, in 1999, she returned to Université Paris 7 as a professor and the director of IREM. Dr. Artigue's scholarly writings have represented important contributions to the study of many fields in mathematics education, from teaching in elementary schools to teaching in institutions of higher learning. Her studies on the use of technology in mathematics education deserve particular mention. In 2013 she was awarded the Felix Klein Medal by the International Commission on Mathematical Instruction (ICMI).

Michèle Artigue has done much for the development of international collaboration in mathematics education—she has been president and vice president of ICMI, and headed or participated and advised many important national and international projects.
CHAPTER 1

The interview was conducted and annotated by Alexander Karp (Teachers College, Columbia University). The references and some notes were provided by Michèle Artigue.

BEGINNING. SCHOOL YEARS

**Interviewer:** The first set of questions is about your personal history and your own school education: Could you please tell us about your own secondary, middle and even elementary education? How was it? How was your mathematics education? When did you become interested in mathematics?

**Michèle Artigue:** I think that I was always interested in mathematics, even in elementary school. I was born in a small village and I attended the elementary school of my village. My mother was an elementary teacher, but she was in charge of the kindergarten so I never had my mother as a teacher. I had quite typical teaching at that time focusing on numbers, operations, magnitudes and the solving of classical problems. At the end of elementary school, I took the examination required for entering secondary school, succeeded and entered the *lycée*. At that time, just a limited number of pupils took this examination, and, if successful, entered secondary education, either in a *lycée* or in a *cours complémentaire*. The majority stayed in primary schools and ended their education preparing the *certificat d'études primaires*. The *lycée* was in Tarbes, the nearest city from my village. It was about five to six kilometers from the village, so I biked, as did many others.

At that time in France, junior secondary education was not unified. The *lycées* typically welcomed pupils from the “bourgeoisie” and the *cours complémentaires* pupils from common classes (it offered only four years of junior secondary education). Latin and Greek were only taught in *lycées* and without Latin, choices were limited for senior secondary education. Despite the fact that I was from a modest origin, my parents sent me to the *lycée* and I learnt Latin. I remember that during the first years of junior high school, we had more hours of Latin than of mathematics, or nearly the same. I studied in the same *lycée* during all my secondary schooling. I was a good student in nearly all the subjects, not in drawing and arts (laughs), but especially good in mathematics and physics. My mathematics teachers wanted me to go on with mathematics, the physics teachers wanted me to go on with physics, but I had a feeling that even if I always got very good marks in physics, I did not understand as clearly the way physicists think and work. I felt more in tune with mathematics thinking.

When I entered my last year in the *lycée*, my mathematics and physics teachers told me that the best thing for me to do would be the *Math Sup* and *Math Spé*, that is to say to enter a *Classe Préparatoire aux Grandes Ecoles*. I had the perfect profile for such a class. The nearest classes were in Toulouse and Bordeaux. We decided for Toulouse. It was just one hundred and fifty kilometers from home, and my older sister worked in the region as a secondary teacher. It was a good choice, and I did the two years of *Math Sup* and *Math Spé* at the *lycée* Pierre de Fermat in Toulouse. These classes were (and still are) located in *lycées*. At that time, boys and
girls schools were separated and the scientific preparatory classes were in the boys lycées. So I was living in a girls’ lycée, sleeping there, and every day I went to the lycée Pierre de Fermat. Among the 35-40 students of the class, we were five female students. This was a usual ratio.

In the first year, the year of Math Sup, my math teacher was a young teacher who had graduated from the École Normale Supérieure of Saint Cloud. With him I began to discover modern mathematics, sets and algebraic structures, and I was very enthusiastic about that. In the second year, I had a more classical teacher, but also a good teacher. In fact there were very good teachers in these classes. Also, in my secondary classes I always had rather good teachers. When I entered the second year, the year of Math Spé, my mathematics teacher told me that, according to him, I was able to enter the École Normale Supérieure of Sèvres. He said that it would be great for me and also for the lycée because during the last thirty years none of their students had entered the Écoles Normales Supérieures of Ulm (for boys) or Sèvres (for girls).

I was very well treated during these two years (laughs). Many students complained that these years were very hard for them, but for me it was not a bad time, well cared for by my teachers, learning a lot of new topics, with many new friends and so on. After these two years, I took the competition for the École Normale Supérieure of Sèvres, I was successful and I went to Paris.

UNIVERSITY YEARS

Michèle Artigue: Entering the École Normale Supérieure, I really had the feeling that I was entering a new world. Many students had prepared for the competition in famous lycées in Paris and had been taught much more mathematics than me. They spoke about objects I had never heard about. Most came from families with a university tradition and from the “bourgeoisie”. We were very few from a very modest origin and we became friends (laughs). For me, it was quite a big change but also something very exciting. As students, we were registered at the Faculty of Sciences in Paris and prepared there the Licence of mathematics. We had as teachers the best mathematicians, for instance Gustave Choquet taught the course of topology, Henri Cartan that of differential calculus and complex analysis, and Laurent Schwartz the mathematical methods of physics. Beyond that, we had complementary courses at the Ecole Normale Supérieure, and it was there that I met for the first time André Revuz who would play an important role in my professional life, at the beginning at least, and also Pierre Samuel who was teaching algebra.

Interviewer: My computation demonstrates that you were still doing your education in 1968…

Michèle Artigue: Yeah, in 1968, I was in my third year. As students of the Ecole Normale Supérieure, we had an accelerated program at the university and were allowed to take all the courses of the licence in the first year. In the second year, we took more advanced courses corresponding today to the level of Master, and in the third year, we prepared for Agrégation, a competitive examination to become a
secondary school teacher. So in 1968 I was preparing for Agrégation, and my mother
helped me by taking care of my first son, Olivier, born in August 1967.

In May, there were the famous political events in France; we had already taken
the written part of the Agrégation and got its results but most of those admissible
decided not to take the oral part of the examination. Nearly all the students of the
Écoles Normales Supérieures, both the boys and girls decided the same. Initially the
movement called the “agrégibles” was not limited to the agrégation of mathematics,
but the mathematics students were first in line. Their demands were mainly the two
following: to establish a single competition for recruiting secondary teachers (instead
of having two competitions the CAPES and the agrégation leading to different
duties, salaries and careers), thus a demand in line with the equalitarian spirit of May
1968, and to include a pedagogical dimension in the competition and the preparation
of teachers. In fact, the movement did not survive the end of the May events, except
in mathematics. Some practice was thus organized for us in 1969, and then a specific
oral with the same proofs as usual, and also some questions about our practical
experience. However this movement had an interesting consequence as it contributed
to the creation of the first IREM (Instituts de Recherche sur l’Enseignement des
Mathématiques) in January 1969. In June 1968, the representatives of the math-
agrégibles were received several times at the Ministry of Education. Visibly, the
new Minister, Edgar Faure, a very clever politician, wanted to calm the rebellion
of these elite students. He met himself with their representatives accompanied by
leading figures of the APM (Association des Professeurs de Mathématiques). In the
discussion, they reiterated the demand, already made to the previous Minister, for
the creation of IREMs. This was a part of the “Charte” adopted by the association
in January 1968 in Chambery and further known as the “Charte de Chambéry”.
This demand had also been endorsed by the Ministerial Commission Lichnérówicz
created in January 1967 for renovating secondary mathematics education. According
to Daniel Perrin, a representative of the agrégibles at this meeting, Edgar Faure
immediately sized the occasion as a way of showing that he was listening to the
pedagogical demands of the agrégibles, and he created three IREMs in Lyon, Paris
and Strasbourg

Interviewer: Do you remember any other discussion connecting policy with
mathematics education at this time? Or it was more or less separate for you then?

Michèle Artigue: I would say that when I was a student the issues of mathematics
education were not in my area of concern. When preparing the agrégation, I had a
few weeks (two or three I do not remember) of practice at the lycée Lakanal, a very
good lycée. I attended classroom sessions, discussed with the teacher of the class,
Marcel Condamine, and finally was asked to prepare and give a lesson to grade 12
students. Everything worked well and the teacher complimented me. I would only
discover later that I had the chance to be supervised for my practice by the co-author
of a famous series of textbooks of the New Math period. We also prepared lessons
for the oral part of the competition but these were not planned for real students, just
for showing that we knew the mathematics. Of course this was a time of intense
INTERVIEW WITH MICHÈLE ARTIGUE

activity in the APM and in the Commission Lichnérowicz. French mathematicians and teachers were very active in the New Maths Reform Movement. Remember that the first ICME Congress took place in Lyon in 1969. André Revuz was a very active member of the APM developing courses on modern mathematics for teachers and also a member of the Commission Lichnérowicz. He became a member of the Executive Committee of ICMI under the presidency of Hans Freudenthal from 1967 to 1970. As he told us when we interviewed him for the ICMI Centennial in 2008, during these years he worked very closely with Freudenthal. This close relationship with Freudenthal perhaps explains why he asked him to be one of the reviewers for my Doctorat d’État in 1984. Personally, I discovered the existence of all these discussions and activities when I began to be in contact with the newly created IREM in 1970-71.

Yet as a university student I discovered modern mathematics, and I discovered logic, which would become my field of research. Logic was something so new for us, and we had fantastic professors, as there was already a strong team in logic at that time. Jean-Louis Krivine who taught set theory and model theory especially impressed me. I remember that when I entered the maternity ward in August 1967, I brought with me the Kreisel-Krivine text on model theory. For me, but not only for me, it was discovering a new mathematical world, and it was fascinating. My interest in mathematics education came later. At that time, it was more the fascination of discovering new mathematics.

BEGINNING OF THE WORK AND RESEARCH IN MATHEMATICS EDUCATION

Interviewer: So in 1972, if I’m correct, you got your Ph.D., right?
Michèle Artigue: Yes. I did my Ph.D. on issues of recursivity under the supervision of Daniel Lacombe. However, in 1972, I was already involved in the activity of the IREM. I was recruited at the math department of the University Paris 7 just after leaving the École Normale Supérieure in September 1969. At that time it was possible to enter the University without a doctorate. The IREM had just been created and André Revuz was the director. Two of my friends at the École Normale Supérieure were already working there, and quickly André Revuz proposed me to join the team, and work part-time in the IREM. Six positions had been created in the math department for people working temporarily and partially in the IREM, so part of my duties were connected with the IREM.

Interviewer: Could I ask you to tell a little bit about IREM? As you know it’s a very unusual institution for many countries.
Michèle Artigue: It was unusual, and it is still unusual. The IREM were created with four missions: to participate in pre-service teacher education, to contribute to in-service teacher education, to pilot experimentations and develop pedagogical research, to collect, produce and disseminate resources for the teaching of mathematics. They were university institutes, autonomous but close to the mathematics departments. In Paris for instance, positions had been allocated by the Ministry of Education to the
math department with the idea that some mathematicians of the math department
would work part-time for some years in the IREM. The Ministry had also partially
detached twenty secondary teachers: they worked half time in their high school and
half time in the IREM; and we were organizing training courses, at that time it was
called “recyclage”, for thousands of teachers. From the very beginning there was
the principle that the functioning of the IREM would be based on the collaborative
work of university mathematicians and secondary teachers. Later on, when research
in math education developed, some became experts in didactics or in history of
mathematics. In line with the spirit of May 1968, the relationships in the IREM
wanted to be free of any kind of hierarchy, I mean formal hierarchy. This was a bit
utopian because in fact there is always some kind of informal hierarchy, even if there
is no formal hierarchy; but this was the basic idea and we have tried to maintain it up
to now. We worked together to plan the teacher development activities. Together, we
were also trying to innovate, experiment, analyze, observe, and understand. At that
time we also began to work with physicists and with some biologists at the IREM, so
it was not just among mathematicians, it became something bigger. I think that it was
really a fantastic experience to have the possibility of beginning my career working
both in the math department and in such an institution.

Interviewer: Let’s talk about the beginning of your research in mathematics
education. I know that you started teaching analysis and started doing some research
in teaching analysis and calculus.

Michèle Artigue: No, that is not quite right. At the beginning at the university, I
did not teach just analysis. I was certainly teaching first and second year students,
the first courses, but these courses were not just analysis. There was a part of
analysis, a part of linear algebra, a part of group theory, etc. Teaching was organized
by semesters and each semester, the students had a main course covering different
domains of mathematics.

I began to research in math education when, thanks to André Revuz and François
Colmez, an elementary school was associated with the IREM as an experimental
school. It was situated about fifty kilometers from Paris in the South near Melun,
and Revuz proposed Jacqueline Robinet, a colleague and friend from the time
of the École Normale Supérieure, and I to take responsibility for the teaching of
mathematics in this elementary school together with François Colmez.

