To understand a living system, such as a tree, in an ecologically systemic way involves more than simply reducing the tree down to its parts or by analyzing the tree from part to whole. Not only does one need to study the tree’s leaves, stems, branches, trunk, root system, and its interaction with the environment but from many vantage points to make sense of how each part exists in dynamic relationship with the others as an integrated system. The same is true about the purpose of this book. It is not meant to be a recipe for how to teach mathematics well or to serve as simply a descriptive account of a teaching practice. It is in essence, a systemic exploration into the embeddedness and co-emergence of theory and practice in mathematics teaching. This book is ideal for undergraduate and graduate courses in mathematics education and curriculum studies. With its up close and contextual forms of data and a variety of interpretive methods used for the analyses, this book is highly suitable for courses in research. The audience includes professors, teacher educators, and in-service teachers who are interested in ecological theories and how these inform mathematics teaching and learning.

“This book allows the reader to see children and their teacher really grappling with important mathematical ideas. Thom reveals how children occasioned by evolving promptings within the classroom might live mathematics through the invocation of hypotheses and the exploring of these, which lie for me, at the heart of contemporary mathematical culture. Thom successfully brings together embodied and dynamical ideas of knowing with current thinking on mathematics knowing and understanding. While the book has deep theoretical bases, Thom has written and developed it in ways that make the practices of mathematics knowing vividly evident and presents new ideas for living out mathematics teaching.”

Thomas Kieren
University of Alberta

“Thom offers a vibrant introduction to how an eco-complex approach to mathematics teaching might feel. Playing across individual and collective levels of knowing, she presents “living” accounts of the emergence of mathematical knowledge. She challenges us to attend to the complexities of subtle and seemingly mundane shifts in mathematical understanding as she confronts us with key issues in both responsive and participatory research. Few researchers have managed to speak so compellingly to what it might mean to carry and eco-complex sensibility in the classroom.”

Brent Davis
University of Calgary
RE-ROOTING THE LEARNING SPACE
NEW DIRECTIONS IN MATHEMATICS AND SCIENCE EDUCATION
Volume 21

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Scope
Mathematics and science education are in a state of change. Received models of teaching, curriculum, and researching in the two fields are adopting and developing new ways of thinking about how people of all ages know, learn, and develop. The recent literature in both fields includes contributions focusing on issues and using theoretical frames that were unthinkable a decade ago. For example, we see an increase in the use of conceptual and methodological tools from anthropology and semiotics to understand how different forms of knowledge are interconnected, how students learn, how textbooks are written, etcetera. Science and mathematics educators also have turned to issues such as identity and emotion as salient to the way in which people of all ages display and develop knowledge and skills. And they use dialectical or phenomenological approaches to answer ever arising questions about learning and development in science and mathematics.

The purpose of this series is to encourage the publication of books that are close to the cutting edge of both fields. The series aims at becoming a leader in providing refreshing and bold new work—rather than out-of-date reproductions of past states of the art—shaping both fields more than reproducing them, thereby closing the traditional gap that exists between journal articles and books in terms of their salience about what is new. The series is intended not only to foster books concerned with knowing, learning, and teaching in school but also with doing and learning mathematics and science across the whole lifespan (e.g., science in kindergarten; mathematics at work); and it is to be a vehicle for publishing books that fall between the two domains—such as when scientists learn about graphs and graphing as part of their work.
Re-Rooting the Learning Space
Minding Where Children’s Mathematics Grow

By

Jennifer S. Thom
University of Victoria, Canada
DEDICATION

To my Ba Chan, father, and mother, for situating me in their certain visions of the world.
We Are Connected To This Earth: 1–3

Significances in the Flow of Interacting: 5–10

Metaphorical Mattering of Mathematics: 73–76

Encircling Our Perceptions: 13–29

Space Wanted: 11
To understand a living system, such as a tree, in an ecologically systemic way involves more than simply reducing the tree down to its parts or by analyzing the tree from part to whole. Not only does one need to study the tree’s leaves, stems, branches, trunk, root system, and its interaction with the environment but from many vantage points to make sense of how each part exists in dynamic relationship with the others as an integrated system.

The same is true about the purpose of this book. It is not meant to be a recipe for how to teach mathematics well or to serve as simply a descriptive account of a teaching practice. It is in essence, a systemic exploration into the embeddedness and co-emergence of theory and practice in mathematics teaching.

The nature of this inquiry and the theoretical realm in which it is situated necessitated the work to be expressed in a form that possessed an ecological sensibility. Upon first glance, this book appears to be a collection of separate compositions. Although each piece is an entity unto itself, the intent was not to render the book as a piecing together of theory, data, and analysis but instead, to bring a multiversal perspective to it and expose the co-existence and co-evolution of theory and practice. Thus, the embodiment of this book’s thesis is also evident in the organization of the text as a whole and the diversity of writing structures within it.

The organic way in which this book is organized can be likened to a tree: that any one leaf is neither directly connected to the other leaves nor does one need to view them in any particular sequence. Yet at the same time, all are interconnected as essential parts of the tree by way of its stems and branches and therefore, are necessary for making the tree a coherent whole. These compositions which were sourced by video and audio taped sessions in the classroom, journal entries, students’ work, and running field notes are not necessarily directly linked to one another or sequential in order. Each piece is considered to be a particular yet integral way of thinking that in turn forms larger conceptual clusters within an ecological mind-space. And together, these interrelated conceptual clusters as manners of knowing seek to in-form the mathematical learning space.
In the same way that one must explore a tree from many different vantage points in order to understand it, this book as an exploration of mathematics teaching also needed to possess different viewpoints. Moreover, it was critical that each piece of writing in some way highlighted the inevitably personal particularities that are involved in researching and teaching mathematics. To do this, different characters were developed. The characters in this book are my students, a teaching colleague, and myself. In order to examine and interpret my teaching from multiple perspectives, my character took on several personalities. In some of the compositions, I am the main character and describe events as I perceived and experienced them either in the moment as they came to be or from a reflective stance after they had happened. In other instances, I do not exist “in person” or am another character altogether. These latter traits allowed me to interpret my examination from a connected yet more distant perspective. And in still other places in the book, the reader will find me in conversation with another character. These vignettes reveal the ongoing questioning and assessing of the theoretical coherence concerning my exploration.

All of these different points of view are further communicated through the following expressive structures:

**Metaphors** and visuals are used to reflect how I was conceptualizing the theory and my teaching.

**Featured quotes, ideas, or questions** positioned on a blank page are the theoretical artifacts that at the time, served as provocations for new inquiries. In this book, these quotes, ideas, or questions are intended to interrupt the reader and signal the opening of a new conceptual space.

**Juxtaposition of text with other text** visually expresses the recursive and emergent manners in which my thinking arose during this study. Sometimes, juxtaposition was used to set ideas with each other so that I could discover what theoretical movement, friction, or generative spaces might be occasioned for my further consideration. While at other times, the juxtaposing of ideas enabled relationships to grow between one author’s thinking and that of another’s.

**The use of black and white or shades of grey** for certain texts and images distinguish these theoretical underpinnings
that I considered to be clearly defined and those I perceived to be fluid and indeterminate. For example, black on white is used in the text collage and portrait for constructivist notions to convey what I conceived as theory that was clear-cut whereas shades of grey were included in the ecological text collage and enactive portrait to express theory that I understood as imbued with qualities not so clear-cut but more unpredictable and ever-changing.

The actual figuring of the compositions — for example, as a newspaper article, conversation, or free form poem — as well as how each piece is organized in a left to right, top to bottom, back and forth, sporadic, or even whirling manner sought to capture the conceptual and metaphorical essence of the ideas being discussed.

**Bolding** of text is used to bring forth ideas in the conceptualization of theory while **fading** of text is used to preserve the contextual background from which these ideas emerged. This form of writing gives the reader a sense of the different visual fields as well as the depth of (those) fields that evolved in my thinking.

In addition to the characters, personalities, and writing styles, the **actual fonts** of the text distinguish between the different tones or perspectives taken on in my examination and interpretation of this work.

And lastly, although this book could be read in a conventional manner, I invite the reader to engage in the same spirit as one would exploring a living tree; perhaps examining the tree's integrated and integral being from its base and climbing up, swinging from one branch to another and gazing around, or even looking down at the always emerging tree and its surroundings from a distant hill.

*Jennifer S. Thom*
*February 2012*
ACKNOWLEDGMENTS

I owe a great debt of gratitude to many people who supported and contributed to the actualization of this book. I especially wish to acknowledge those not mentioned here who helped me during critical periods of the work.

I am extremely grateful to Susan Pirie who went beyond collegiality to encourage me to pursue this work. Her genuine interest in all aspects of the project, from the theoretical topics and practical instances of teaching mathematics to the actual styles in which these are expressed, made definitive contributions to the evolution of the book. The generative conversations we had occasioned so many insights that proved to be invaluable as I worked out both the conceptual and writerly details on various drafts of the manuscript. Just as integral was Wolff-Michael Roth's open invitation for me to explicate further, the concepts of the work and make it available to a larger audience. I have had the benefit of Tom Kieren's thoughtful readings and commentary on this work from its very inception. His theoretical prompts and intense, unwavering inquisitiveness were fundamental in the completion of this book; initially, in helping to define the direction that the work would take and later on, in articulating several of the ideas around which the book is structured. And without Karen Meyer, I wonder what my exploration into the discourse of ecology would have been like. Sincere thanks for setting me on my way and for helping me to find a place of resonance within it.

Ann Anderson's interest in the beginning stages of this work was important in furthering my understandings about classroom mathematics and my actual teaching of mathematics. Werner Liebert and Chet Bowers, each in his own way, altered my perceptions about mathematics teaching and learning. On several occasions, their comments interrupted my thinking and caused me to re-examine my assumptions.

The conceptualization of particular ideas and compositions in the book also grew out of conference presentations and publications, both of which I received valuable feedback that was so important to the further writing of this work. Our Cultural Ways of Being includes ideas that evolved after and between presentations at Learning And The World We Want,1 the University of Victoria,2 and Narrative Matters,3 as well as an article published in Educational Studies,4 Parts of Surfacing and Noticing and New Furniture emerged from a theoretical paper published in Mind, Body, and Society: Emerging Understandings of Knowing and Learning5 As these writings continued to take shape, they were combined with empirical data of my mathematics teaching, presented at the annual meeting of the North American Chapter of the International Group for the
Psychology of Mathematics Education and following the conference underwent further theoretical and interpretive revisions before assuming their current forms in the book. Scripting an Unscripted Lesson, Mathematical Language, Langaging, & Residues of Learning, and Arisings began as a presentation at the Complexity Science and Educational Research Conference. Considerable alteration and conceptual development of these compositions was carried out for the book. Mathematics Beyond the Classroom includes some excerpts previously presented at the National Council of Teachers of Mathematics Regional Conference, the 2nd Meeting of Enactivism and Post-structuralism: Implications for Development of Educators, and an article published in The Numerator. And, Opening Spaces Of Their Own started out as a presentation at the annual meeting of the American Educational Research Association and later became an article published in Journal of Mathematical Behavior. Like these other pieces of writing, it too experienced revisions, particularly in the conception of the narrative form and the visual layout that it now takes.

In addition, I wish to extend my immense thanks to others who have helped in various ways and especially, the Richmond School District for its dedication as an educational community. Not only was the district my educational home where I grew as a teacher but in a significant way, provided a rich environment for the co-emergence of my teaching and research to take place. During this time, it was the students who peaked my curiosity and compelled me to search for deeper understandings of what it meant to teach and learn mathematics. The teaching I did with Donna Jenner was truly a collaborative learning experience and indeed, a pleasure. I also want to express my appreciation for the financial support that I received from The University of British Columbia, the Richmond School District, and the Social Sciences and Humanities Research Council. These grants greatly aided my research and writing of this work. And, it was through Laurie Baxter’s perceptive persistence and spirited, artistic ways, that the cover of the book emerged, visually embodying the meanings inside it.

Most importantly, I wish to thank my family for their kindness and guidance to see this work to fruition. In particular, gratitude goes to Pat for spending many a conversation helping me to craft my ideas and get them down on paper. I am especially grateful to Lucas for his care, patience, and constant encouragement. And finally, this journey would not have meant so much or been as wonder-filled without Nolan, Clara, and Myles.
A Letter To The Author

Susan E. B. Pirie
France
February 7, 2012

Dear Jennifer,

You asked me to write something as an “introduction” or “afterword” to your book and this is a problem for me. In your preface you have said all that I would have said to introduce the book to the reader.

Anything I write would detract from your work and your thinking. In addition, it does not seem to me that an “afterword” is what is needed, since the piece is not to be read from cover to cover with a “and now what?” from me, to follow. I hope that the reader will hop about within the text as you recommend, will go back to what has been read with an insight from what has come later. It is not a book with a conclusion. There is no “end point” to understanding. It does not need further pointers to connections with other thinking because it is all about doing one’s own thinking and path-lying or finding, isn’t it?

This letter is both personal and yet public as is the book itself, which has taken many
years to reach the public domain. I feel I have accompanied your manuscript from its embryonic ideas to maturity. Yet in absolutely no sense am I its parent. You conceived it; you gave birth to it; you were its guardian through awakening youth, awkward adolescence to this momentous stage when it is ready to leave your hands and set out alone. A tough moment for a parent, saying “Fare well” to one’s child, full of hope for its future success. Perhaps I can be considered its godmother, hovering over its cradle, following its progress, celebrating its achievements, always wishing mother and child a bright future. For the book will always be your “baby”, just as will Nolan, Clara and Myles when they reach adulthood. You can never be sure you used all the right words at the right moments, gave sufficient thought to the actions that accompany a child’s upbringing. It is with trust that you must launch your progeny into the world. Have faith that the public will receive with curiosity and intellect the personal sides of Jennifer which they are offered.

This is a book about growth; personal growth, pedagogical growth, mathematical growth and the growth of children’s understanding. Metaphors have often played an important part in the communications between you and me and education is certainly in need of a new metaphor. The ecologically compatible approach of complexity theory encompassed by enactivism that you explore
A LETTER TO THE AUTHOR

offers a powerful and exciting alternative to the existing, mechanistic, structural thinking underlying current curricula. It was Lakoff and Johnson’s, *Metaphors We Live By* (1980) that first opened my eyes to the power of a metaphor as an analytical tool. Since then, much has been written by the mathematics education community around metaphor. For a period, the word itself created a bandwagon onto which many jumped using and abusing the notion in a variety of ways until its central power became dissipated and, at times, lost. We cannot avoid the fact that our existing concepts structure the way we interpret what we say and hear. Unaware, we are affected by the personal images and connections that language calls forth for each individual. We live our lives, albeit mostly unknowingly, immersed in a language of metaphor. Pause for a moment and reconsider part of my previous sentence; we live immersed in language. ‘Immersed’ calls upon the conceptual metaphor of an all-encompassing fluid, reflecting and influencing the smallest movement and thus the least expression of thought. Substitute the word ‘surrounded’ — we live surrounded by language — a metaphor of place, of physical situation, and immediately we have the feeling of a little freedom: others may talk at us but we have the choice of heeding or ignoring their implications. However I might have said “we are trapped by language” evoking a hostile environment within which we have no power;
we have no choice but to use words unaware of the images they may conjure up for others. For that is the tyranny and the power of metaphor, the image it evokes, the mental connections it engenders for the user and the receiver. Yet we can turn this power of the metaphor to influence thought to our own advantage, and use it as an analytical tool, a way to see differently, deliberately. Stay with my metaphor of fluids, of immersion in a language. What do we know about fluids that might help us to explore the use of language? What might one mean by “hot” language? “cold” language? “shallow” language? “the tide” of language? “rocks beneath the surface” of language? Do any of these new ideas of language invoke images that could shed light on how language is used? Perhaps we can play with these ideas sometime together!

Meanwhile if I were writing for your readers I would say, “pay close attention to the metaphors that Jennifer uses. Allow the images to arise that she conjures with her carefully chosen words. Use these images to see deeper into the complexity of understanding mathematics and children. Although highly theoretical in her thinking, her work is firmly grounded — she would probably say “rooted” — in her classroom. This book is no theoretical treatise, accessible only to academics. It is highly readable by the teachers she wishes to engage. Description, argument and interpretation are expressed through forms of
portraiture, narrative writing and metaphorically enacted through layout and surprise. She offers vignettes which portray the co-emergence involved in creating an ecologically rooted sense of place for children's learning of mathematics, juxtaposed with diagrams and photographs which serve as visual images for her conceptual development of theory. These are no theoretically conceived teacher and children, but the author herself and her real students, with all the human quirks and imprecisions that constitute an everyday classroom. It is rare to find a work that combines, so effectively, theoretical originality and the reality of the classroom in such an enticing and readable form."

Since retiring to my small corner of French countryside, where fruit trees and vegetable plants, bees, chickens and lambs replace video recorders and students, my learning – because we never stop learning – has taken me down new exciting roads. My intellectual meanderings no longer intersect with yours directly but it gives me enormous pleasure to be able to watch the launching of your book because I see you moving off under your own impetus, taking our thinking together and that of others to new realms. You are laying down your own path and need no guide. The thoughts, ideas, diagrams, etc., I have left behind are there for you to carry, explore, throw away as you see fit on your journey. Do what you will with them.
All I feel entitled to say to your readers is, “once I was fortunate enough to have her as my student and now she is journeying alone, carving paths and planting trees through the wilderness of learning. Take her hand and travel with her. That way you will find your own new and challenging paths to understanding.”

What I really want to say to you, Jennifer, is, “Thank you for enriching my thinking and ways of being during our time together. Your readers have a treat in store.”

Very best wishes,
Susan
WE ARE CONNECTED TO THIS EARTH
WE ARE CONNECTED TO THIS EARTH

As we continue to pour chemical cocktails into the environment and move fast and furiously from one technological adventure to another, it is no longer a matter of choice but a matter of fact that in order for living systems to survive on the earth, we must live within its limits of sustainability.

Upon our clumsy awakening to the environmental crisis, we are presented with the realization that the impact of our actions cannot be contained and the effects of them reach farther and deeper than we ever anticipated. The ongoing devastation of the world’s natural and cultural systems makes this point clear. The results of how we live are not only felt by the local human community and our neighboring communities, but what we do affects all that is on this earth with us — the land, water, air, and every living being that depends on these sources for their existence. We are not independent beings. We are “just one particular strand in the web of life,” says Capra.1

Yes, but we recycle!