In this elementary school, we had very good conditions, not the same as
Guy Brousseau1 had in the school Michelet attached to the COREM (Centre
d’Observation et de Recherche sur l’Enseignement des Mathématiques) he had
just created in Bordeaux, but something in the same spirit. We had more teachers
than classes, which reduced their teaching load and allowed them to be involved
in research. We were in charge, with my colleague Jacqueline, of grade two, grade
three, and perhaps grade four, but I don’t remember exactly. We had a large degree
of freedom as long as our pupils were learning well, I would say.

François was a friend of Guy Brousseau. They both were from Bordeaux and
François’ father was the director of the IREM of Bordeaux to which the COREM was
François was in close contact with Guy and so he regularly received the situations that Guy designed for his elementary school. We began testing didactical engineering developed by Brousseau for the teaching of numbers and operations. For instance, we introduced multiplication through the counting of rectangular grids and the method per *gelosia*, which proved very effective. We also introduced grade three pupils to probability through the famous bottle situation; at the time, students only met probability in grade eleven. We also reproduced the famous Brousseau’s didactic engineering for the introduction of rational numbers, comparing the thickness of sheets of papers, and so on. My first task was to reproduce and use these approaches. Progressively, we also developed some specific constructions for these elementary students, especially in geometry, approaching the idea of circle with them from a variety of perspectives, and studying how their conceptions of this object developed. The beginning of the research for me was at the elementary school, as was the case for many mathematics educators at that time. My first publications were about numeration in the first grades and pupils’ conceptions of circle, in the late seventies and at the turn of the eighties.

Then, in 1979, an experimental section Mathematics-Physics at the University Paris 7 was created. The project was again under the umbrella of the IREM and was piloted by André Revuz for mathematics and Jean Matricon for physics. As I mentioned earlier, we had contacts with physicists since the creation of the IREM, and I have worked with two didacticians of physics: Laurence Viennot and Edith Saltiel on the respective status of graphical representations in mathematics and physics. We created a team mixing researchers and didacticians of the two disciplines, and together we organized this experimental section. Our idea was to connect the teaching of math and physics as much as possible. We organized a series of joint lectures for the students on topics of particular interest for the two disciplines, prepared and taught by two of us, and everyone on the team attended these lectures. We developed common tests with a part of math and a part of physics, and we marked them together. We took the examinations of the other discipline and we compared our results with the students’ results, which was very insightful and made us more modest (laughs). It also made it difficult to declare that your students are lost in mathematics or physics when you see your colleagues in the other discipline making the same mistakes, and have to discuss with them. It was really a nice interdisciplinary project. Also, we began to use computers as a part of teaching and for students’ projects. It was the turn of the eighties, the use of computers was quite innovative at that time, and it was very successful too.

For me, this second experience was also very rewarding. In fact, the only resisting difficulty we experienced was with the organization of the joint lecture planned on the idea of differentials. On this theme, the mathematicians and physicists of the team were not able to come to an agreement. With Laurence Viennot and Edith Saltiel, we decided to take this as a topic of research and to investigate the respective positions of physicists and mathematicians regarding integral and differential processes, and their effects on students’ conceptions. This then became a bigger
project that involved the team of Marc Legrand in Grenoble and was supported by the CNRS (Centre National de Recherche Scientifique) where a Group of Research in Didactics had been created\footnote{5}. It was the moment when I began to work on the Didactic of Analysis.

Also at that time, I was in contact with Adrien Douady, who was a specialist in dynamical systems. He was working at the University of Orsay and his wife, Régine, was a colleague of mine at the IREM. Moreover, during the weekend, we used to climb rocks in the forest of Fontainebleau and Adrien was a member of our group of climbers. He was trying to introduce third year students at the University to the qualitative study of differential equations, and helped me discover this domain. At the IREM, we had very good computer equipment, and especially a big Hewlett Packard drawing table. Adrien and his sister Véronique Gautheron, who was an assistant in our math department and also involved in the IREM activities, used it for drawing phase portraits and exploring the behaviour of dynamical systems. I joined them and with Véronique prepared an exhibition of phase portraits of autonomous systems of order 2 and wrote a book presenting an elementary vision of the qualitative study of differential equations\footnote{6}. I began to use it in a course for second year students specializing in biology and earth sciences. Then, with Marc Rogalski, who was creating an experimental section at the University of Lille and his colleagues, I developed a didactical engineering for first year students on the topic. It was implemented during several consecutive years, and systematically investigated\footnote{7}. This is how I began to work on the didactic of analysis.

\textbf{Interviewer:} That leads us to several topics, one of which is your research in technology. But before going into that, I would like to ask the question about former and current students. You were for many years in this business of teaching university students, could you comment a little bit on any changes that happened with students? Did any change occur in their preparation or in their attitude?

\textbf{Michèle Artigue:} It is true that the students we had at the University at that moment had a lot of hours of math in secondary school, much-much more than students have today. They were acquainted with more formal mathematics, and when I look at what we were teaching in the experimental course, in this mathematics and physics section of the University, it would be impossible to teach the same today. But if I look at the methods of teaching we tried to develop at that moment, I would say that if I was about to do the same today, it would be still valid — the values, the principles in this experimentation, the different forms of assessment we used, the importance we attached to students’ projects and writings, the use we made of computers for supporting visualization and algorithmic thinking are still valid.

It is true that today the students are no longer the same. The population is different for a lot of sociological reasons. It’s not easy to explain in a few words the complexity of the changes that have occurred in the last thirty years with the change in the demography of students and the societal changes. For instance, thirty years ago, there were just a limited number of students entering tertiary education, either in the classes préparatoires aux grandes écoles or in universities. The situation began
to change in the eighties with the policy of having 80% of a class age reaching the level of baccalauréat, either in general, technological or vocational high schools. The number of students in universities and the number of classes préparatoires increased a lot. Today after secondary school, students have a lot of possibilities, many that include some form of selection, except for the universities, which are not allowed to select their students. I would not say that students today are less able than the students we had in the past, but they certainly know less mathematics, and they know mathematics differently. For those entering university, it is often a default choice, they do not know what they want to do with their lives, so it's quite different. There is no doubt that the existing dual system with the cohabitation of attractive selective institutions, because tertiary technical schools are also selective, and a university, which is not selective, contributes to the difficulties we meet in the first university years.

RESEARCHING TECHNOLOGY IN MATHEMATICS EDUCATION

Interviewer: Now moving to technology, could you please tell more about your work in using technology in mathematics education?

Michèle Artigue: I began to work with technology in the experimental math-physics section. At the beginning, we even did not have graphical possibilities, and so we were using computer technology to do some programming, to develop some algorithmic vision and make some experiments, but without graphics. Then we got graphics and it was at that moment when I began to work on differential equations, trying to make their qualitative study accessible to first year students. It was clear for me that technology was a way of making this possible. Indeed, it worked. My second experience with technology was very different. It was in an IREM group too. I worked with a junior high school teacher, Jacqueline Belloc, and we used technology in order to help a group of grade eight students that were having very serious difficulties. These students were at the point to be excluded from the school and, with Jacqueline, we proposed to give them a new chance, using technology for reconciling them with mathematics and more globally with schoolwork. Thanks to the governmental plan IPT (Informatique Pour Tous) of 1985, the school was equipped with a network of Thompson computers and we especially used the software Euclide based on Logo that included geometrical macro-procedures. Students had to write programs, this was not the kind of dynamic geometry we would become familiar with just a few years later, but we used it in a productive way, and when they ended grade nine, nearly all these students had a scholastic future. It was my first experiment with technology in secondary education and in these two years, I began to discover the complexity of the work of the teacher in computer environments, a complexity that the training sessions organized for them seriously misunderstood.

Following these first experiences, in the early nineties, I was asked by the Ministry of Education to join a group that was reflecting on the change that would be necessary if computer algebra systems (CAS) entered the secondary education. It was a group
CHAPTER 1

of teachers and experts in technology and CAS. I was not at all expert in CAS. They were doing a lot of interesting experiments but were having difficulty making their results clear and providing precise advice to those in charge of technology at the Ministry of Education, so I was asked to join the group and contribute. I observed their work for about six months and then we began working together. Some colleagues of my research team, especially Maha Abboud-Blanchard and Jean-Baptiste Lagrange, joined me in the enterprise. We developed questionnaires in order to see what was the use of these CAS systems, mainly the software DERIVE, by all the regional experts and by those who had been trained by these experts. We also began to make observations in the classes of the experts of the national group asking them to show us situations that were, according to them, especially representative of the potential of CAS for teaching and learning mathematics, and we developed a literature survey. After one year, we wrote a report for the Ministry of Education, showing that CAS technology had clear potential for mathematical learning, but that this potential was not easily actualized, and that the misleading character of the ordinary discourse about CAS technology, the underestimation of instrumental issues contributed to the difficulties met even in the experts’ classes. This was the beginning of the instrumental approach that was developed in the following decade.

For instance, it was commonly claimed that, thanks to technology, students could avoid technical work and concentrate on conceptual and strategic activities, that the learning of algebraic techniques was no longer necessary. This was a big mistake from an instructional point of view. Without negating that the nature of the technical work changes with CAS, we tried to promote another vision: a vision based on the assumption that techniques play a crucial role in mathematical conceptualizations and that the relationship between techniques and concepts is really a dialectic relationship. Relying on research in cognitive ergonomy, we also claimed that we should not underestimate the need of the instrumental geneses, which are necessary for transforming a digital artifact into a mathematical instrument, and the specific mathematical demands of such geneses, which are not necessarily part of the curriculum.

The results were not those that the Ministry was expecting but they were interested in the analysis and explanations. A second project, a bigger project, was launched and at that time (it was in 1995), we worked with the first TI symbolic calculator, the TI 92. We were also supported by Texas Instruments, so we could develop our research about these issues, both theoretically and practically.

OTHER DIRECTIONS OF RESEARCH

Interviewer: Could you please comment a little bit about other directions of your research? We discussed technology in mathematics education, teaching analysis, teaching elementary school. Would you please tell us about any other areas of your interest?

Michèle Artigue: I would say that I have also had a long term interest in historical and epistemological issues. This interest emerged when I was working on differential
issues, trying to understand the respective positions of mathematicians and physicists. I worked then with a secondary teacher, Maryvonne Hallez, who was a very active member of the history group at the IREM Paris 7. Together we analyzed the educational debate regarding differentials and derivatives, at the beginning of the twentieth century, in the journal L’Enseignement Mathématique, the international study launched by the CIEM in 1911 and piloted by Beke around the introduction of differential and integral calculus in secondary schools, and the curricular evolution from that period. It was really helpful for understanding the problems we met in the experimental section. Then I worked with Jean-Luc Verley, a colleague in the mathematics department, and a historian of mathematics who had created the IREM history group. With Régine Douady and Jean-Luc Verley we created a course in the master of mathematics program entitled: Didactic and historical approach towards mathematics. Jean-Luc Verley did not publish very much but he had an immense mathematical culture and also possessed a fantastic personal library. It was really with him that I discovered the world of the history of mathematics. I never became a historian of mathematics, but I have had, since that time, a real interest in historical and epistemological issues. I wrote an article entitled “Épistémologie et Didactique” in the journal Recherches en Didactique des Mathématiques inspired by the work developed for this course, which became more or less a reference text. I then supervised two doctoral theses jointly with the epistemologist Michel Serfati who also worked at the IREM, those of Caroline Bardini and Véronique Battie. For me this was an important component, even if I am not at all a specialist.

I have also worked on issues of transition in math education, and for that I used the anthropological theory of didactics developed by Yves Chevallard. This interest for transition issues began when I supervised the doctoral thesis by Brigitte Grugeon on the transition between vocational and general education. The lens provided by the anthropological theory radically changed the approach of the students’ difficulties. Instead of focusing on students’ cognition, we focused on the discontinuities between the algebraic cultures of vocational and general high school, and this move opened new doors for productive action. Then with other doctorate students, Frédéric Praslon, Analia Bergé and Ridha Najar, we used the same approach for understanding the difficulties of the secondary-tertiary transition. In the last decade, I have also been involved in several projects trying to overcome the fragmentation of the field of math education by building connections between theoretical frameworks and research practices. This has been achieved mainly in the frame of the European network of excellence, Kaleidoscope, and then the European project ReMath, and within an international group that emerged from the fifth CERME Conference of European Research in Mathematics Education. A special issue of Educational Studies in Mathematics and a book in the Math Education Series of Springer will be published soon as the result of this collaborative work. I have the impression that my ICMI responsibilities made me more and more sensitive to the necessity of establishing such connections because I discovered, being vice-president and then president of ICMI, up to what point the fragmentation of the field is an obstacle to making clear to
people (particularly those outside the field) what has been achieved, what we exactly know in math education, and how it can serve the cause of practice, to express our knowledge in a way that is scientifically consistent and widely accessible.