Deep changes cannot happen if our actions to reduce pollution and decrease stress on the earth’s natural systems remain entrenched in our desires to improve human health and maximize profit at the cost of all other forms of nature. If we are to prevent further damage to the environment it is imperative that we change our mechanistic perceptions of the world to ones that are ecological.

Simply put, this means abandoning the reproduction of our Cartesian-self-assertive-Newtonian manners of being in order to cultivate a more integrative existence on this earth. It entails re-rooting our thinking so that we may comprehend the world in nonlinear, connected, intuitive, and holistic ways; ways which contrast with those that are linear, analytical, rational, or reductionist in approach. Furthermore, values needs to be placed on cooperation, quality, and conservation instead of anthropocentric, exploitive, or competitive acts of domination and mind-sets that focus on the bottom dollar. It is likely that these changes will involve more than just exchanging our current ways of living for ecological ones. The mechanistic, anthropocentric traditions that we embody in our culture today have been evolving since the Industrial Revolution and so too will it take time for eco-centric practices to become taken for granted patterns in our thinking, actions, and identities.

For eco-centric thinking to bring about changes that possess depth, longevity, and integrity, it cannot be restricted to environmental cleanliness. In fact, eco-centric think-
WE ARE CONNECTED TO THIS EARTH

ing must become our natural, everyday way of being. Further still, learning to be ecologically mindful cannot be treated as an additional component in students' education but has to be an integral part of each and every classroom. In short, this means that teachers and students of mathematics may not be excused.

**QUESTION:**
So, what does classroom mathematics have to do with this?

**ANSWER:**
Everything!

One place to start could be to look for spaces in which to propagate nonlinear, connected, fluid, and holistic patterns of thinking mathematically. At the same time, educators could begin the process of assessing the embedded and taken for granted linear and mechanistic rituals practiced inside the classroom and within the larger field of mathematics education.

By not excusing ourselves from this task, we can work towards reconnecting and developing ecological ways of teaching and learning mathematics.

**What Is A Tree?**

A tree, we might say, is not so much a thing as a rhythm of exchange, or perhaps a centre of organisational forces. Transpiration induces the upward flow of water and dissolved materials, facilitating an inflow from the soil. If we were aware of this rather than the appearance of a tree-form, we might regard the tree as a centre of a force-field to which water is drawn. The object to which we attach significance is the configuration of the forces necessary to being a tree... It is not necessarily the case that whatever exists must be sharply bounded. In fact, rigid attention to boundaries can obscure the act of being itself.

- Neil Evernden, *The Natural Allen*

This redefinition of something as familiar as a tree at first rings strange. But we must recognize the more-than-tree-form. It describes just as well we know that a forest is more than just the trees that grow there, and that our intercourse with the world extends beyond the edges of our skin. Our language falls short of our apprehension because of the way we have been taught to identify the world. We belong to are made of that world that surrounds us, and we respond to it in ways beyond knowing.

(continued from page 32)

Today we are calling on the nations and the peoples of the world to change personal attitudes and practices.

We adopt the ethos for living sustainably, people must reassess their values and alter their behavior. Society must promote values that support this ethos and discourage those that are incompatible with the sustainable way of life.

(continued on page 30)
Reflection

A mirror reflects an image
seen as the image is seen.
It does not change the
looking.

To reflect on what we do,
or are, is something else.
It reveals what we could not
see.

~ Pille Bunnell"
We are constantly engaged in the flow of interacting but often, it is not until much later that we appreciate the significance of it.
Significances in the Flow of Interacting
It happened years ago. Jennifer could not remember exactly when but as a young child she was drawn into the enchantment of the "enveloping and sensuous earth". Eyes wide and bright, Jennifer stories out just some of her treasures to me. She speaks of wild landscapes just beyond her grandparents' orchard, tunneling on her stomach and disappearing into the tall, sweet grass, lying on her back and watching the night sky for cascading meteors, and feeling the cool dampness underfoot as she crept through the grass and listened hard in search of that mysterious chirping cricket. Among these treasures are many more: ones of forests and of the ocean, others stotted to her by her Ba Chan's tanka poems, as well as the Chinese proverbs told to her by her father.

"I suppose," said Jennifer, "because my family life has always been rooted in a kind of living that looks to nature for metaphors and life lessons. I also want to understand the world as a unity in which everything is interrelated. And I guess that's why I wonder how ecological forms of thinking might help me to better understand this place we call the mathematics classroom.

"Classroom mathematics and ecology? Interrelated? Tell me you're not serious," I said.

"I know, I know," she said, "But listen."

Anticipating a long, winding, twisting, turning kind of response, I straightened my posture and took a deep breath as Jennifer led me down her explanatory path.

"You see," she said, "ecology and classroom mathematics have everything to do with each other."

Jennifer told me that the word, *ecologie*, came from the Greek word, *oikos*. It meant the family household and the maintaining of its daily operations. Eventually, *oikos* was integrated into the term, *oeconomie*, which was coined by Ernst Haeckel in 1866. Described as the study of the environmental conditions of existence, *oeconomie* was eventually shortened to what we know today as *ecology*.

"I remember exactly what I was doing when I first learned about the history of the word," said Jennifer. "To be quite honest, even though I'd heard what had been described, I didn't give it much thought at the time because I was preoccupied with what I assumed to be a completely different matter. You see, I was taking a summer graduate course and we were discussing the origin of the word. I remember smiling and nodding as I listened only to return to my
SIGNIFICANCES IN THE FLOW OF INTERACTING

previous thought. It wasn’t until much later that I realized the significance of the encounter.”

Jennifer explained that the dilemma preoccupying her thoughts concerned her awkward attempts to communicate to others, the importance of being ecologically mindful in the mathematics classroom.

“You see, at the time it was easier for me to say what being ecologically mindful didn’t mean. Being ecologically mindful wasn’t necessarily about taking environmental issues and turning them into mathematical problems. It wasn’t about conducting scientific investigations... you know, taking ecological ways of thinking and using them as a magnifying lens to examine issues in mathematics education and then perform experiments in the classroom; that is, it wasn’t about forming a hypothesis, replicating procedures, generating conclusions or formulating a unifying theory that could be transplanted into every classroom. What I struggled with was that I didn’t have a simple explanation of what being ecological did mean for the mathematics classroom. My descriptions were cumbersome – that knowing and acting were embodied within one’s way of being – or, a mindful comprehension of the integrative, holistic, and nonlinear nature of teachers’ practices and students’ learning of mathematics. So while I spent my time trying to sort out the ideas that I viewed to be critical in developing an ecological sensibility for teaching and learning mathematics, I was completely ignorant of the fact that I did have a way to express my understanding of ecology and mathematics education. Oikos,” said Jennifer.

Jennifer told me that upon reflection, the term, oikos, captured exactly how she understood her mathematics class to be. For her, she imagined the mathematics classroom to be very much like a family household. As teacher, she saw her role as caring for and sustaining the mathematical relationships and interactions of her students.

“So just as ecological thinking about the environment focuses on human relationships with nature,” Jennifer smiled, “I have a similar focus for my teaching and my students’ learning of mathematics. As the students’ teacher, my focus is on examining and assessing the kinds of mathematical relationships as well as the forms of mathematics that emerge in the classroom, and responding to them in my manners of teaching mathematics.”

“And your reason for wanting to be ecologically mindful?”
"My wanting to be an ecologically responsive mathematics teacher comes from caring about how mathematics exists in the classroom, my teaching, and the students' learning of it. This kind of caring necessarily involves a commitment to nurturing relations that are not only ecologically coherent in the classroom but also ones that promote a sense of coherence within the larger educational communities."

Jennifer picked up a book that she had been reading before I arrived. Opening it to page 78, a composition written by M. C. Bateson, she read aloud:

We are living in a time of both creativity and concern about education, and the decisions that are made for the classroom will feed directly into the way graduates

"and students," she added,

participate in society and the way they impact on the natural

"and social-cultural," she said,

systems around them.

"And so you see," Jennifer said, "the choices we make as mathematics teachers not only affect the kinds of mathematics students learn in school but more profoundly, the ways in which they are taught to learn and the ways in which they interact with mathematics outside of school affect the world they live in. We, mathematics education and ecology do not exist in separate households but rather, we share a common space."
SPACE WANTED
Looking to share a space with ecology. Interested in what ecologically coherent forms of teaching and learning mathematics could mean for the classroom. Can move in IMMEDIATELY.

(continued from page 30)

3 FACES OF ECOLOGY

M. C. Bateson identifies three “faces” or realms of ecology: empirical, environmental, and systemic. The author defines empirical ecology as biological, meteorological, and geographical studies that focus on understanding how the planet is changing and how these changes affect the interrelationships of the world’s natural systems. The environmental face of ecology is concerned with identifying the level of impact that our ways of living have on the earth’s systems. This area of ecology also involves the development of solutions for environmental problems that minimize harmful stress on the earth. It is within the systemic realm of ecology where mathematics teaching and learning can be most radically explored. This is because systemic thinking focuses on seeking “the pattern which connects” a system or systems together as interacting wholes.

A “system” could be a teacher or a student. It could also be a collective group such as a mathematics class, a school, and so on.

The connecting pattern or patterns that interrelate one system to another encompass the forms of knowledge, actions, and identities that result from the ongoing interactions within the systems as well as the ways in which these dynamics are sustained by the systems themselves.

Focusing on relational qualities makes it possible to view the world as an integrated whole; a dynamic and fluid unity in which all living and social-cultural systems are interconnected. The world is not conceptualized as a collection of separate entities but instead, as a highly complex unity in which all systems are interrelated and therefore, interdependent.

It seems sensible that when looking from an ecological perspective, mathematics teaching could be conceived as similar to that of students’ mathematical learning. Mathematics teaching then, as a fluid and complex process, implies that it exists always in relation to the ongoing interactions of the students, the mathematics, and the material and nonmaterial environment of the classroom.

In light of this, it is from a systemic perspective that the following question is posed:

In what ways can systemic manners of thinking about mathematics education enable forms of teaching and spaces for learning mathematics to possess an ecological sensibility?
Yes, but what gives rise to a systemic, ecological view of the world? or the mathematics classroom for that matter?
Our perceptions of the world and the mathematics classroom...
Our individual ways of being...
I think that if we start by looking at how we are as individuals and then connect this to how we exist as collective groups, you'll be able to appreciate why a systemic understanding of the world and the mathematics classroom is really about the process of living. This kind of understanding defines a conceptual space in which we can make visible what often remains invisible — the co-emergent, complex nature of our biological, structure determined, social, cultural

Okay, let's begin.

Well, as humans, we exist in the world as what Maturana and Varela would call, autopoietic, or self-making systems. What this means is that we possess both organization and structure. It's our organization that distinguishes you and me as people and not, say, fish or goats! And it's our structures — our internal dynamics and relations which enable you and me to develop ways of knowing, acting, and being that are uniquely our own. Simply put, your structure isn't the same as my structure and it's because of our structural diversity that we can distinguish you and me as being different people.

But how is it that we are structurally different?

Maturana and Varela describe structural coupling as the process by which our structures evolve. The changes that occur in our knowing, actions, and identities arise from the recursive interactions between two or more living organisms.

A nice sounding definition but what does this mean?

Well, if we take this idea of structure and think of a person's understanding of mathematics to be his or her mathematical structure, in a way similar to how a person's forms of knowing, acting, and being are impacted by life experiences — a person's mathematical structure too undergoes changes as a result of his or her mathematical interactions.
ENCIRCLING OUR PERCEPTIONS

And so, because one’s structure is dependent on the kinds of mathematical interactions one has and how they then feed into what the person already understands, these differences in experience and impact create differences between one person’s mathematical understanding and that of another person’s... therefore, diversity in mathematical structures.

Yes. And recursively, how we go on then, to teach or learn mathematics will be shaped by these structural differences. This also means that as a class engages in mathematics, structural coupling may arise in the structures of the individual students and their teacher. The growth that results from this process is dynamic and continuous. It happens in us moment to moment as we experience what you could call, human and nonhuman perturbations in the environment.

So it’s the perturbations that make for structural changes?

No, not exactly. It isn’t the perturbation that determines how our structures change. And, perturbations only exist if the person perceives these as perturbatory. Rather, it’s the individual based on his or her structure, who specifies what will or won’t be a perturbation, whether or not coupling will occur, and if so, the kind of internal changes that will arise in his or her structure. Knowing this, we can say that we exist in the world as autopoietic and structurally determined systems.

Would this mean then, that in the mathematics classroom, it’s the child who determines based on his or her internal structure, what will and will not serve as occasions for learning mathematics?

Yes, and the child’s mathematical understandings — his or her structure, shapes and is shaped by future understandings.

But what about the teacher? Isn’t it the teacher who teaches the class what mathematics to learn?
Of course it should be expected that a teacher attend to children's mathematical learning in ways that are invocative and provocative. However, with a systemic view, we can't assume that the teacher exists as the only source for occasioning children's mathematical perturbations. Engaging in mental reflection about mathematics or taking part in mathematical interactions with the human and nonhuman environment also serve as possible sources for structural changes to occur.

So, even if a teacher intends to have the class learn, say, a new strategy for adding three-digit numbers together, it's the child, not the teacher who determines if and what kind of learning will arise?

Exactly. And when structural changes do take place, new pathways or relationships emerge and impact the child's existing mathematical understandings. So it's impossible for us to predetermine how our individual structures will evolve since they are ever-changing because of the coupling process. This is what Davis, Sumara & Luce-Kapler mean when they say that learning is dependent on, but cannot be determined by teaching. Mathematical learning takes place with the environment: as unpredictable yet recursive growth of one's mathematical structure of understandings.

Okay, I can see how we as individuals are shaped by the interactions we have with the environment but it seems to me to be a very inward, insular way to view the mathematics classroom, don't you think? This kind of thinking moves in only one direction — from the environment to the individual child.

And up to this point it has. However, a systemic view does bring forth a co-emergent worldview. If you will, in that it recognizes the interdependence and complex circularity that exists between the environment and living systems. Just as our internal structures are ever evolving through our interactions with the environment, the environment is also undergoing structural changes. These changes within-and-in us and the larger environment recursively shape what's possible in terms of future interactions and how each responds to the other. Lewontin elaborates on this complex circularity when he explains that:

The organism and the environment are not actually separately determined. The environment is not a structure imposed on living beings from the outside but is in fact a creation of those beings. The
ENCIRCLING OUR PERCEPTIONS

environment is not an autonomous process but a reflection of the biology of the species. Just as there is no organism without an environment, so there is no environment without an organism. This co-evolution that takes place as the environment and we interact raises an important issue when considering the mathematics classroom. It’s not only the environment that shapes the teacher’s or a child’s mathematical ways of knowing, acting, and being, but it’s also the teacher’s or child’s interactions that affect what future events and responses take place within the larger classroom.

Each needs the other.

“That’s right. Now can you begin to see how it’s possible for the world to be viewed as an integrated whole by recognizing the interdependence of living systems and their environments? Life arises by way of natural drift — as a result of the recursive interactions between living systems and the environment. This co-evolutionary view of the world differs from other perspectives that project images of evolution as being a linear process of competitive domination whereby species and their environments are not interdependent but separate from each other.

Yes. A subtle yet important difference, I suspect.

What’s more, a systemic, ecological view doesn’t portray mathematics teaching or children’s mathematical learning as being individualistic and linear in nature. Each arises fluidly in relation to the other and with that of the larger environment be it a mathematics class, a school, or even the educational system. An ecological perspective brings attention to understanding the interrelationships that exist within the mathematics classroom.

In the beginning of our conversation, you mentioned that we are also social and cultural beings, yes?

Yes, that’s right. Keeping in mind what we’ve discussed in terms of how we are as individuals and how the environment and we co-emerge, let’s move outwards to the broader, social realm. By doing so, we can continue to discuss the pattern which connects our living as individuals to our collective actions, identities, and ways of knowing as social systems.
Our social ways of being...
ENCIRCLING OUR PERCEPTIONS

Maturana characterizes social interaction as being:

... when two or more structure determined systems interact recurrently with each other in a particular medium, they enter in a history of congruent structural changes that follows a course that arises moment after moment contingent on their recurrent interactions, to their own internal structural dynamics, and to their interactions with the medium, and which lasts until... they separate. In daily life, such a course of structural change in a system contingent on the sequence of its interactions in the medium in which it conserves organization and adaptation is called drift.  

In terms of the classroom, this would mean that social mathematical interactions arise when two or more children work mathematically together. Importantly, learning occasioned from these mathematical interactions not only shapes the further development of each child’s understandings but also, the collective understandings of the partner or group and the larger mathematical environment in which the interaction took place. These collective forms of knowledge, actions, and identities and how they're created through social interactions are what Maturana refers to as drift.

If we understand human social systems to be what Gregory Bateson and Maturana refer to as systems that evolve through the cohesive, collective interactions of the members, then what we know, how we act, and who we are can’t be taken as happening only within the realm of the individual. Such growth also needs to be recognized as emerging from our collective manners of living — the relations that are created through ongoing interactions and that which connects us as interdependent, social beings.

And are social phenomena unpredictable like our individual structures?

Yes. Just as we can’t predetermine the evolution of an organism or its environment because they are dynamically interactive, we can’t predetermine the collective mathematical activity that will take place in a mathematics classroom. In terms of a class’ mathematical learning, it’s naturally unpredictable because children’s internal and collective structures are constantly changing from moment to moment.

I see. So a child isn’t only a structurally determined learner but he or she is also a member of larger social systems... such as a mathematics class?
Exactly. And in the classroom it’s not only the child but also the children and their teacher interacting at the same time... as individual and collective wholes, responding to environmental perturbations — shaping and being shaped by the mathematical learning that emerges. Like our individual understandings, collective forms of learning aren’t thought of as being transmitted from an external entity. They are constantly emerging and co-emerging through social interactions. Because of this, children’s mathematical growth can be described as being, much like paths that exist only as they are laid down in walking.”

Can you explain in more detail the nature of social interactions and what forms they might take on?