ON FRENCH SCHOOL OF MATHEMATICS EDUCATION

**Interviewer:** My next questions will be about national and international in mathematics education. Let me start with national. You are a part of the French school of mathematics education. Could you please comment a little bit about the development of this school of mathematics education and how it influenced your studies and what you would say are its most important features and characteristics about which people outside of France would better learn?

**Michèle Artigue:** Yes, I think that it has been a great opportunity to live and to grow in this environment, and very exciting to contribute to the development of the French school of didactics of mathematics. Of course, this is an a posteriori reflection because when I was a young researcher growing in this culture, I did not think about that, I did not envisage other possibilities… Reflecting now on the past, I am very grateful to the visionary fathers of this French school, especially researchers such as Guy Brousseau and Gérard Vergnaud. From the seventies, they wanted to have the field of mathematics education develop as a genuine field of research, not just as an extension of an existing field such as psychology or mathematics. They claimed that the field of mathematics education, what we call didactics of mathematics, addressing specific questions, required specific theoretical constructs and methodologies, and should be developed both as a fundamental and applied field of research. Very early on we also adopted a systemic perspective thanks to Guy Brousseau and his theory of didactic situations. Our aim was to understand the functioning of these didactic systems in which students and teachers interact between themselves and with mathematical knowledge, because we thought this essential for understanding learning processes. Thus from the beginning we considered learning as a social process, and a subtle combination of adaptation and acculturation processes. We tried to find ways of optimizing the functioning of such systems, and with the methodology of didactical engineering we created a qualitative methodology fully in line with this systemic view, and put the complexity of the classroom at the center of the experimental work.

There was also the important idea that didactic research should maintain close contact with mathematics and mathematicians. Our community is today much more diverse but still we consider this connection with the mathematics world of today important. The epistemological interests that we discussed earlier were also connected with this idea.

Another point is that didactic research typically emerged in the IREM, and for that reason it did not develop as laboratory research. We were working with teachers. The school terrain was a permanent source of questions, and conversely research ideas were tested in school designs and practices. The privileged role given
INTERVIEW WITH Michèle Artigue

for a long time to didactical engineering as a research methodology illustrates it\textsuperscript{24}. However, we considered it our priority to increase the understanding of teaching and learning processes. The pitfalls of the modern math reform had made it very clear, we needed fundamental research, and action based on research advances. All these characteristics made our approaches and forms of thinking different from those developed in research in mathematics education in many other countries including US. Today with the development of socio-cultural approaches, the development of design-based research; these ideas are no longer so original. But in the late seventies and even the eighties, they were, and they contributed to the emergence of a strong didactic culture.

Also, we questioned very early on the math knowledge itself and the complex processes that led from some reference knowledge to the knowledge actually taught in classrooms, adopting an ecological posture. Yves Chevallard with the theory of didactic transposition played a fundamental role\textsuperscript{25}. Then, embedding this theory into a more global anthropological approach, he helped to extend the systemic vision to institutions and to take into consideration the diversity of conditions and constraints affecting the functioning of didactical systems. There is no doubt for me that all these approaches were really great tools for thinking about and working on mathematics education.

ON ICMI AWARDS IN MATHEMATICS EDUCATION

Interviewer: Turning to the International collaboration, I would like to start with a very specific question. You were at the birth of ICMI awards — Freudenthal and Klein awards. Could you elaborate a little bit on them? How these awards emerged and what were the challenges for their creation?

Michèle Artigue: I became a member of the ICMI executive committee in 1998, and apparently the idea of creating ICMI awards had already been evoked, but no consensus found. In 1999, our EC decided to appoint a committee of distinguished scholars to study the idea and make a recommendation. We discussed their advice, which was positive, at our annual meeting in 2000 in Makuhari at the time of ICME-9, and in line with their recommendations, we took the decision of creating two awards for research in mathematics education: one for a lifelong achievement which we decided to give the name of the first president of ICMI, Felix Klein, and one major program of research in the last ten years, which we decided to give the name of Hans Freudenthal, its eight president and a pioneer of research in the field.

We were all conscious that it would certainly be difficult to make comparisons and select candidates among all those deserving such a recognition, yet at the same time, through these awards, we wanted to officially acknowledge the maturity acquired by the field of research in mathematics education, and to contribute to the visibility of its achievements. Then we defined the policy about these awards. An Award Committee of six scholars in the field would be appointed by the president of ICMI, after consultation with the EC and with other scholars in the field, and this
committee would be given the task of selecting the awardees, in total autonomy and confidentiality. For limiting the pressure on committee members, only the name of the chair would be made public. Three members were nominated for 6 years and the three others for 3 years.

It was also decided that no member of the current ICMI EC could be nominated in the award committee for ensuring total independence. An exception was nevertheless made for launching the process, and it was decided that I would be the chair of the first award committee.

The first Award Committee ended its term in December 2006 after having attributed two Felix Klein and two Hans Freudenthal awards. After discussion with the members of the committee, I produced a confidential report for the EC presenting a retrospective reflection structured around the following points: nature of the two awards, criteria of selection for the awardees, number and frequency of awards, dissemination of information, official presentation of the awards, committee functioning, ethical issues. This report was discussed at the first meeting of the new executive in June 2007 in London. It will be part of the ICMI Archives.

WORKING IN ICMI EXECUTIVE COMMITTEE AND BEING A PRESIDENT OF ICMI

Interviewer: Now regarding the years of your presidency. Could you tell about major issues, major accomplishments, major problems which you faced as a president?

Michèle Artigue: I was a president only three years (laughs). For me, my life as president was an extension of the eight years spent as vice president during which I worked very closely with Hyman Bass and Bernard Hodgson, the president and the secretary-general of ICMI at that time. I remained also a member of the executive committee, until 2012, as past-president when Bill Barton became president and we had close contact. Thus for me it has been fourteen years of my life and I do not make a strict distinction between the time I was president and the time I was vice-president of ICMI. I remember very clearly the time when I became a member of the executive committee in 1998. I met Hyman Bass and Bernard Hodgson at the International Congress of Mathematicians in Berlin, just after the election, and I discovered there that the relationships of ICMI with its mother institution, the IMU, were not so good. There were evident tensions, as I mentioned in my closing lecture at the symposium organized in Roma in 2008 for the centennial of ICMF. The program for the section of the Congress on mathematics education and popularization of mathematics, traditionally planned by ICMI, had been partially rejected by the organizers of the ICM. A round table had been imposed with some mathematicians from the US who apparently wanted to export their Math Wars on the international scene, I would say.

At that moment there were also questions raised in some quarters of the ICMI community: why stay under the dependence of mathematicians, why doesn’t ICMI become independent from IMU? It is mature enough for that. We seriously discussed the issue and decided that ICMI should stay inside IMU, but that the current state of relationships was not acceptable. This was in line with my vision of the
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Field from the very beginning of my career. I know that creating and maintaining productive relationship between mathematicians and math educators is not easy, but I remain convinced that our fields need to be closely connected. Moreover the ICMI Community involves all those with professional interest in mathematics education. It does not limit to the community of didacticians or mathematics educators.

It was decided that we would make a lot of effort to improve the situation, because otherwise there would be no future. It was a part of our agenda, and Hyman Bass played a decisive role. His high status as mathematician combined with his engagement in education, and his political and diplomatic expertise made the evolution possible. Progressively, the situation improved, thanks also to the support of members of the IMU executive, and notably the successive presidents John Ball, László Lovász and Ingrid Daubechies, the secretary Martin Grötschel, Manuel de León and Cheryl Praeger, who successively were the specific ICMI contacts in the IMU executive. They perfectly understood the importance of mathematics education and of ICMI. Collaborative projects were launched such as the Klein project and more recently the CANP project. Mutual confidence developed. When I compare the quality of relationships now with what I experienced during my first years in the executive, I can measure the evolution. Moreover institutional changes have also been achieved; since 2008 the ICMI Executive Committee is elected by the General Assembly of ICMI and no longer by the General Assembly of IMU. There is still a lot to do because these relationships at the EC level do not reflect the actual situation of the relationships in many countries in which mathematicians and math educators poorly collaborate when they do not conflict.

The second point I would like to mention is that we have tried to extend the outreach of ICMI and make the voices of the periphery more audible. For a long time, ICMI has been an affair of developed countries and dominant cultures, and this also had to be changed. We have taken a lot of incentives in that direction, as was made clear at the Centennial Symposium in Roma. For instance, we have reinforced our regional networks by creating two new regional networks: the AFRICME network in Anglophone Africa and the Francophone Mathematics Space (EMF) which is also helpful for Francophone Africa. EMF is based on a new idea of a region sharing one language, but issues of language cannot be underestimated. We can ask researchers to speak some kind of international English understandable by others as my poor English (laughs), but we cannot ask it of teachers. And it doesn’t make sense to think that only researchers can be able to participate in the activities of ICMI. So the language issue, having more understanding of the differences, and giving more voice to developing countries was, certainly, a major dimension for ICMI in the last decade. For extending the ICMI outreach we have prioritized developing or emerging countries for the ICMI Study Conferences, and also organized, for the first time, the ICME Congress in a developing country. This was in 2008 in Monterrey, Mexico. This dimension of our activity has been supported by the renewal of our relationship with UNESCO, which began in 2002 with the participation of Minella Alarcon from the Division of Basic and Engineering Sciences in our EC meeting in Paris, and the
ICMI support to the realization of the UNESCO travelling exhibition *Experiencing Mathematics!* presented for the first time at ICME-10 in 2004. From that time, UNESCO regularly supported, even if it was modestly, some of our activities in developing countries. Then in 2009, after an international seminar organized by UNESCO in Paris, I was asked to pilot the publication of a UNESCO document on the challenges of basic mathematics education\(^{10}\) (during this period, I collaborated closely with Beatriz Macedo, who was in charge of mathematics and science in the division of Basic Education). One of the consequences of this publication was the launching, with the support of UNESCO, of CANP, the Capacity and Networking Programme, a program for reinforcing the mathematical and didactical preparation of teachers and the development of regional networks of teacher educators. In CANP, in line with the ICMI spirit, we want to connect and involve all those engaged in the education of teachers, the mathematicians, the math educators, the expert teachers, and other professionals and institutions. This program, which is also supported by IMU, has had one major event per year since 2011, first in Mali, then in Costa Rica, and in Cambodia just recently. Two new networks have been created, and ICMI has launched a new series of publications. For each major event, the countries involved prepare a report on the situation of teacher education in their country.

One more direction of ICMI activities is the ICMI Studies\(^{11}\). We have launched a lot of new ICMI studies, the last one being the ICMI study twenty-three entitled “Primary Mathematics Study on Whole Numbers”. We have also initiated partnerships for ICMI Studies, for instance with IASE (International Association for Statistics Education) for ICMI Study 18 on statistics education and with ICIAM (International Council of Industrial and Applied Mathematics) for ICMI Study 20 on the educational interfaces between mathematics and industry. During the last decade, we have reflected a lot about what we want to achieve through these studies. We want to produce state of the art research and practice, and delineate perspectives for future research and development. We want to include significant achievements in a diversity of contexts and cultures. We would like to have them written in a language that is widely accessible making them useful beyond the sole community of researchers. It’s really a challenge.

**Interviewer**: What do you think are major problems which we as a community, — the national communities, communities of researchers in mathematics education, communities of practitioners — are facing and are about to face in the nearest future?

**Michèle Artigue**: Yeah, yeah, yeah. There are so many problems! I have been working in the field for four decades and I have the feeling that whatever have been our efforts, we are still facing the challenge that mathematics education is considered something that is hard for the students, something that is not for everyone. We understand much more about better teaching and learning processes, we have accumulated knowledge, we are able to produce successful experiments in a diversity of contexts and cultures, but improving mathematics education at a large scale is still a major problem. We have a lot of problems with teacher education in many countries — pre-service education and even more in in-service education,
which is not well organized in most countries. We have the fact that life conditions, social relationships are changing creating new challenges everywhere in the world. Technology is changing a lot of things, and we need to think about how we can benefit from that in education. We had some years ago the second ICMI study on technology. At that time technology was already quickly moving, deeply impacting the modes of communication and access to knowledge, but at the same moment, we had to admit that even what we can call today the old digital technologies, such as pocket calculators, dynamic geometry systems or spreadsheets, were not productively integrated in most classrooms. Technology is changing the ways in which teachers and students interact, and their interaction with mathematical knowledge. The classroom is no longer something closed but a place where information is entering from the outside through a diversity of channels. How can we make that beneficial for the future of mathematics education?