In terms of their nature, I think of these as Maturana does — like conversations in progress. Maturana explains that social interactions can be brief, withdrawn from and then re-entered again or, they can be continuous. It is these conversations within social systems that he considers necessary in how it is we come to know and be in the world. Social dynamics are what bind us as a pair or group of living beings together as a collective, social system. Co-emergence takes place as we interact with and in relation to one another... we are able to coordinate and re-coordinate how we think, our actions... basically, how we are, in order to maintain cohesive ways of being with one another. In this way, social relationships that keep a collective unity intact can be seen as similar and just as critical to the co-evolution that takes place with individual organisms and their environments.

Maturana’s idea of languaging is useful because it describes the process by which social systems function and evolve as collective unities. Now, it’s important that you don’t think of languaging as simply individuals engaged in verbal conversation with themselves or others. Languaging involves the physical, verbal, and mental ways we humans think and interact among one another, but it’s also the understandings that arise from such linguistic interactions. It’s how we are able to coordinate and re-coordinate our ways of being so that we can continue to interact within groups and develop collective forms of knowing. In other words, languaging in the mathematics classroom entails the mathematical understandings that emerge from the different ways in which members of the class think and engage mathematically with one another. Because we exist in language and have the potential to be languaging agents, it’s possible for new understandings to arise. Knowledge systems evolve then, as a result of our social activity.

So does this mean that it’s through the interactions of the class that collective mathematical understandings that are different from personal ones come into being?
ENCIRCLING OUR PERCEPTIONS

Yes. Now, let's talk about our cultural ways of being.
Our cultural ways of being...
ENCIRCLING OUR PERCEPTIONS

The practices of teaching and learning school mathematics are two examples of cultural behaviours in our society. Generally speaking, cultural behaviours are social patterns that span generations. It's as a result of our living in the languaging process of cultural drift that we establish collective ways of being that are passed on and evolve from generation to generation.

So cultural ways of being are social phenomena that continue over generations?

Yes. And the historical transformation that happens is a result of the recurrent interactions and languaging between the older and younger members of a cultural group. In the same manner that drift is explained by Maturana and Varela as necessary for us to evolve with the environment and socially with others, so too is cultural drift necessary for the continuity and evolution of cultural systems.

Okay. I see how cultural drift provides a systemic way to understand, say, how human-centred and mechanistic social patterns established in the Industrial Revolution have continued into today's culture. But what I don't yet understand is what ecological thinkers such as Bowers, Capra, Naess, and Orr mean when they say that our cultural ways of being shape how we perceive and therefore, live in the world.

We do much more than simply exist in the world — remember our conversation about coupling?

Yes.

Well, I believe that connected to this understanding is the fact that THE WORLD(S) we bring forth isn't only created through our individual and social existence. It's also SPECIFIED through our cultural knowledge, actions, AND identities. Cultural phenomena, when CONSERVED, are carried forth seamlessly from one generation to the next and create a world from our living WITHIN CULTURAL COGNITIVE CIRCLES. These cognitive circles arise from our cultural ways of living and we justify the patterns that they occasion as a matter OF TRADITION OR, less reflectively and more acceptingly expressed, we say it's simply JUST THE WAY THINGS ARE. In this way, OUR CULTURAL PATTERNS embedded in our thinking ARE US, DISAPPEARING FROM THE SURFACE OF OUR CONSCIOUSNESS.
Surely, our cultural actions, beliefs, and identities aren’t that invisibly specific!

Hmmm... well, consider the images we as members of Western culture attach to the idea of what it means for a person to be an individual. When we think about what makes a person an individual, often embedded within this is the notion of independence. As a teacher, I find that parents often express to me that it’s important for their children as individuals, to be self-sufficient, able to think for themselves, make independent decisions, and be their own people. In valuing these qualities, we teachers and members of older generations encourage younger generations to develop their independence by providing learning opportunities that focus on the individual or autonomous child. Within our culture, independence and individuality serve as distinguishing qualities of being successful. They engender a sense of freedom, self-reliance and standing out from the crowd.

Well, isn’t that what we should be doing? Encouraging students to become independent individuals?

Hold on for a moment. Let’s contrast this image with what it means to be an individual in Japanese and Chinese cultures. Traditionally, within these two cultural circles, the image of an independent individual is not the image that comes to one’s mind. This is because in Japanese and Chinese cultures the older members teach the younger members that an individual is not defined in terms of self-reliance or self-sufficiency but more in how the individual contributes to the wellbeing of his or her family. You see, a person’s identity exists in the collective sense of the family. This can be seen in how people address one another. Unlike in Western society where we are distinguished on a first name basis such as Jennifer or in a first-name-last-name order as Jennifer Thom, people in Japanese and Chinese cultures are addressed by their last name or in a last-name-first-name order such as Thom-san or Thom-Jennifer. Identity isn’t derived from the validation of one’s self, but from the respect for the family as a collective whole. Conservation of these relations is carried forth through one’s values and action that foster the wellbeing of the family as a collective whole. Given instances such as these, we can better understand what Maturana speaks of when he says that it’s in the implicit or explicit accepted premises under which their different kinds of discourse, actions, and justification for actions take place6... that cultures create taken for granted and, hence, invisible yet distinctive cognitive circles.

Okay, yes... how cultural beliefs create blinders that shape how we experience the world... but, if we’re truly blind to our cultural ways of being, is it even possible for us to become aware of them?
ENCIRCLING OUR PERCEPTIONS

Another way for us to examine just how culturally embedded our lives are is to consider the cultural experiences, beliefs, and values that emerge from recurrent interactions of a group and metaphorically become what Gregory Bateson and Bowers refer to as a culture’s map.

A map?!

Yes, a map. Simply put, the term and concept borrowed from Korzybski means that a culture’s map identifies what its members will and won’t perceive as having significance by rooting these features within the culture’s temporal, spatial, spoken, written, and symbolic language. Because these cultural distinctions permeate the group’s languaging, a culture’s map as Postman says, does much more than construct concepts about the events and things in the world; it tells us what sorts of concepts we ought to construct.5

Can you give me an example of a feature or concept that we create or recreate in our living out of these metaphorical maps?

Just look back at how the idea of the individual is distinguished and played out as a feature of Western and Asian cultural maps. The former very much influences a person to value ways of thinking and actions that enable an identity of independence while the latter, emphasizes a person’s connection to her or his family and imbues a sense of interdependence. So, they both share the same concept — the individual, but are two completely different cultural conceptualizations.

Yes, which lead to totally different ways of being in the world.

Speaking from a systemic, ecological thinking space, it’s here in the envelopment of the cultural realm... nested within our individual and collective ways of knowing and being that we dwell in our practices of teaching and learning mathematics. For me, the mathematics classroom is imagined as an integrated space where individual, social, and cultural systems co-exist and co-evolve. In an ontological manner then, we live simultaneously as biological, structure-determined, social, and cultural beings. Encircled once more to include all other living beings and natural systems on the earth, it’s how we humans come to exist, as Capra says, as just one particular strand in the web of life.6 It’s in this way that the world isn’t perceived as a collection of separate parts but as a dynamic whole: a complex unity of all living and social-cultural systems that are fluidly interconnected and, therefore, necessarily interdependent. And it is here in this conceptual space that a systemically
ecological view of the world and the mathematics classroom resides.
All living systems within the biosphere... Our cultural ways of being... Our individual ways of being... Our social ways of being...
SPACE WANTED
Looking to share a space with ecology-oriented or what ecologically coherent forms of teaching and learning mathematics could mean for the classroom. Can move in IMMEDIATELY.
(continued from page 30)

3 FACES OF ECOLOGY

M. C. Bateson identifies three "faces" or realms of ecology: empirical, environmental, and systemic. The author defines empirical ecology as biological, meteorological, and geographical studies that focus on understanding how the planet is changing and how these changes affect the interrelationships of the world's natural systems. The environmental face of ecology is concerned with identifying the level of impact that our ways of living have on the earth's systems. This area of ecology also involves the development of solutions for environmental problems that minimize harmful stress on the earth. It is in the systemic realm of ecology where mathematics teaching and learning can be

HELP WANTED
*How can we break out of our conventional approaches and imagine more productive alternatives?* Reply to mailbox: T19M9M7S

must radically explored. This is because systemic thinking focuses on seeking "the pattern which connects" a system or systems together as interacting wholes.

A "system" could be a teacher or a student. It could also be a collective group such as a mathematics class, a school, and so on.

The connecting pattern or patterns that interrelate one system to another encompass the forms of knowledge, actions, and identities that result from the ongoing interactions within the systems as well as the ways in which these dynamics are sustained by the systems themselves.

Focusing on relational qualities makes it possible to view the world as an integrated whole; a dynamic and fluid unity in which all living and social-cultural systems are interconnected. The world is not conceptualized as a collection of separate entities but

VACANCY
Looking for teacher to teach grades 4-6. Separate room. Shared facilities.

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instead, as a highly complex unity in which all systems are interconnected and therefore, interdependent.

It seems sensible that when looking from an ecological perspective, mathematics teaching could be conceived as similar to that of students' mathematical learning. Mathematics teaching, as a fluid and complex process, implies that it exists always in relation to the ongoing interactions of the students, the mathematics, and the material and nonmaterial environment of the classroom.

In light of this, it is from a systems perspective that the following question is posed:

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[Image of a table with columns and rows, some entries are partially visible.]

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Settling In

Thinking about how I (it's me, Jennifer) might respond to this help wanted ad, I thought it best to bring it home as it were and invite Stigler and Hiebert's question into this ecological thinking space of mine. However, once inside, I soon realized that although this question certainly belonged to the realm of classroom mathematics, it was going to be difficult if not impossible for me to have an open conversation with it. Explained another way, it is like when you spot THE sofa in a furniture store but as soon as you get it home and put it into your living room, the sofa does not look so fabulous anymore. Instead, it is clearly out of place because it does not go with any of your existing furniture. For me, (and in spite of the authors’ good intentions) this seemed to be the case with bringing this question home; neither the question nor the ecological space suited each other. You see, the manner in which the question is posed:

How can we break out of our conventional approaches and imagine more productive alternatives?  

puts forth for me as a teacher, an 'end-result' mindset of improving productivity in the mathematics classroom, the need to dispose of or discard teaching practices that are perceived to be old or commonplace and to acquire new teaching tools so that we may increase or maximize children's learning of mathematics.

Before I am able to think about possible ways to consider and respond to this question, its linear, disconnecting posture has already told me that manners of teaching are commoditizable relationships, ones to which we essentially become engaged, eventually marry, and if necessary, divorce ourselves from. What is more, is that within the question's reductionist confines, Stigler and Hiebert's query closes itself off from the opportunity for deep changes to take root.

You see, even if we changed from one teaching style to another, radical shifts in the mathematics classroom would not likely occur if our thinking continued to be fashioned from the mechanistic pattern of productivity. The persistence of such a mindset is disabling in that it denies the very possibility of Stigler and Hiebert's query to be one that provokes teachers in becoming more integrative and creative.
And because the authors’ question does not allow for an examination of the ecological coherence of the mathematics classroom, it creates the impossibility for mathematics teaching and learning to be conceived as holistic, organic, recursive, and co-emergent.

Determined to re-conceptualize Stigler and Hiebert’s query within an ecological realm, I set out to find a question that would make sense in such a space. This question had to be one that encouraged a systemic way of knowing: one that necessitated the development of an understanding for the ways in which our processes of living are always and necessarily shaping how we teach mathematics.

Instead of provoking a knee-jerk reaction from the reader or myself, the question needed to be an open invitation to look deep and make visible what often remains invisible — the cognitive circles and cultural maps — we embody and enact as we teach. Once visible, these experiences, beliefs, and values can be examined and assessed for the kinds of mathematics teaching that they enable and disable, the ways in which they become embedded within the language of the classroom, and the impact they have on how children come to know mathematics. While I wondered about these points, a similar yet radically new question invited itself into my thinking:

*How might we as teachers consider our patterns of mathematics teaching? And, if necessary, how might we re-root the learning space so as to nurture and children’s mathematical growth?*

As I reflected on this question, I realized that not only does the question share Stigler and Hiebert’s concern for how mathematics teaching and learning takes place in the classroom, but it also makes sense within a systemically ecological thinking space. The question provides the necessary structure for the re-conceptualization of mathematics teaching to occur while the ecological mind-space allows a place for such an exploration to arise. Together they engender an examination into how what we know and who we are emerge and become our manners of teaching mathematics.

Feeling more settled in this new conceptual space of mine, I wondered what I should do next. Not having a clear direction in which to move my thinking, I looked to Stigler and Hiebert’s question again but this time, how I might use it to prompt an exploration of my eco-centric one. It was in the midst of this back and forthing that I realized both questions spoke of concern for this place called, the mathematics classroom. It made sense for me then, that the place to begin was to begin with, place.
Sense of place complex? We tend not to think so, mainly because our attachments to places, like the ease with which we usually sustain them, are unthinkingly taken for granted. As normally experienced, sense of place quite simply is, as natural and straightforward as our fondness for certain colors and culinary tastes, and the thought that it might be complicated, or even very interesting, seldom crosses our minds...

~ Keith Basso℠
Beginning with Place

Basso's description of the ways in which we create and sustain our relationships with places brings forth just how strong our connections to place are. In doing so, he reminds the reader how taken for granted, forgotten, unnoticed, or ignored the actual contexts and patterns that make a place a place really are. Primary, basic, and essential, sense of place is undeniably and always a critical part of every mathematics classroom.

If we perceive the world as a place in which we live as systems within systems, then the mathematics classroom, as place, is not constituted simply by the presence of four walls, some furniture, a teacher, students, and mathematics. Necessarily, this place includes the relations that evolve from the intermingling of teacher, children, their surroundings, and mathematics. If we think about how we come to know places, then our sense of a mathematical place in the classroom emerges from the spaces in which we perceive mathematics to arise and the forms it takes on. Put another way, the classroom as a mathematical place and how we connect with it not only comes from what we know and feel, but the kind of place it becomes grows out of the interactions we have with it. Thus, we do not define places; they do not define us. Rather, in dynamic interplay, we come to form together.

As a teacher, I have always been committed to a holistic way of thinking about mathematics in the classroom, one that facilitated my growth in teaching mathematics and children's learning of it. But it was not until I had read Basso's book, *Wisdom Sits in Places* that I began to think ecologically about the mathematics classroom as place. Moreover, as ecological ways of being are not an everyday practice in our society or its educational systems, it is understandable that one would not think of the mathematics classroom in such a manner, much less be able to imagine what an ecologically coherent mathematical place might mean. Sense of place and ideas associated with place do not come about naturally or consciously for us. As a result, they remain hidden or invisibly embedded within our taken for granted manners of teaching mathematics.

Through my experiences in gaining a deeper understanding for how my teaching shapes the mathematics classroom and trying to create a sense of place that embodies ecological notions such as recursion, co-emergence, and fluid integration, I have learned that this kind of work cannot be achieved by what we think may be breaking out of conventional approaches. To do so in the attempt to get rid of all that undermines an ecological sense of place in the mathematics classroom would be naive and superficial. Systemic, ecological changes need to begin by first considering what place means to the mathematics classroom. It involves looking
deeply and engaging in the complex, recursive process of identifying and questioning one's taken for granted conventions of thinking about and teaching mathematics — inquiring into and learning how these ways of being contribute to the sense of place that exists in the mathematics classroom.

Why is this so important? Does it really matter? I think so. Let me explain by describing the different senses of place for mathematics that I have come to know as both a learner and a teacher. These vignettes chronicle my growing understanding of place. They provide revealing glimpses into how deeply mechanistic, commoditized, linear, and disconnected my common sense of place for the mathematics classroom was and the challenges I faced in making it a cohesive whole.
Reflections Revealing the Fragility of Classroom Mathematics
It [mathematics] requires silence and neat rows and ramrod postures that imitate its exactitudes. It requires neither joy nor sadness, but a mood of detached inevitability: anyone could be here in my place and things would proceed identically.

— David Jardine\textsuperscript{10}
A Student’s Place

Through my years of elementary schooling, I grew to believe that the spaces where mathematics existed in the classroom, the forms it assumed, and my relationship with it were clearly marked out by my teachers and had little if anything to do with my peers or myself. I came to know that at school, it is the teacher who makes mathematics happen. Just like TV programmers, I thought to myself, teachers ensured that mathematics began, ended, or reran at exactly the same time each day. They were conscientious not to let mathematics spill into any other programs of study such as science, socials, art, or language. And the only time when mathematics did extend past its designated slot was after school — if we had not completed our exercises during class.

Our lessons were similar to that of learning to catch a ball. First we watched the teacher demonstrate how to do the mathematics and then ready or not, the teacher threw problems up onto the chalkboard or to us in the form of a textbook. Scrambling to catch the mathematics, we madly recorded the mess of numerals and symbols in their correct linear fashion, practised, practised, practised, and then hopefully, tossed the mathematics correctly back to our teacher. On other days, we awaited the moments when timed drills, pop quizzes, and tests became the stage where we performed our proficiency and ability to juggle addition, subtraction, multiplication, and division facts.

I also learned that the teacher liked it best when mathematics happened not with other classmates but rather, silently in our heads, and figuring out solutions should not involve fingers, drawings, or counting. Never questioning but always reproducing, this is how my teachers and we students busily created and maintained a place of anonymity for mathematics in the classroom.
Encountering familiar issues in a strange setting is like returning on a second circuit of a Möbius strip and coming to the experience from the opposite side. Seen from a contrasting point of view or seen suddenly through the eyes of an outsider, one’s own familiar patterns can become accessible to choice and criticism. With yet another return, what seemed radically different is revealed as part of a common space.

— Mary Catherine Bateson
Back on the Strip:  
My University Years

Towards the completion of my undergraduate degree in education at the University of Victoria, I was required to identify a specific teaching area that would become my focus for the remainder of my program of study. Based on my interests, it was a toss up between visual arts and mathematics education. I found it difficult to choose one over the other and so I chose the area that offered me boundless possibilities for complex and creative exploration. The two years of mathematics education and mathematics courses that followed made for what I considered to be two more journeys around the Möbius strip. The first trip, which I have already described, was my childhood experiences learning mathematics in school. The second, from the perspective of a teacher-to-be, and the third, from a learner of mathematics again is what I describe for you now.

During each of these returns, I found myself questioning and shifting my conceptions of what it meant to teach and learn mathematics. However, it was only afterwards that I realized it was on these journeys that I was visiting and revisiting the notion of mathematics’ place in the classroom.
From the Side of Pedagogy

Now enrolled in mathematics education and approaching it not from that of a mathematics student but coming at it from the opposite side — one of pedagogy — provided me with the contrasting point of view necessary to reveal taken for granted conceptions I held about classroom mathematics.