So for me, there are a very many problems. And, of course, I should mention equity issues, gender issues, and linguistic issues. ICMI Study 21 is devoted to linguistic diversity. When Jill Adler proposed this theme to the Executive Committee of ICMI, some members thought that such issues were relevant for some countries only, but when we discussed it, we quickly discovered that we all had to address, in some way or another, these issues of linguistic diversity. Traditionally, linguistic diversity has been treated as a problem for learning. In the study piloted by Mamogkethi Setati from South Africa and Maria do Carmo from Brazil, we have tried to show that this is a very limited view and that linguistic diversity can be a resource for learning, not just a problem.

ON HISTORY OF MATHEMATICS EDUCATION

**Interviewer:** We turn to the final section. Since our Journal is devoted to the history of mathematics education, I would like to ask what would be of particular interest for you in the history of our field.

**Michèle Artigue:** As I said before, for a long time I have been interested in history of mathematics and in history of mathematics education. When, for instance, I worked on differential and integral processes, I needed to understand the history of this subject and also the history of the debates in education around this subject, so both the history of mathematics and the history of mathematics education were very important for me. Since 2000 in Paris my laboratory is part of a doctoral school that includes a laboratory of history and philosophy and epistemology of science, which is one of the biggest in France, and a laboratory specialized in history of education. I have been the director of this doctoral school where we organize a special day each year with a series of lectures on a theme of common interest. I have regular contacts with research in that area and I find it inspiring. Personally, I am more interested today in epistemological reflection on the nature of mathematics and mathematical practices, in learning about the diversity of these practices in different contexts and cultures, not just in the history of mathematics.
In our doctoral school we have specialists of Chinese mathematics and Indian mathematics. There is also some work in ethnomathematics in connection with education. In two days from now, Christine Proust who is a member of this doctoral school, will present her research on the Babylonian mathematics and scribe schools at the annual Colloquium organized by the French subcommission of ICMI. For me what is really exciting in all these works is that they show the diversity in the ways mathematics has developed in the different cultures, the diversity in the ways mathematics has been taught in the different cultures, and for me, it’s really a source of inspiration to see this.

Interviewer: Thank you for this interview.

NOTES

1. École Normale Supérieure – an educational institution created during the French Revolution, which is a center of scientific research in many fields and, in particular, in the field of mathematics. It is a center of the preparation of professional mathematicians at the highest level.

2. In France, along with universities, there is a system of so-called Grandes Écoles. Since the time of the French Revolution, they have been considered the most prestigious educational institutions. classes préparatoires provide intensive preparation (usually for two years) for enrollment in the Grandes Écoles.


8. More information about this period can be found in Barbazo & Pombokourc, 2010. See also the interviews by Maurice Glaymann and André Revuz accessible at the historical website of ICMI: http://www.icmihistory.unito.it/clips.php


10. More detailed information about the IREM network is accessible on the portal www.univ-irem.fr, which also gives access to the websites of the different IREMs.


12. The COREM was created by Brousseau in Bordeaux in 1973. The data collected there during more than 20 years are still studied by researchers. Detailed information is accessible at the following url: http://guy-brousseau.com/le-corem/acces-aux-documents-issus-des-observations-du-corem-1973-1999/


15. See Allibert et al., 1988.

16. Artigue & Gautheron, 1983


19. Artigue, 1990b

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21 Information about this project and the different deliverables are accessible at www.remath.cti.gr. See also Artigue & Mariotti, 2014.
22 The special issue of Educational Studies in Mathematics was published in March, 2014.
23 Gérard Vergnaud is a French psychologist and mathematics educator.
24 See Brousseau, 1997.
25 See Artigue (2013) for a synthetic vision.
27 Artigue, 2009.
30 http://unesdoc.unesco.org/images/0019/001917/191776e.pdf
31 http://www.mathunion.org/icmi/conferences/icmi-studies/introduction/

REFERENCES

CHAPTER 1


The following interview with Heinrich Bauersfeld was conducted in the Summer and Fall of 2012 by e-mail. Heinrich Bauersfeld received his doctorate in mathematics from Hannover University but devoted his life to the teaching of mathematics. His work made a seminal impact both on the practice of mathematical teaching and on shaping the entire field of mathematics education (Didaktik der Mathematik) as a scientific discipline*. He worked as a professor and a director at the Institute of Mathematics Education at the University in Frankfurt am Main and later as a professor and one of the directors of German Federal Institut für Didaktik der Mathematik in Bielefeld. He was in charge of several major German National research projects in mathematics education including the very first large-scale experimental studies (also, he was a leading author of the series of textbooks based on these studies).

* Although German Schools of Education (Pädagogische Hochschulen) enjoyed academic status already after WWI, their faculty was responsible only for teaching the basics of the educational methodology rather than conducting their own research. That changed in the Federal Republic during the 1960s and 1970s when Schools of Education became parts of the universities which implied involvement of the faculty in the research and supervision of doctoral theses. (This note is by Gert Schubring)
Highly internationally recognized and influential, Dr. Bauersfeld has served as an invited speaker at many conferences and as an invited professor or consultant at many universities. He has authored more than 200 publications (complete list of all his publications is available at http://wwwhomes.uni-bielefeld.de/hbauersfeld). He served as a member of the German Educational Council from 1969 to 1974.

The interview was conducted by Alexander Karp (Teachers College, Columbia University). The Notes are by Heinrich Bauersfeld.

BEGINNING: SCHOOL YEARS

Interviewer: Traditionally we start with personal history. My first question is about your own school years. What do you remember about your teachers and about the atmosphere of the school?

Heinrich Bauersfeld: Born in December 1926, I entered an all-boys school at Easter 1933 and had the very same teacher, W., the whole time from grades 1–4 without any change. As a former socialist he lived under the Nazi regime, as an inconspicuous non-Nazi (I learned about it 30 years later, when I met him again as a professor of education at the teacher training college in Hannover). His teaching was free of any Nazi ideology. Yet the unavoidable encounters with the then new Nazi system happened at all other school affairs: at the many parades (Appell) in the school yard with long addresses of the school’s headmaster (Rektor) and the new swastika flag; trainings in air raid protection with sirens howling and tear gas in the school floors (in 1935/36 already); uniforms growing everywhere and with all Nazi organizations.

The political indoctrination started early and heavily.

One frightening experience from 1934, when I was just 7, haunts me even nowadays: One morning many schools were led into an old theater to see a political show that Hitler’s brown troops (“SA”) had organized, a Nazi indoctrination using recitations, songs, and short sketches. One piece began with about a dozen young men on stage, in line, hand-in-hand, singing one of the aggressive Nazi songs, when suddenly loud shots were fired, and one man after the other sank down. I was sitting beside a teacher who noticed me trembling and said: “Don’t worry, it’s a play only.”

The disadvantage of four years of learning from one key person arises from the total exposure to the person’s strengths as well as to their weaknesses. My teacher W. was a freak in keeping all formalities under control, arithmetic, orthography, general properness, but there was no geometry, no music, and no singing. Often we had to line up in front of the classroom for a checkup of our ears and fingernails. An ever-present cane was ready for punishment under a rigid face, besides all punches, slaps, wrenching cheeks and ears, etc. The daily mental arithmetic started with all kids standing. Right answers were followed by a “sit down!”, while those giving wrong answers were held standing. Clearly, the very same kids formed the little group of last “uprights.” These kids got tasks down through “1+1=?” in order to enable their final “sit down!” or the cane produced a rigid abbreviation of the procedure. He never hit me and tolerated my sometimes non-standard ways of thinking, yet his bias towards maltreating my poor classmates shocked me.
INTERVIEW WITH HEINRICH BAUERSFELD

My elementary teacher W. preferred a method of teaching the concept of number, which Johannes Kühnel had developed in the twentieth century (this method was spread widely at that time). Kühnel thought that linear representations, like a number line, were too difficult for children and chose to represent ten as a 2x5 arrangement of ten discs (“Zahlbild”), which were counted in zigzag. Figure 1 represents 15, for example. An important feature of this representation is the eye-catching difference between even and odd numbers.

![Figure 1. Presentation of 15 in a Zahlbild.](image)

In the same way, he formed a hundreds table from such tens as a 5x2 vertical rectangular arrangement of tens. This device (Kühnelsche Hundertertafel) found wide acceptance and is still in use in some schools. Figures 2 and 3 demonstrate how numbers were represented. These representations for which we had to use a special piece of paper (Abdeckblatt) caused serious difficulty already when adding two numbers – it could require some kind of rearrangement. Multiplication and division usually were even more difficult. These procedures were particularly time-taking for the slower classmates, who were permanently in danger of mistakes and subsequent punishment. Their way out of these formalistic drills was my way too: we learned the elementary facts about numbers by heart, though with different speed admittedly. Knowing the solution more or less by heart enabled students to imitate the official procedure, an outcome that comforted both teacher and students.

![Figure 2. Representation of 27](image)

![Figure 3. Representation of 32.](image)
Another special feature of my teacher was that he enjoyed giving us chains of tasks in our daily mental arithmetic. Through the pace of his utterances he claimed to raise our attention and concentration. Typically, he discussed cutting into pieces pies and round-form cakes. Importantly, for this kind of activity he allowed us to work while seated, not standing. He started giving us tasks with fractions quite early (although they were limited to addition and subtraction). Examples were taken from everyday experiences. My advantage in these games arose from my mother's needlework. She was a trained plain seamstress, so I had ample chance, early, to play with cards of a dozen buttons each. These experiences enabled me to solve most of the simple tasks with fractions via transformation in button counting: for instance $\frac{1}{6} + \frac{1}{3}$. Of a button card of 12 makes $\frac{1}{6}$ just 2 buttons and $\frac{1}{3}$ makes 4 buttons; $2+4=6$ and 6 of 12 is exactly $\frac{1}{2}$. (Indeed, I did not know that this method was already in successful use in mathematics education decades earlier.) Moreover, soon the frequent use made the results of operations with simple fractions memorized. This frequent practice, I think, had given reason for the fairly early disappearance of any kind of devices or embodiments or graphical representations in my personal mental arithmetical operations. I don’t “see” number lines or groups of dots or any other concomitant phenomena when operating, except when the task requires it explicitly.

**Interviewer**: You mentioned your mother, could you tell more about your family?

**Heinrich Bauersfeld**: My father came from a rather poor family as the second of four children. He received a stipend at the best grammar school (Klinger-Schule) in Frankfurt am Main, but he had to give up his subsequent studies of law at the University after two semesters. His older brother died early and his sister suffered from severe poliomyelitis and survived crippled. Thus my father was forced to work to support the family. He started in a bank and slipped soon into a steep career as an auditor. One of his customer enterprises hired him and in the 1920s, he was a well-paid director already. Not for long, since the worldwide economic crisis began in 1929. His enterprise went bankrupt and a larger enterprise in Hannover took over, including his own. There he had to start again as an executive secretary.

As an only child I received an overload of care and anxious limitations from my beloved mother. Late and by force I joined the Nazi youth organization in 1938 as *Pimpf* (for 10- to 14 year-olds there was *Jungvolk* and for 14- to 18 year-olds, *Hitlerjugend*). This organization in many ways had been organized similarly to the famous “Pathfinders” (with military drill, singing, scouting games, etc.), and being politically naïve, I often viewed participation in it as a welcome escape from my mother's supervision. I succeeded to escape into a special music unit, where I learned to play a ceremonial trumpet (*Fanfare*). The automatic change into HJ (*Hitlerjugend*) enabled me to join another special unit. This amateur dramatic group (*HJ-Spielschar*), saved me from the usual paramilitary drill in the regular HJ-formations. I am sad to confess in retrospective, that we were victims of a careful planned seduction and integration into the Nazi system. The awakening happened not earlier than late in wartime, when the end came near, and the deadly illusion became obvious.
My parents avoided political discussions with me, so I was left without critical information and seducible. My father, member of the Nazi party (NSDAP) since 1934, had several encounters with the party leadership, but never talked about it. It is no late excuse to say that my father was not a real Nazi. The board of directors in his company forced the “newcomer” to join the National Socialist party in order to keep them free from additional political pressure: “At least one has to do it!” Indeed all board members saved themselves as “innocent” after wartime and remained in function, while the committee for denazification sentenced my father as “nominal party member” to three years loss of income. That meant he did not lose his job, but his income was cut down substantially. This hit the whole family and my first years at university, and, as bombed-outs, we lived together in one room.