It began in my first mathematics education class. In this course where the focus was on teaching mathematics, the professor engaged us teachers-to-be through modeling possible ways to develop children’s conceptual understanding, actually experiencing hands-on minds-on activities for ourselves, and assessing student understanding through video analysis. In doing so, this professor dispelled many of my taken for granted assumptions about school mathematics; ones that included it as being an activity of simply doing tasks and solving problems quickly in one’s head and that mathematics is best learned in isolation.

What became apparent to me was that as a teacher I needed to be creative in thinking about my manner of teaching as well as thinking creatively about mathematics itself. Forms of teaching which communicated to students that there are many ways to solve a problem, avoiding what was referred to as heavy-handed teaching that implied teaching-by-telling, and developing open-ended activities such as simple games or riddles that draw children into the complexity of the mathematics instead of repelling them from it became important faci in my growth as a teacher. Moreover, I realized that just as mathematics should be brought into being through the teacher, the students, and a variety of settings in the classroom, it was also important for teachers to enable learners to develop meaningful connections between the mathematics they study at school and the mathematics that occurs in their daily activities at home and in their community.

Learning from the opposite side of the Möbius strip — from the perspective of a teacher-to-be, I began to understand the impact that teachers have on the kinds of mathematics that arise in the classroom. And it was here that I began considering how I might enact a pedagogy that embodied a sense of connectedness for mathematics in the classroom.
Third Time Around

The following September, I found myself taking yet another trip around the Möbius strip. This time, again, from the perspective of a learner. Here in two of my mathematics courses I experienced first hand how it felt when mathematics took place in the dynamic ways that the first professor had described. These two professors made it clear to me that I would neither be given nor expected to memorize formulas or procedures. Feeling panicked, my first reaction was that I had registered for the wrong mathematics classes — how on earth was I to play the game if these professors did not show me which mathematics I was to toss back and forth?

Fortunately I persevered, continued to attend the classes, and for the first time in all of my years of learning, I was convinced that real mathematics was not a game of catch but rather something that is created and creative. Suddenly, for me, mathematics no longer took place in an anonymous world but with the world of human and natural contexts. We spent our time examining, questioning, and watching mathematical patterns emerge in different areas such as biology, economics, and everyday life. Learning in this manner provoked and enabled me to explore, devise, and create self-generated methods and mathematical formulations in place.
Common Space

By encountering familiar issues as M. C. Bateson describes and coming at them from the opposite side(s), we experience them as different or new. Then recursively, upon examination and questioning what we think to be distinct events, like the Möbius strip, we come to realize that the apparently disparate issues do not exist on separate planes but rather, exist within a common space.

For me, as a result of moving along what I perceived to be completely different planes was that my experiences of elementary mathematics learning, mathematics education classes, and university mathematics courses existed within a common conceptual space. Whether my experiences were that of a learner or teacher-to-be, they were all situated within the realm of sense of place for mathematics.

As a beginning teacher, I did not enter the classroom with a fixed image in my mind of mathematics as an activity that consisted of teacher demonstrations and student reproductions but instead, an image of mathematics as an ongoing engagement in which children and their teacher adventure in a world of knowing together.
SPACE WANTED

Looking to share a space with ecology interested in most ecologically coherent forms of teaching and learning. Mathematics could mean for the classroom. Can move in IMMEDIATELY.

(continued from page 46)

3 FACES OF ECOLOGY

M. C. Hatemo1 identifies three "faces" or venues of ecology: empirical, environmental, and systemic. The author defines empirical ecology as biological, meteorological, and geographical studies that focus on understanding how the planet is changing and how these changes affect the interrelationships of the world's natural systems. The environmental face of ecology is concerned with identifying the level of impact that our ways of living have on the earth's systems. This area of ecology also involves the development of solutions for environmental problems that minimize harmful stresses on the earth. It is within the systemic realm of ecology where mathematics teaching and learning can be most radically explored. This is because systemic thinking focuses on seeking "the pattern which connects" a system or systems together as interacting wholes.

A "system" could be a teacher or a student. It could also be a collective group such as a mathematics class, a school, and so on.

The connecting pattern of patterns that interrelate one system to another encompasses the forms of knowledge, actions, and identities that result from the ongoing interactions within the systems as well as the ways in which these dynamics are sustained by the systems themselves.

Focusing on relational qualities makes it possible to view the world as an integrated whole; a dynamic and fluid unity in which all living and social-cultural systems are interconnected. The world is not conceptualized as a collection of separate entities but

VACANCY

Seeking one teacher to teach grades 3/4. Separate room. Shared facilities.

Instead, as a highly complex unity in which all systems are interconnected and therefore, interdependent. It seems sensible that when looking from an ecological perspective, mathematics teaching could be conceived as similar to that of students' mathematical learning. Mathematics teaching, then, as a fluid and complex process, implies that it exists always in relation to the ongoing interactions of the students, the mathematics, and the material and nonmaterial environment of the classroom.

In light of this, it is from a systemic perspective that the following question is posed:

In what ways can systemic thinking about mathematical education enable forms of teaching and learning that can provide us with the pedagogical spaces for teaching mathematics to protect our ecological integrity?
Many, and perhaps most teachers begin their careers with the conviction that they will avoid those teaching practices that they found unhelpful or inappropriate when they were students. ... However, ... most beginning teachers quickly find themselves settling into patterns of teaching that are strikingly similar to the ones they intended to avoid.

~ Brent Davis, Dennis Sumara & Rebecca Luce-Kapler
Traditions, Tensions, and Transitions
Turning The Soil:  
The Beginning Years of Teaching

I began teaching second and third grade children in Richmond, British Columbia. When I entered the teaching field, I distinctly remember being eager to inspire a more connected sense of place for mathematics in the classroom and at the same time, being careful not to become another Mrs. Fibonacci. Mrs. Fibonacci, a storybook character, is an elementary school teacher who loves and lives mathematics to such an extreme that she makes learning an unbearable nightmare for one particular child in her class by turning everything into a mathematical problem that needed to be solved. For the main character, learning mathematics becomes a curse he cannot escape:

“What if this keeps up for a whole year? How many minutes of math madness would that be?”
“What’s your problem” says my sister.
“365 days x 24 hours x 60 minutes,” I snarl.6

Like all beginning teachers, I devoted enormous amounts of time to preparing my lessons. I read journals for mathematics teachers, went to workshops searching for new ideas, collected real world materials to connect the children’s mathematics with familiar contexts, and designed interactive mathematical tasks that would engage every child in my class; all the while, being careful not to cast any curses. But even though the children, their parents, and my colleagues seemed to be pleased with my efforts, I did not feel as if I was accomplishing what I had set out to do. My teaching and the children’s mathematical learning still seemed disconnected.
What we conserve, what we wish to conserve in our living, is *Insight*, I believe, refers to that depth of understanding that comes by setting experiences, yours and mine, familiar and exotic, new and old, side by side, Learning by letting them speak to one another. what determines what can and what cannot change in our lives.
In the months that followed, I searched and scrutinized my mathematics program to find the source of my unease. I pored over the curriculum guides to be certain that I was teaching the correct concepts and the skills for the different grade levels. I revised the order and adjusted instructional sequences so that they moved more efficiently. I continued to tweak or elaborate the content of my lessons, depending on the needs of my students. Looking at the program as a whole, I felt that I was engaging the children in mathematical work that enabled their learning to be both hands-on and minds-on and that I was making space for mathematics to be integrated with other subject areas. Unable to find any obvious problems, I continued to proceed along the current course.

Then, several months later, I started to question the kinds of relationships that existed between my teaching and the children's learning of mathematics. I took a reflective step back and examined my mathematics program for a pattern or patterns that connected the children's mathematical learning spaces together as a whole. In doing so, taken for granted ways of teaching began to emerge. I discovered that these were not only rituals unique to myself, but surprisingly, they were matter of fact ways of being for my colleagues too... even those of my schoolteachers. For me, these teaching practices had simply become THE way to facilitate children's learning in the mathematics classroom. Intrigued and wanting to take a closer look, I wrote these manners of being down on paper. And as I did this, it became apparent to me just how matter of fact they were and how deeply embedded in my teaching these shared facilities had become.

* **Before beginning any lesson, sort the children according to their grade level. Then, teach each group a different mathematics lesson.**

* **When planning the curriculum for the school year, simply divide the mathematical concepts and skills for each grade into ten equal parts. Now, allocate one of the ten school months to teaching addition, one month to subtraction, another month to multiplication, and so on, until the end of June.**

* **Always make sure that consistent amounts of time are given to mathematics lessons. Schedule it in regularly each day (e.g., everyday between recess break and lunch hour).**

* **Mathematical concepts should be taught sequentially; from an informal, concrete stage to more formal, abstract ones. Teaching should facilitate the student's (the autonomous child) construction of knowledge in a conceptual to procedural to relational order.**
After reading this “must do” list, it was apparent that the sensibility of wholeness and flow that I desired for the mathematics classroom did not exist. Instead, was one that possessed rigid, mechanistic, and disconnected qualities. These could be seen, enacted in my scheduling of lessons at same time every day, my taking inventory of the mathematics curricula and then packaging them up into discrete units of instruction, teaching separate grade-specific lessons, and my always doing so in a manner that proceeded from the concrete to the abstract. The kind of place that I had intended to root and the one that had actually become embedded were in contradiction to each other. What served as tried and true rituals for teaching mathematics had unthinkingly become that which was furthering of what Berry refers to as the cultivation of discrete parts without respect or responsibility for the whole.10

My teaching actions not only dismembered mathematics for the children but on another level, I had also dismembered mathematics from itself. I say this because one might argue that my efforts to teach the students’ conceptual then procedural then relational knowledge could be viewed as in keeping with facilitating connected understandings. However, despite the fact that I did this in my teaching within each of the concepts and procedures, I was still teaching the concepts as separate parts and attention was not paid to enabling the students’ connections among concepts, procedures or mathematical topics. One might also argue that putting together these individual units of instruction came to form a complete mathematics program. This might be true; however, the parts still were not fluid or dynamic but rather, discrete and static.