**Interviewer:** Could you please tell about your education in the upper grades?

**Heinrich Bauersfeld:** Moving into grammar school (*Deutsche Oberschule für Jungen*) at Easter 1937 functioned for me like a wide opening of the world. Suddenly there were special teachers for each subject including English, Latin, mathematics, music, gymnastics, etc. Headmaster Kühnhold was not Nazi, and we had many good teachers, no one of whom ever appeared in a Nazi “costume.” The history teacher was a Nazi; even more his brother acted as cabinet secretary in the Nazi government (*Reichsminister*). He was a weak teacher, anyway, and we did not take him too seriously. More than half of the staff members were veterans of the First World War, some war-disabled. This caused a certain climate of military discipline, with physical punishment, and nationalist mentality. I remember only one female teacher: this young woman served as substitute English teacher for only half a year. The teachers of mathematics and gymnastics were especially brutal. Slaps into the face were normal reactions to mistakes and to misbehavior. The mathematics teacher, Dr. Ruperty, was a muscleman with very hard hands. With us he exercised a permanent training of stereotyped mathematical language with proofs, formalized geometrical constructions, and other formal argumentations. He loved to call his victims to the blackboard for a written demonstration of their solutions, with him standing nearby. He accepted only solutions similar to his own and punished right away even the slightest deviation. I had no problems with him, but I feared and hated him as my classmates did. Even about a decade later, during my final examination in mathematics at university, Ruperty’s recurring image made me stutter, when asked to define continuity (*Stetigkeit*). As a professor already I came to meet him by chance in town, retired but still the very same type. He remembered my name and asked about my doing. He disappeared without a word shaking his head.

After two years when war began these school conditions changed abruptly. Our school building became a military hospital. We had to share rooms with another grammar school, and our staff changed dramatically. The veterans as well as the younger teachers were drafted again and replaced by reactivated retired teachers, often very old and frail already. Increasingly school became less important, if not annoying, and overshadowed by the omnipresent war affairs.
CHAPTER 2

Remarkably, even then the pro-Nazi pressure did not come from the teachers. It arose from my classmates as soon as they climbed up in the hierarchy of Jungvolk and Hitlerjugend, becoming leaders with ranks and titles. A few loved to appear at school in uniform as a kind of protest against this “old-fashioned” and “backward” organization. Headmaster Kühnhold did not appreciate it. Anyway, learning meant no difficulty for me; mathematics was my favorite subject.

My life was changed much deeper later. In February 1943, right after the turn in Stalingrad, my whole class was ordered to serve as a kind of pre-soldiers in a battery of anti-aircraft guns (“Flakfelder,” with a special uniform). This happened to all school students born 1926/27, and all over Germany. For a while our teachers of German and mathematics (Ruperty!) came by bicycle for short lessons in the morning, often interrupted by actual bomber alarms. The bombing of Hannover in November 1943 erased our flat in a four-story compartment house. At the battery we saw Hannover burning down at the horizon, anxious about our relatives. My parents had a narrow escape. The complete compartment burned down into the cellar. Father’s rich library, as well as my own books, my homemade models made of wood and metal and everything else, I found myself left with the few items I had with me in the battery. This catastrophe marked the end of my youth. Due to other bombing damages, the conscription to the German Labor Service, “RAD” (Reichs-Arbeits-Dienst), reached me belated just one day before Christmas in 1943. Thus I came to sit under a Christmas tree together with 150 other RAD draftees in brown uniform in a camp near Hamburg. So my learning at grammar school dwindled to a limited period of a bit more than five years.

After three months I had to change uniform again, the German Navy had me. As officer candidates we soon felt that our commander-in-chief kept us away from all risky fronts, since he refused our applications for transfer to frogmen or one-man torpedo units. Apparently, the admirals found themselves in a fight for a lost cause, as I learned after war¹. Thus I never came into the situation of firing in combat. Instead we passed one training course after the other, and in May 1945 I marched into an internment camp of the British Forces, as just appointed midshipman (Fähnrich). The British released farm workers and students early, preferring the internees. I found my parents in Hannover, and had to work there as translator and “civil labor manager” for 300 German workers with the British “Mechanical Equipment Park Coy, RE.” When I left, Sergeant Major Finham said “goodbye” to me in fluent German, although he had never said a single German word before.

HIGHER EDUCATION

Interviewer: How did it happen that you returned to your studies and continued to Higher Education?

Heinrich Bauersfeld: In autumn of 1945 I joined a special six-month course for ex-soldiers in order to gain matriculation standard, replacement for the Abitur. When leaving school in 1943 I had received only a so-called Vorsemestervermerk,
INTERVIEW WITH HEINRICH BAUERSFELD

no maturity certificate, but a paper promising later matriculation at universities. New regulations after war made this paper worthless. Special courses became compulsory. These required studies in German, mathematics, and one additional elective subject; I took natural science, and ended with an oral exam. Again, this was not enough. Right after the ceremonial handshake with the rector magnificus at the Technical University in Hannover I had to pass another Vor-Semester, compulsory also with lectures on German, and ancient (Greco-Roman) history, and again a final examination – written this time. After all we were allowed to take a few lectures in the field of our final aim of studies, in parallel to the compulsory lectures. My final aims were mathematics (as major subject) and physics and chemistry as minor subjects for teachers of grammar schools.

The double-tracked studies started in summer 1946 and turned into desperate difficulties for me. The “Swiss cheese-like” instructions in wartime had left me full of holes; even sines and cosines were only shadows. I had to work hard to fill my gaps to pass the many written recitations of the beginning intensive studies in mathematics. The busiest five years of my life had begun with uninterrupted studies, no holidays, and we all were consistently hungry. Yet the ray of hope and comfort came with my first and most influential mathematics professor, Lothar Collatz. I was fascinated with him – he was young (and due to health problems, not drafted), extremely fast, strictly task-oriented, and no Nazi. Although I was one of the youngest students, he picked me as his assistant (part-time). So I came to produce transcripts of his lectures, teach students just one course behind of myself, and correct their written assignments. Collatz’ lectures treated nearly all areas of mathematics. More an applied than a pure mathematician, he was more interested in practical methods than in proofing, definitions, and “epsilontics.” He presented a lot of problems and made us look for solutions, on closed/strict or approximate paths. As a hobby he used to run a very popular column with mathematical problems in our best weekly “Die Zeit,” called Einstein’s Logeleien.

The Technical University, located since 1879 in the large castle of the former kings of Hannover, had been bombed heavily and was partly in ruin, temporarily repaired for a few thousand of students, windows covered with thin plastic foil (perforated by student pencils), most rooms without heating, the library poor and old-fashioned (if at all, one copy per book). Thanks must be given to the “America House” in Hannover! Their library was excellent in actual humanities. During the extreme cold winters in 1947 and 1948 we put large icicles at the table in front of the blackboard. All through Collatz’ two hour lectures the icicles did not melt. Thus, calculus, theory of functions, number theory, etc., did appear as both hot and cold subjects to me. My colleagues and myself – we were four assistants at Collatz’ chair – we were hungry, constantly. At work in our room on occasion we had red beans boiling on a little fire on the marble floor, with singed wood from the ruins around. One of us seemed to have an inexhaustible source of these little jewels.

Another fascination for me in terms of scientific brilliance was Hans Daniel Jensen, professor of theoretical physics. Each one of his lectures was of top quality
and remarkably deep, and also attracted us like happenings in geniality and dry humor. He used to “work” at his broad blackboard with a piece of chalk in his right hand and a wet long-handled scrub brush in his left hand, filling more than 10 meters in length with his formulas, simultaneously erasing with his left hand what he meant not to need any longer in order to win more space. All that writing and commenting happened at an incredible speed, which left us with severe difficulties in our records. Most fascinating was his treatment of formulas, especially perhaps for the few mathematics students among his large auditory. In order to arrive at sharp results he loved to use physical argumentations for his surprising abbreviating and simplifying of large parts of his terms (with a smile: “easy to see, this term converges to zero because ...”). In the end, sometimes, an expected factor $1/\pi$ or so was missed somewhere: “You will find the mistake easily when you check it at home!” We were quite sure to be learning from a future Nobel Prize winner. Indeed Jensen received this highest honor in 1963, the Nobel Prize in physics. We had to miss him in my third academic year when he followed a call to Heidelberg.

In my fourth year another crisis began. Although we were on the teacher's course of studies there was no course in education, nor psychology, nor philosophy at the Technical University. The dominance of formulas and logic, the poverty of the technical language games in mathematics and classical physics, and the endless piling up of reaction rules and their exceptions in chemistry seemed to exist too far from real-life conditions. I began to miss language and philosophy, etc., mental challenges which I had in abundance in my father's library, where Kant, Nietzsche, both Humboldts, and many more, were accessible. I have to confess that my choice of mathematics as main subject was owed to the absence of alternatives for students of teaching in Hannover. Although mathematics was my favorite subject at school, I would have chosen studies of languages and linguistics as well. But alternative facilities were not in reach right after war.

Luckily the ministry realized the general deficit and assigned, as lecturers, professors from the Teacher College (Pädagogische Hochschule) in Hannover, e.g., Otto Haase for education and Hans Wittig for psychology. Concurrently, Haase was head of the school department at the ministry for culture, science, and education (of the Land Lower Saxony), and he was an early member of the Youth Movement from the beginning of the century, and, last but not least, victim of Nazi persecution. Through many excursions he introduced the small group of his students to progressive schools and excellent boarding schools, while Wittig introduced us into contemporary psychology and philosophical anthropology. A little later Wilhelm Kamlah, philosopher at the University of Göttingen, completed the humanities' team. These extensions opened a new world for me. From then my further studies followed at least two different tracks, though I did pursue both with all energy and intensity.

In spite of poverty and famine, we celebrated large parties in our ruins. The physicists were the most creative group among the students and between Porter, Gershwin, Cole, and alcohol-free drinks they presented excellent political revues. One scene opened with the European countries in line, represented by their symbol
figures: Netherlands's “Meisje,” the French “Marianne,” English “John Bull,” German “Michel” with his pointy cap, etc., all dressed originally. And in front of this unit the US-American “Uncle Sam,” the commander, wrapped in stars and stripes, shouting sharply: “Stand still! Volunteers for the defense of Europe one step forward!” And in a united movement, “clack-clack,” all Europeans took two steps back, leaving “Michel” alone in front. The Germans were expected to begin their rearmament relatively soon, in 1948 already, and all we, “old soldiers,” were upset about it.

After the First State Examination for secondary school teachers, early in 1951, I thought about continuing my education in philosophy, in Göttingen with Kamlah as supervisor. Yet, the big classical department of philosophy in Göttingen required three of their subjects for the final disputation, allowing only one out of mathematics and natural sciences. So I resigned and took the mathematical path with Collatz as supervisor. Since I had done some deeper studies with partial differential equations, we agreed for a thesis on methods for the approximate solution of partial (hyperbolic) differential equations of second order. Collatz stood for the opinion that a doctoral degree in mathematics had to lead into a university career; every alternative meant a waste of time to him. Indeed, all of his former assistants and doctoral candidates became university professors of mathematics later. I regret to say, that the clearer my way became into school and into the staff of a teacher college – i.e., teaching mathematics instead of developing mathematics itself – the more decreased Collatz’ interest in my thesis. Moreover he had accepted a call to Hamburg in 1952, and was then engaged in founding a new and larger institute. It was here already that I encountered the disregard if not disdain of didactics and education as enacted by professional mathematicians. After two years, in 1954, the conclusion happened in Hannover, not in Hamburg, without any disputation. I finished as Dr. rer. nat. (equivalent of PhD), cum laude, and delivered three copies to the library of the Technical University in Hannover. Later I found a footnote about my thesis in one of Collatz’ books, but no entry in the listing of his doctoral students in his detailed obituary.

FIRST YEARS OF TEACHING

**Interviewer:** When did you start your teaching?

**Heinrich Bauersfeld:** In 1953 Otto Haase appointed me as an assistant at the Teacher College (Pädagogische Hochschule) in Osnabrück. Under supervision of my master professor, Walter Breidenbach, I taught mathematics in different schools and studied mathematics education (Mathematikdidaktik). The majority of the faculty at the Teacher College had been professors at the famous reform institutions for teacher preparation: the Pädagogischen Akademien in Prussia, a few years before the Nazis came and closed them down. This was a unique and impressive experience in a holistic teacher education culture for me. In 1954 when a vacancy in Hannover opened, I moved there, still as assistant (lecturer in 1955), and as a
successor of Professor Gustav Rose, who had written a first book on psychology of learning and teaching mathematics (Rose, 1928). The teacher college trained student teachers for the compulsory part of the educational system grades 1-8, and later, 1-9 (Volksschule).

There were hundreds of students, many of whom were older than I. I was the only faculty member teaching mathematics. My budget (for books) was 50 DM annually. My teaching load included more than 12 hours of lecturing per week, and all oral and written examinations were in addition to that. Yet additionally I had to run four to six teachers’ refresher courses (in-service) each year, each of one week at that time. There was little time left for to do small scale research and related publishing. Nevertheless, I was involved heavily in writing mathematics textbooks for grades 1-9. (My name as a co-author and co-editor appeared in more than five million copies eventually.) Anyhow, miraculously, I found time to marry Heinke Barbara Fischer in 1957. We have two daughters Anette (born in 1959) and Franziska (born in 1961).

In 1958 a possibility occurred for an exchange between our teacher college and the Pädagogische Hochschule in Potsdam in East Germany. With a group of students and two professors, I had been there twice. It was a dip into a strange and poor world, though the college was located in the large New Palace in the famous park of Sanssouci. We could have a few contacts with students only under the control. Secretly, colleagues from their faculty warned me that other faculty members were activists of the Socialist Unity Party (the leading party in the East Germany). Looking back I can describe my feelings in those days as to be in a virtual world, spooky. In return we received small groups in Hannover -- the “students” were many years older than ours, apparently a cadre selected carefully. Still, about half of them did not return back to East Germany. As a result, East German authorities cut short the exchange abruptly after two turns. I remember an interesting detail of the two-faced East German world: one of their professors, an active Socialist Unity party member, saw Milovan Djilas’ “Die Neue Klasse” (New Class) at my desk, a critical analysis of the communist system, that just had appeared (at Munich, in 1958). He grabbed it and asked me to lend it to him for reading. Right before crossing the border home he mailed the book back to me, with thanks.

Things developed slowly, though successfully: I received tenure as lecturer (Dozent) in 1955, became associate professor in 1961, and full professor in 1965 after several calls to other universities. When, at Hannover, the number of students had risen to more than 2000 and my budget to 800 DM annually, the call to Johann Wolfgang Goethe University in Frankfurt am Main in 1966 brought the long-desired opening and realization of long-stalled ideas for research. It was a hopeful change, not only because the Land Hessen (State of Hessen) had just integrated the teacher colleges into universities as new Departments of Education with remarkably better budgets and the chance for promotions, but also because of all other facilities and connections.
Heinrich Bauersfeld: In Frankfurt I found my institute in a separate little villa (a former pension; many of the university institutes were scattered over the town), with its own library and a few assistants and lecturers, that my predecessor Hermann Thyen had left. After the first weeks, I had already received a phone call from the Teacher Union’s funding agency offering 80,000 DM for my expertise about the modernization of teaching mathematics in the primary grades, 1-4. Faced with the recent reform activities in many developed countries, the Teacher Union was interested in the modernization of our somehow outdated primary curricula in Germany.

The first experiments started in 1966 with a few highly motivated teachers in their classes, using ideas which had emerged during the last decade at the teacher college, also using materials from Max Beberman, David Page, Zoltan Dienes, George Papy, and some Piaget’s tasks. In autumn 1967 we began a pre-project (Vorprojekt) with ten grade 1 classes. With their students and teachers, we developed the first version of our mathematics curriculum, starting with grade 1 and planned until grade 4. Our team included two professors (psychologist Valentin Weis and me), two lecturers (Ursula Lubeseder and Gerhart Homann), and two assistants (Hendrik Radatz and Knut Rickmeyer). Set language, logic, and relations were very popular at that time. However, geometry, the most neglected part of mathematics in the elementary grades, was my favorite, specifically, constructive work with paper and many different tiles, shapes, and solids. Additionally, as a tribute to the spirit of the epoch, we used games with structured materials (following ESS models). That new start enabled all children to join classroom discussions independently from their individual preparedness. Many of them knew a variety of facts about numbers, though nothing about geometrical properties. On the other hand, geometrical structures are fundamental for every embodiment of numbers. Therefore we postponed the treatment of arithmetic. Our goals included intrinsic motivation, social learning, cooperation, and developing useful language games (inspired by Wittgenstein). Also we organized small group work from the very beginning, following ideas of compensatory education, with special attention to language development. This small development group functioned as creative pre-group to the later Volkswagen project. Here we developed the early versions of the new curriculum, step by step from grades 1 through 4.

Many more teachers were keen to participate, even parents of “our” kids made advertisements for our approach. Thus, it was easy to shape an application to the “Volkswagen-Stiftung” for a larger development project (called Frankfurter Projekt) on New Math in primary education. My application released several weeks of painstaking inspections, expert evaluations, and debates. Anyway, after careful deliberation I received in 1968 an allowance of about one million DM for four years. This, indeed, was the first mega-DM funding of mathematics education research over here, and, more remarkable, for a longitudinal project that aimed at
the reform of mathematics teaching in the primary grades, as such different from the later cross-section analyses of selected factors in SIMMS, TIMMS, PISA, etc., that used prepared clusters of tests for formal hypotheses testing. Nevertheless, the VW-Stiftung expected as outcomes only “hard” (i.e., quantified) psychological facts, bound to the current ideas of scientifically controlled curriculum development. The foundation had no interest in our reform curriculum, later published as “aleph”-curriculum.

The main project (Hauptprojekt) started with 42 experimental classes and 40 control classes, all volunteers, distributed all over Hessen. With regard to their cooperation, we preferred to run two experimental classes per school, albeit not more. We succeeded to have most of the teachers lead their class through from grades 1-4. The experimental classes followed our curriculum “aleph,” printed in my institute. The teachers of the control group followed their own favored methods, and knew they were competitors. Very helpful: The school department (ministry) had freed us from the syllabus; we were bound only to reach the compulsory level in mathematics at the end of grade 4.

Max Beberman, who was running a large curriculum development project at Urbana, IL (UICSM), came to visit us in 1968 already, and returned several times together with members of his staff. We had met earlier with Edith Biggs in Nottingham. Late in 1968 he organized for me and Gerd Homann a four week tour through the most famous curriculum projects in the US, so we learned from David Page (EDC, Newton, MA, UIAMP and author of Manoeuvres on Lattices), Patrick Suppes and Edward Begle (Stanford, CA, SMSG), Max Beberman (UICSM), Robert Glaser and Lauren Resnick (LRDC, Pittsburgh, PA), and Robert B. Davis (Madison Project, later Max's successor at Urbana, Champaign, IL). Intensive exchange worked well also with the two Shell Centres in London (Geoffrey Matthews and Kath Hart) and Nottingham (Hugh Burkhardt and Alan Bell), where I was active later in return as consultant, and not at least with the Ladbroke teacher centre of London's Local Educational Authority (LEA). Sad to say, my friend Max died far too early at Heathrow Airport 1971 on the way to us, misdiagnosed and treated for coughing, but actually the stitches of his pacemaker were broken.

Computers at that time were shaky and very delicate machines. We had to process our data at the Technical University in Darmstadt as our university's machinery was still too weak. All data were programmed and handled on Hollerith cards. Even the larger mainframe in Darmstadt was often in need of repair and only worked reliably about a third of its active time. Even worse, this technological miracle destroyed our whole set of data twice (1972)! VW-Stiftung would not believe this and asked three experts, who confirmed the disaster – not without chiding me that this happened just to a mathematician. We administered and used tests for intelligence, language performance, problem solving, and cognitive style dimensions. Weis and his co-worker Edith Wolff have produced many frame-referenced tests for the achievements. Aside from that, they organized the extensive formal testing and evaluation. Inside of experimental and control group, variance was reasonably higher than between
both groups. As hoped, the outcomes of the formal evaluation showed our project students as being significantly better in problem solving and language performance. (Admittedly no news, since in large samples just about everything gains significance, as Lee Cronbach had stated much earlier.) More important for us: students from lower socio-economic strata in the experimental group were better achievers than those in the control group.

We have learned much more from our efforts in soft evaluation. Our student teachers were involved in observing small group work and were very effective in recording the children’s cooperative activities. Edith Wolff, a trained Freudian psychologist, conducted intensive interviews of standing duration with more than half of our experimental teachers. These protocols helped us to arrive at a much more subtle and broader understanding of teacher’s personal problems, particularly of how exposed teachers feel within their school community and as project members. Soon we became aware of a grave deficit: we did not know enough about the social perspective. Teacher-student interaction, but also student-student interaction and teacher-teacher interaction presented a host of unknown “factors.” At that time sociology did not find much attention in the mathematics education community; it was not popular. However, we could benefit from the famous Institute for Sociology (directors Horkheimer and Adorno) just across the road, since Ulrich Oevermann used to come over with his group of doctoral students and discuss our problems with us. Consequently, the social perspective became my main concern across the concluding years in Bielefeld. Additionally our case studies in trait-treatment interaction (Witkin’s field dependency and Kagan's reflection vs. impulsivity) contributed to this shift of perspectives through the interesting interactions of teacher and student(s) with opposite traits. Hendrik Radatz analyzed related cases in his doctoral thesis (Radatz, 1976). As the worst combination he identified the interaction between an impulsive teacher teaching a reflective child (true with both genders likewise!). The typical teacher’s comment was: “He doesn't say a word, but accidentally he comes up with surprising remarks – his neighbor must have told him!”

Soon, the remarkable shift of the relation between teachers and project staff emerged as a chief characteristic of the project. For the involved teachers the four years seem to have functioned as an intensive in-service training. When initially teachers used to look upward to the knowledgeable authorities, the researchers developed slowly into a kind of assistant, so we both learned, though from different perspectives, and we shared this knowledge cooperatively, including the discussions of the staff’s mistakes as well. The initial distance faded away; interest and motivation grew on both sides, the most valuable outcome probably. By the way, the vice-president of VW-Stiftung liked to call me up deep in the night, when he had difficulties with solving his son's mathematical assignments. About his funding agency's final effects – who profits? – he remarked once: what we factually further with our educational funding is the improvement of the project team’s reflectiveness, rather than the improvement of schools. That reminds me of the heap of complaints
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published occasionally in AERA's Educational Researcher about teachers' neglect of educational research.

In 1968, a conference of the school secretaries of all twelve German Länder (Kultusministerkonferenz) had decided to reform the elementary mathematics syllabi and had declared set theory (Mengenlehre) as a compulsory subject from 1972 on. This caused severe disturbances with teachers and, hastily, the lower school administrations provided for a series of teacher refresher courses in abstract set theory and – no wonder – with poor advice for teaching practices. In our project we felt to be on a productive way out of these troubles, and we found wide attention for our approach. Even Japanese teachers came repeatedly in buses of 40 and took pictures of every item in the institute. But in other areas angry parents, who could not help their children, protested increasingly, the media multiplied the protesting, and a spectacular meeting of school authorities, together with the educational elite, cleaned and cut back the syllabi to their former status. This happened in 1972, when the Frankfurter Projekt terminated, and we felt able to offer better solutions. Thus, coming too late, Aleph never became a bestseller, but many teachers had a copy on their shelves. Anyway, many of our geometrical tasks began to appear in every schoolbook, more or less modified.

Frankfurt was the place where in 1968-69 the students' protests, strikes, and riots were heaviest (next only to Berlin). However, I could not refuse the honorary appointment in 1968 as member of the German National Educational Council, a body of twelve elected persons, one of each Land, (Hessen's representative, a bishop, had just died). The Council's task was to produce a recommendation for the necessary reform of the German school system from Kindergarten to grade 12. My academic duties continued unchanged, as director of both the institute for mathematics education and for the mega-DM project, and in total with a staff of 32 scientists and secretaries. A free half year of exemption saved me. Yet, an important task at the Council was worth the troubles: as head of a subcommittee I had to prepare a “recommendation for the furthering of curriculum development in a new school-near mode”. Together with Hans Brügelmann, my assistant at the Council and later professor in Siegen, and with a group of experts and officials of the school ministries of the Länder, this became a three year long weekend job. We integrated the international experiences with teacher centers, and visited institutions in England and Norway, in particular. I had visited the promising EDC in the United States with similar functions before. There was strong hope to increase teachers' participation and influence on curriculum development, combined with an in-service advanced training. Unfortunately the Council's official recommendation appeared in the hottest period of the protesting and riots at the universities. That caused many school administrators to take contra-functional decisions through enacting more control and by detaining new allowances and any liberalization. Only two Länder did create one experimental institution each, but closed it down after a few years. The very successful work of the model center in Lower Saxony (Land Niedersachsen) had reached a third of the teachers in East-Frisian during its three years of life. It was the
INTERVIEW WITH HEINRICH BAUERSFELD

YEARS IN BIELEFELD

Interviewer: How did it happen that you moved to Bielefeld?

Heinrich Bauersfeld: Following the new attention to mathematics at school, in 1969 the VW-Stiftung offered a nine million DM-grant to universities for the foundation of a federal research institute for mathematics education, similar to the IPN (Institut für die Pädagogik der Naturwissenschaften), existing since 1966 at Kiel's university. The state councils for universities and schools (Wissenschaftsrat and Bildungsrat) used to meet once a year. On the flight to the 1970 meeting I came by chance to sit next to Karl Peter Grotemeyer, just elected Rektor of the new university in Bielefeld and member of Wissenschaftsrat. I informed him about the published offer, also that my endeavor for an application to the Frankfurt University had failed already, due to a categorical NO of the faculty for mathematics. Spontaneously Grotemeyer adopted the idea as an important enrichment for his new university, and, against a wall of troubles, difficulties, and intrigues, Bielefeld won the battle and received the grant. The new institute IDM (Institut für Didaktik der Mathematik) began its work in 1973. I had been involved intensively in all of the founding activities, yet my call arrived delayed, so I left Frankfurt not before 1974, right in the final period of my (extended) Frankfurter Projekt.

As head of the founding council, Rektor Grotemeyer had organized Michael Otte's call and appointment as first professor and director early in 1973 and authorized him to start the development of the institute IDM. More than half a year later the other two elected professors and directors, Hans-Georg Steiner and me, found an institute in a rather elaborated state; it already had coworkers full of plans and busy secretaries, which left not much chance for a joint effort in terms of cooperative work and common projects in the future. At least the latecomers could hire their own assistants. A distinct political bias made it soon necessary to split into three independent research groups, characterized by the labels Primary Mathematics (Bauersfeld), Secondary Mathematics (Steiner), and Theoretical Foundations/Philosophy (Otte). The VW-Foundation had planned another six million DM for a new building for the institute, which appeared to be not necessary in Bielefeld. From that we received as additional support three million DM for three research projects, one for each director. It also provided funds for ten postgraduate scholarships and the completion our library. Since the fascination with great curriculum projects was fading away, it was time for more fundamental considerations. This made me think about the problems with the students' entrance into secondary education after grade 4, the problematic threshold in Germany. The transition from primary to secondary education marks a point of long-term discussions in Germany. This transition means not only a change of institution, but requires a choice among institutions which leads...
to different qualifications. And in Germany this functions more or less as a decision about your future career chances. In order to settle with these problems the Land (Nordrhein-Westfalen) had just created a new “intermediate” type of comprehensive school (Orientierungsstufen) with grades 5 and 6 only. Their main goal was softening the transition, and increasing social integration, similar to the goals of the few comprehensive schools (Gesamtschule). These new schools suffered even more from their initial phase, since their students came in from up to 20 different primary schools, with extremely different knowledge and routines particularly in mathematics. With my group I prepared for a new project, therefore, an analysis of the difficulties in this transitional phase and how to overcome it. As a next step this led to comparative studies of the many different modes of differentiation, as were practiced internationally.

Observations in comprehensive schools made clear to us that neither refinements of separation nor variations of teaching techniques would help. Instead, we saw as core problems the communication between teachers and students, with the failing of their mutual understanding and attributing of related consequences and reactions. Teaching and learning appeared as psychological and social events that come about in the subculture of a classroom. For too long psychology had served as the major reference science for didactical disciplines; with a focus on the individual, the single teaching teacher as well as the single learning student, however not on their reciprocal, mutually reflected cognition and actions. Thus, the missing sociological perspectives in mathematics education came into our main focus, again, and we deepened into intensive studies of teacher-student interactions in mathematics, in theory and practice.

Fortunately we could learn a lot from an excellent sociological research team at our university. Indeed, it was the first group over here that was just adapting this new “soft” sociology movement from the US. In social psychology, pragmalinguistics, and sociology, especially in ethnomethodology (Garfinkel; Mehan) and symbolic interactionism (Blumer), we found helpful information. Not much later a British group started interesting investigations, e.g., Doug Barnes. We selected interesting episodes from many videotaped situations in classrooms and discussed the interpretation of both tape and transcript, in workshops together with Fritz Schütze and Jürgen Streeck, both members of the Arbeitsgruppe (ABS), and with a few interested colleagues from reference disciplines. Soon we found that understanding speech – different from written text – requires information beyond the verbal utterances. With the reconstruction of meaning, as emerges in the mutual understanding among students and teachers, the need arose for a more detailed recording of the non-verbal parts of communicative actions, of paralinguistic characteristics (pitch, intensity, melody of speech, turn-taking, etc.), and non-verbal characteristics (body language, eye movement, etc.). Important and meaningful in social interaction is not only what speakers say, but also how they say it, and why just this and just to this partner, and in this special situation. Our related transcripts became more and more complicated. Clearly, answers (as well as findings) are interpretations and relative truths rather
than hard facts. Quantifying is hardly possible with this kind of research; it provides no contribution to statistically approved scientific knowledge or rules. Therefore, soon it was criticized by our institute's advisory board (the critique was pretty typical: similar concerns were expressed even earlier in the US regarding Beberman's work).

On the other side, and more important perhaps, teachers welcomed such findings, since it opened new perspectives and sharpened their understanding, enriched their repertoires with more alternatives in action and improved their ascription of effective reactions. Slowly a new dimension beneath the well-known surface of classroom activities won contour and enabled a much more sophisticated and subtle understanding of these subcultures. Indeed, these subcultures reasonably differ from classroom to classroom.

Surprisingly fast we encountered severe philosophical and epistemological problems. Since the soft sociology disciplines had done their investigations mostly in everyday life situations, we were left with the new task of transferring and adapting the concepts and methods to our special situation, the asymmetric communication in an institution like school, an institution established with hierarchy and power for the educational purpose. Familiar psychological notions got another meaning, e.g., interaction means interaction between persons rather than interaction between variables; learning at school appeared as subjective constructing of meaning in social interaction. And new concepts appeared like emergence, negotiation of meaning, shared meanings, viability, working consensus, etc. From these “soft” sociological perspectives new language games “emerged” for the description of our mathematics education realities as well as for our related distancing and critical reflections.

As an important effect the new insights led to a different style in the intercourse with teachers, particularly in refreshing courses. We replaced the usual verbal instruction in such courses by presenting challenging video clips and short protocols of interesting classroom episodes, and awaited reactions, protests, and discussions. The teachers never disappointed us. We had many clips that reliably elicited spontaneous emotional reactions, revealing striking dissent among the teachers and leading into hot discussions of motives, convictions, routines, and traditions, etc. For the participants, such indirect approaches to deeper routed problems used to release surprise, if not strong emotions. What had been taken as undoubted tradition, as truism, or as shared conviction ran into harsh contradiction from colleagues. So they could win critical distance from their accustomed reactions and attitudes, and possibly arrive at self-motivated corrections. This perspective gives serious reason for the positive effects of such learning via examples; neither rules nor techniques are learned, but useful skepticism and the abilities of distancing and self-correction improve. Kant would name it an indirect sharpening of judgment. Half of a century ago Kagan (1967) had written a philippic for “more relativism,” however it was in psychology, not didactics.

After the elections in Nordrhein-Westfalen, the government changed from social democrats to conservatives (CDU - Christdemokraten). As one of their first actions
they closed down these intermediate schools, institutions, anyway, not beloved by Gymnasium and Realschule: indeed, these had lost their first two grades which were given to the intermediate schools. All of a sudden our project lost its basis in reality. What now? Since we were sure to be on an important and promising path, we decided to continue our research work in even more fundamental directions, albeit at smaller scale. Moreover, the first oil crisis (1973) had made governments think about savings and budget cuts. Consequently our government used the end of the initial five-year grant as a reason and tried to close down the institute, in spite of the guarantee given for continued financing. Due to heavy protests, nationally and internationally, the attempt failed, albeit at further reductions of our budget.

MORE ON RESEARCH

Interviewer: Could you please tell more about your research in the later years and your collaborators?

Heinrich Bauersfeld: For the interpretation of classroom communication we found very useful theoretical and methodological information with Bateson (1972), Blumer (1969), Mehan (1975), and von Glasersfeld (1974). The illusion of transmitting knowledge fails because of the subjective ascription of meaning, the fundamentally functioning filters on both sides, teacher as well as student. Among our transcripts we identified surprisingly long periods of diverging meanings, situations in which the student keeps his deviant meaning though reacting with the teacher’s correct words, which, on the other side, reinforces the teacher's assumption of a correctly learning student. Such counter-productive progress with hidden split meanings can survive over weeks, sometimes even over months, until a striking mistake may reveal the illusion to both sides. The constructivist perspective allows us to search for such events. When I came to meet Ernst von Glasersfeld personally at PME 5 in Grenoble, France in 1981 he was surprised at our projects and their manifolded theoretical bases. I think part of this surprise was owed to the difference between his radical constructivism and our developed social perspective (anyway, he included the social perspective later explicitly). As a result of our exchange I found myself invited as a speaker at the Gordon Research Conference on “cybernetics” at Wolfeboro, NH in 1986, with von Glasersfeld and Heinz von Foerster as organizers. I presented a report on our analyses of hidden regularities in classroom communication. It was a unique chance to meet many important participants, among them Heinz von Foerster, Bärbel Inhelder from Geneva, and Humberto Maturana. Even more important was the following and last meeting on cybernetics with Ernst von Glasersfeld at Oxnard, CA, where I had to chair a group of presenters. Von Foerster's cybernetics with “positive feedback” and his key notion “observer of second order” became very useful in our classroom observations, also Maturana's concept of “system” (in comparison to Niklas Luhmann's concept of “system”; Niklas was an often read, yet rarely seen colleague at university Bielefeld). There were two more
impressive meetings with Heinz von Foerster, in 1990 at the ASC Conference near Oslo, Norway, and in 1995 at the ASC Conference in Chicago, at both as invited speaker.

At the Gordon Research Conference on cybernetics at Wolfeboro, NH in 1986 I met also Paul Cobb, at that time a professor of mathematics education at Purdue, “my very best doctoral student” as Ernst von Glasersfeld had remarked. Paul found our approach so interesting that we decided to run a joint research project. Before Paul had seen “mathematical development as an individual process of conceptual construction,” while my preference was with “the social and interactional aspects of mathematical activity” (Bauersfeld & Cobb, 1995, preface of our project report). Paul's co-workers at Purdue, Erna Yackel and Terry Wood, had produced a large number of videotapes in experimental elementary classes. Together with them, and at the German side with Götz Krummheuer and Jörg Voigt, we analyzed relevant episodes, searching for pattern and hidden rules. We prepared many informal (not-published) papers about our theory and methods. The Spencer Foundation financed these three years of a very effective collaboration, and, last but not least, of a remarkable change of perspective with our American partners. The final report appeared in a series directed by Alan Schoenfeld, who deleted my final chapter of the book that discussed possible consequences for teaching mathematics: “We know all that!” Many years later, I found an article in the Educational Researcher that rebuked him for the neglect of similar recommendations; a letter of excuse was his reaction to me.

Soon the relationship with Ernst von Glasersfeld intensified to friendship. His outstanding wide range of personal experiences and studies enabled his depth of reflection as well as the richness of his perspectives. Among other disciplines he had studied mathematics. Later, when the Nazis occupied his homeland Austria he emigrated from there to Ireland to work there as a farmer. He had organized language experiments with primates at Yerkes, and he had worked with Silvio Ceccato at his Center for Cybernetics in Milan and in the US in research projects on computers and language, before he settled as professor of psychology in Athens, GA. Aside from all that, he was a giant on skis. We met at many conferences, not only in the USA, but also in Germany, when he gave invited lectures. He held a long laudatory speech at the symposium for my sixtieth birthday at Bielefeld University. He was a very supportive friend, not least of which by passing to me many literature references and extensive theoretical advice. His highly estimated quotation about human interaction was, “Le seul donné est la façon de prendre,” which he assigned to Roland Barthes, and translated into, “The only given is the way of taking.” Radical constructivism in a nutshell.

**Interviewer:** I have a personal interest in Eastern Europe and, specifically, the former Soviet Union. Did you have any professional contacts with colleagues from there?

**Heinrich Bauersfeld:** During a period of political thaw, in the middle of the 1980s, the federal government was in favor of an exchange with the Soviet Union about
their educational system and charged the Land Nordrhein-Westfalen to organize a
delegation for a first visit. We started with two officials from the ministry and two
experts, Helmut Fendt, a professor of education, and me. In Moscow we stayed at
the top hotel, “Ukraina” (still, I remember being once called up about midnight by a
tender voice: “I'm so lonesome!”). The next day a conference at the Soviet Academy
of the Pedagogical Sciences took place with selected members. The content was an
extended comparison of systems and I found no news. Only later we learned that
these persons were the appointed members of the Soviet delegation in return. More of
interest were visits to different schools and the teacher training college. Meetings with
regional educational authorities left a lasting impression by their literary informedness
as well as by their pragmatic perspective. Most interesting was the visit to Specialized
School No. 18, where selected highly gifted 15- to 17-year-old students from all
over the Russian Republic lived and learned their mathematics from their teachers
and faculty of Moscow Lomonosov University, including such a celebrity as A.N.
Kolmogorov as a teacher. I observed a class of some 20 students studying algebra.
Their communication proceeded incredibly fast and in exact mathematical language
only, accompanied by rich notes at the blackboard. The concluding visit in Minsk,
capital of the Belarusian Republic, was more like a demonstration of economical power
rather than of education. Research at their Academy of Pedagogical Sciences seemed
to focus on the tiny differences between Russian and Belarusian language, each proud
of their autonomous status. We also experienced a special school of fine arts, with
dozens of rooms with grand pianos (!) and organs in skillful use. We were also taken
to the large war memorial at Khatyn, which was under reconstruction as a memorial
for the hundreds of Belarusian villages that had been burned down by German troops.

Interviewer: Could you give some examples of other research projects in these years?
Heinrich Bauersfeld: Teaching student teachers a bunch of microstructural
regularities in their conversations with children rests upon the illusion of improving
teachers’ reflection. But as we learned, such knowledge fades away under stress and
regress, not rooted in practical experience becoming something of value at exams
only. More and more our orientation shifted, therefore, from detecting universal rules
and traps in classroom communication to a more sophisticated understanding of the
interactive processes and their conditions. Less verbal instruction, more indirect
learning via own practical activities, “pick-a-back”. With our teachers and student
teachers we have used such episodes as openers in order to elicit a discussion about
adequate interpretations and reactions among themselves. Thus, participants were
encouraged to organize their repertoire of options for actions through fighting for
solutions of concrete problems rather than piling up verbal techniques and rules for
intervention. Thus they could improve their judging rather than their techniques. In
other words we changed our research orientation from a structuralistic into a more
functionalistic one, from search for universals to furtherance of personal development.

One of my small scale projects was the pursuit of a beginner’s class for grades
1-4 (1988-91). At an elementary school in a difficult migrant environment, I found
an experienced female teacher who agreed to take over a new first grade class and to keep it through the end of grade 4, at least in mathematics. Moreover she gave her assent for videotaping each of her mathematics lessons, and to cooperate in changes to the curriculum using tasks that I had provided. The headmaster also agreed. It was an ideal collaboration: she adjusted my suggestions to her students' needs. An intensive collaboration and experiment developed. Parts of the resulting pile of tapes were transcribed for our research in classroom communication, particularly for identifying productive types of interaction, together with Jörg Voigt as co-worker. Many tapes proved to be useful examples also for discussion in our in-service teacher courses and in seminars with teacher students. Of special value had been several occasions of two weeks' free time with the whole class and their teachers on an island in the North Sea with plays, swimming, singing, collecting all kinds of objects, and tinkering with little runners with cork wheels for racing competitions at the beach. I had to revise my notions about many of the kids, who revealed surprising abilities in their actions in a totally different environment, especially those with immigrant parents. Again, I learned to act much more cautiously and guardedly with my judgments.

Under a decreasing birth-rate and the related shortage of labor, especially of skilled workers, in the late 1980s, the public debate focused again on the concept of "elite" and the furthering of the gifted, a deficit that had been neglected over the decades. There was little research available, as the themes of contributions at scientific meetings and conferences confirm. A further requirement arose from inside the university. The new approach “kid's lectures” blossomed with more than 400 children per lecture; limited access became necessary. Soon the parents as well the children themselves asked for connected furtherance and more detailed programs. Together with a few interested colleagues we offered a series of two hour workshops per week for high achievers or kids with high test scores. There were courses in mathematics, philosophy, and biology, and soon we added Japanese and Russian language, taught by native speakers. We found excellent foreign graduate students, whom I had prepared, and who performed inspiring courses with these kids. I took charge of the mathematical course. Although successful and in spite of growing interest, the colleagues retreated soon, overburdened though motivated, and sent their best and most interested doctoral students as replacement. With these tutors I ran intensive preparative meetings, before and between the workshops, discussing tasks and research questions and a useful system of written reports. The fast developing approach required more support and funding. Since emeriti are excluded from fund raising, and I had reached this status meanwhile (retirement 1992), I tried to motivate an active colleague of the department of education to become my successor and an applicant for a funded project. The promised take-over failed, and the promising approach faded away. Besides the kids, the student tutors, interestingly, did regret this failure deeply. We had learned together in a directly rewarding and non-commercial experiment.

In 2003 one of my former students, now headmaster of an elementary school (grades 1-4) contacted me and organized an informal continuation. With permission
of the parents I could work with a small group of six children for two hours per week. The children were selected by their teachers as spectacular achievers in mathematics, with a special eye on girls and immigrant children. The students left their classes for these two hours and missed their regular lessons, indeed without disadvantages. Their teachers were happy to have their classes for a while without these ever-active troublemakers, and the children appreciated the different – though quite demanding type of work – that I prepared for them. Not one of them missed a session. The tasks had been prepared for cooperative work in particular. The published collection appeared also in Dutch and in Swedish (Bauersfeld, 2002, 2007).

Interestingly, the numerous publications of my research group found more attention on the international scene than at home. Many invited lectures in the USA, Canada, England, Norway, Denmark, Austria, France, and Greece, as well as the many interested visitors from abroad, including China, Japan, and the Soviet Union, may indicate this fact. At home the Scientific Advisory Board of IDM with its majority of mathematicians was not that happy with our productions. “Where is mathematics in your work?!” asked the very same professors who were members of our Founding Board, where they had tried to prescribe the development of introductory lectures in mathematics at university as a first goal of the new institute’s research work. Later, when we were both emeriti, the president of our advisory board, Rektor G., took me aside and apologized emotionally for the unfair treatment of the reports of my group at these sessions. Only now had he become aware, he admitted, of “the reasonable international echo” of our work.

As a late pleasure my earlier theoretical considerations (on activity theory, mathematically gifted students, and research designs) were required again when a group of cooperating researchers found themselves in discontent with the precision of their theoretical concepts. This caused an intensive dive into the wide relevant literature and led to a lecture at Muenster University that will be published later (Bauersfeld, 2013).

ON INFLUENTIAL SCHOLARS

Heinrich Bauersfeld: Thinking about the scientists who had a strong influence on me, I see, nearly all of them were “travelers” among many departments, or expressing it positively, were permanently crossing the borderlines between departments by creating and pursuing new ideas. The price was high: disregard and neglect by members of related departments. Typical was Ernst von Glasersfeld, who had to break off his mathematical studies in Vienna and did study many subjects later. The outstanding quality of his work and publications took him finally to a professorship at Athens, GA, however without acceptance, or even attention, by mainstream psychology and philosophy. The famous Niklas Luhmann, a belated convinced constructivist, had been treated as an outsider in his department of sociology. Martinus Langeveld, the Dutch educator and psychologist, has treated traumatized children in Israel, born in concentration camps (KZ), among many other concerns, but found little echo with his
publications among our German professors in education. After his retirement, his wide areas of work had to be distributed among three new professors, a rare event. After Hans Freudenthal's sharp revue of SIMMS, naming serious mistakes in design and methods (Freudenthal, 1975), which elicited far too easy reactions and reforms by political authorities, he became homo ingratus by nearly all cognitive psychologists. Yet, the majority of mathematics educators over here agreed with his critique and resistance against these large international surveys, in concurrence with Jerome Bruner’s critique, “The existence of experimental methods makes us think we have the means of solving the problems which trouble us, though problems and methods pass one another by.” (Bruner, 1983, p. 130).

After all I would admit that Jerome Bruner has had the strongest influence on me, both as person as well as through his publications. His many books (1956-1971) accompanied our German activities of curriculum development and far beyond. His “Essays in Autobiography” (1983) are the most impressive text, not in the least through the detailed and, without reserve, self-critical presentation of his scientific development, of his ideas, his projects, and also his errors.

From my own experiences I would recommend this book as a required reading for all students in education and psychology. Our chat in Bielefeld happened under quite remarkable conditions: the department of psychology had invited Bruner for the conferment of an honorary doctorate, yet, the decision to award it was unexpectedly turned down, and a desperate dean hurried to welcome the “non-laureate” at the airport in Hannover, since he had been in the air already. Bruner laughed and came in spite of all. When asked about the strict separation of his famous three steps in cognitive development – enactive, iconic, and symbolic – he shouted loudly: “Bruner was wrong!” and explained that he sees all three as being present in every action, though in different states of fruition. From Bruner I have adopted several important distinctions, e.g., conjunctive vs. disjunctive concepts and functionalistic vs. structuralistic research (Bruner, 1983).

ON A PARADOX IN MATHEMATICS EDUCATION

Heinrich Bauersfeld: Conjunctive concepts (e.g., mathematical and technical concepts) share a set of universal characteristics. Their definition consists of the set of their necessary and sufficient qualities. Disjunctive concepts unite a set of objects that share only some, yet not all, characteristics, like members of a family, and that makes their definition difficult. Therefore, all efforts to make their definitions more precise are doomed to failure. Disjunctive concepts are the normal case “in the junk world of real objects and natural categories” as Bruner said (1983, p.128).

The object of teaching in mathematics education is a perfectly constructed world, abstract, based on axioms, and free of contradictions. The concepts are ideal conjunctive concepts, defined completely by their necessary and sufficient characteristics. No wonder that over centuries philosophers and other scientists tried to frame their ideas and relations in a mathematical mode. Mathematics education
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suffers from a unique paradox: we are to teach conjunctive concepts, but our means, all metaphors and embodiments, appear as disjunctive concepts. Thus, we try to teach precise, sharp concepts of an abstract world through the use of non-sharp concepts as related to our concrete environment. One out of the Weizsäcker family once called success in education the "campus miracle" of producing clean cups by applying muddy water and a dirty towel.

Interviewer: Thank you for this interview.

NOTES


2. The “Materials for Attribute Games and Problems,” containing “Blocks, Color Cubes, and People Pieces,” from E.S.S. (Elementary Science Study), published 1967, seems to be the first material of this type, earlier than Dienes' blocks, which became famous in Germany a little later.

3. Following his “Philosophical Investigations,” we tried to develop rich descriptions of mathematical properties in every day language instead of using the abstract definitions only, as was compulsory in the primary schools of the GDR (former German Democratic Republic).

4. For the comparison we tested SES (socio-economic status) in samples of both groups.

5. Bauersfeld, H. (1972). Einige Bemerkungen zum “Frankfurter Projekt” und zum “alef”-Programm. In E Schwartz (Ed.), Materialien zum Mathematikunterricht in der Grundschule (pp. 237-246), Frankfurt/M.: Arbeitskreís Grundschule e.V . By the way: after my retirement the complete project materials (large computer tapes, correspondence, reports, and so on) that had survived the move from Frankfurt to Bielefeld ended in the waste, because the new university had no archives at that time, as are installed now.


9. By the way: My first impression of such approaches arose with observing Alan Bishop in Cambridge, UK, in 1974 teaching his university students. He used videotaped lessons, stopped the tape at critical events and asked, “What is the matter?” “What would you do now?” “Any alternative, other interpretations?” Also he had published a booklet with short episodes for starts from text (Bishop, 1972).

10. That reminds me of R.A. Laing’s “Knots” (Vintage, 1970) and the pun (I don’t know the author): I wish, I were a bird, then I could fly. I wish I were two birds, then I could fly after me and see me flying. I wish I were three birds, then I could fly after myself and see me observing myself flying. (My re-translation from a German version back into English).

11. In face of the voluminous praise of Paul Cobb’s work (Yackel et al, 2011), I do regret now having renounced the publication of a paper on my analyses of the classroom findings, that was also meant as contribution to the final report, but released emotions with our partners as too critical.

12. There is an old German pun, illustrating just this effect: “Wird es mit Verstand verhau'n, waechst dem Kind das Gottvertrau'n, und so steigt von hinterwaerts, Froemmmigkeit ins Kinderherz.” (“Once a kid gets spanked with sense, grows her trust in God immense; and so moves from backside upward, piouness into the kid's heart.” My attempt at translation.)

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PUBLICATIONS OF IDM

„Untersuchungen zum Mathematikunterricht“, Köln: Aulis Verlag Deubner, a regular book series, 9 of the 20 volumes (published until 1994) were from Heinrich Bauersfeld’s group; internal booklets for reports of conferences, meetings, and projects; distributed free;

„Schriftenreihe des IDM“, Universität Bielefeld, the ‘green’ series of reports of conferences and more voluminous treatments, distributed free, 41 volumes;

„Materialien und Studien“, Universität Bielefeld, the ‘white’ series of internal booklets for voluminous texts, similar to ERIC-documentation, distributed free, 68 volumes;

„Occasional papers“, IDM, the „red“ series of preprints, invited lectures and so on, distributed by the authors.