In taking this reflective step back, I saw that it was not enough for me to design a well, put together mathematics program and I wondered what I might do to engender a clear sense of flow in the mathematics classroom. Even though I could see how some of my invisible or assumed ways of teaching were undermining this, I did not know what kinds of re-rooting (conceptual or otherwise) were necessary.

What I learned however, was just as Basso11 describes, place is not something that can be taken for granted — not even in the mathematics classroom. Place is primary and basic yet at the same time, it is far more complex than had originally crossed my mind. If places are indeed created and sustained through interaction, then the mathematics classroom as place exists only in its being. And, it could be said that what distinguishes one mathematics classroom from another is its sense of place. Further still, it is then the kinds of mathematics that emerge from one’s teaching and from children’s learning that define the actual terrain of a mathematics classroom.
"JUST HAND THEM DOWN THE MATHEMATICS" ... OR NOT?!

WHEN MATHEMATICS is imagined and enacted as objectified, static knowledge that is to be traditionally passed down from one generation to the next, the teaching and learning of mathematics is disabled from ever becoming anything else. Under the air of hand-me-downs, it is easy to understand why mathematics is taught and learned out of a sense of obligation or contempt rather than a sense of open desire or wonder. It is also understandable why mathematics is all too often considered as that which is to be mastered rather than that which is to be understood. In commoditizing mathematics, we make absurd, the possibility for us as teachers and our students to perceive mathematics as anything else but a fixed entity. In this way of conceiving mathematics, we make it inconceivable for school mathematics to become something other than just a collection of hand-me-downs.

The embeddedness of these images within one’s taken for granted ways of thinking about mathematics not only makes it natural for us to assume mathematics to be an inanimate thing, but in doing so, locates mathematics as that which exists — out there. Given this mindset, it is not surprising that a teacher would feel impelled to set the class on a straight and narrow, one-way course to become collectors of mathematics. Given this mindset, it makes sense to ingrain the ritualistic practice of acquiring mathematics into school mathematics unit and lesson plans, methods of assessment, and enacted in the classroom; product oriented practices that focus on desired, expected, or even measurable outcomes of instruction — that after instruction — the students will have mastered the mathematics taught in the lesson before moving on to the next part of the curriculum. Of course, the ways in which students are instructed to take possession of their mathematical hand-me-downs of concepts, skills, and even attitudes may vary. Still, teaching by telling, engaging students in hunting for, having them seek out hidden mathematics in real world contexts, and even explorations de-
TRADITIONS, TENSIONS, AND TRANSITIONS

(continued from page 3)

designed for students’ discovery of mathematics all keep alive, the tradition of handing down mathematics.

Moreover, when product-oriented ways of thinking about school mathematics are coupled with a back to basics mentality, the teaching and learning of mathematics becomes subjected to the weigh scale of how much in regard to the amount of mathematical facts and skills that students are to learn and little or no emphasis is placed on such things as their mathematical thinking or conceptual understanding. Given this mindset, mathematical processes such as those identified by the National Council of Teachers of Mathematics12 as: problem solving, reasoning, communicating, connecting, and representing, would likely be deemed by most teachers as not essential. Viewed as additional knowledge,13 teaching that attends to the development of mathematical processes would then depend on whether students have acquired first, the prespecified mathematical facts and skills with which to process the mathematics.

The point here is that when students are taught to learn mathematics in the tradition of hand-me-downs and as a product oriented matter of collecting, hunting down, or retrieving pieces of knowledge, this creates the impossibility for mathematics to be taught and learned in ways that enable it to arise as living and animate.

Now, identifying the limitations of how mathematics exists in the classroom and the possibility of it becoming something else is all fine and good. But in doing so, means that the conversation does not end here. Rather, it opens up a whole host of questions that require further consideration such as:

• How might an ecological way of thinking help in reconsidering such taken for granted perceptions and re-imagining a more responsive view for the teaching and learning of classroom mathematics?

• What shifts in thinking are necessary to conceive classroom mathematics as something other than a line of hand-me-downs from teacher to student?

• What would it mean if we assumed mathematics to be embodied?

• How might problem solving, reasoning, communicating, connecting, and expressing be understood as integral to the learning of mathematics and not just additional knowledge?
I met Jennifer at the coffeehouse. She greeted me as I pulled up a chair to join her. Reaching for the newspaper that was lying on the table, she leafed through it and then stopped when she got to the Letters to the Editor section. Jennifer began reading one reader's response to an earlier article, "JUST HAND THEM DOWN THE MATHEMATICS ... OR NOT!!"

MOVING THINKING SPACES AND ASSESSING OLD FURNITURE

IN RESPONSE TO LAST WEEK'S EDITORIAL: I TOTALLY agree with the arguments that were raised. And the questions for sure, are important ones in making positive changes in the math classroom. My worry, though, is that real changes can't happen if this job of rethinking and re-imagining classroom mathematics is approached with the attitude of getting rid of or simply adding onto what's already there. What the editorial didn't mention was that it's not about taking ecological ways of thinking and coordinating them like new pieces of furniture into a rundown living room with the hope of updating one's mind-space by making it look more current. Really, a major part of moving from invisible and mechanistic places of knowing to ecological ones is about rediscovering the all too familiar furnishings that have been set about (classroom) mathematics and asking, how well do these furnishings go with the space?

All for new spaces,
Joel

"... getting rid of... adding onto what already exists... no. Definitely not," said Jennifer as she read the letter. And so she continued, reading bits of it silently in her head and every so often, sputtering out particular words or phrases.
"Exactly," said Jennifer as she looked back up at me.
"Enabling deep changes in teaching mathematics isn't about
changing out of certain approaches and slipping into new ones."

She agreed with Joel that what she needed to do was to examine her mathematics teaching from where she (conceptually) stood now. Jennifer wondered what she might see and see differently from an ecological perspective. What kinds of furnishings had become so comfortable and integral to her mathematics teaching that they were now permanent and perhaps, invisible? Jennifer questioned whether they would even suit an ecological mind-space. And more to the point, she was anxious to know what kind of place, what kind of oikos4 she was engendering for her students’ mathematics. But where might she begin?

For several days, Jennifer thought about the specific vantage point from which to position her thinking and to examine these issues. What kept emerging for her was a distinct and recurrent theme. The theme was recursive in the sense that it was the place where Jennifer found her thinking returning again and again. Moreover, it was not until this moment that she recognized the ever presence of this location. Here was the place where she existed both in and away from the classroom. When she described it to me, I immediately named it her in-between space. Not because it was a space of indecision for her but rather, the in-betweeness had to do with how her teaching and her research co-emerged and co-evolved. For Jennifer, teaching and research neither existed as separate entities nor did they move sequentially from one to the other as she once thought.

"As a pre-service and a beginning teacher I understood research to be something that was conducted by others who were not part of the classroom and its function was to produce theory which in turn, became a tool that I could use in my practice of teaching mathematics," she said. "But now, what comes to my mind is an image of teaching and researching as continuous interaction and dynamic growth... whereby teaching and researching flow into and give rise to each other because of each other. So teaching, exists as praxis and not simply the establishment and maintenance of one’s teaching practice."

As I listened to Jennifer describe how her view of mathematics teaching and research had changed, I realized that to characterize her in-between space as the location where the two met or intertwined would be to miss the meaning altogether. They did not meet. They were each other. For Jennifer, teaching mathematics and her study of it were inseparably connected. In a complex yet circular manner, Jennifer considered them to be interacting, co-evolving systems; that is, each system as a necessary part of the other. Furthermore, the distinction she made regarding her shift from teaching as practice to teaching as praxis revealed that mathematics teaching as praxis was not simply a routine that one performs but that which requires one’s active engagement; teaching as praxis then,
 implies a way of being that is critically reflective and reflexively responsive. Evoked from within an ecological realm is the importance of being ever mindful of how our knowing, actions, and identities in teaching mathematics arise recursively from that which we have already lived and therefore, become embodied in how we are living now, and how we will live. In other words, teaching as praxis recognizes the simultaneity and the complex circularity of that which occurs in one's teaching is also necessarily enfolded in all that interacts with it. This space certainly was not an in-between space. It was not a conceptual space located somewhere in the middle of teaching and research. Really, it was another space.

"I see this kind of reflexivity as necessary in attempting to understand how it is that my teaching and research give rise to each other," said Jennifer.

"So, if you had to describe in your own words, the guts of it all, how might you do so?" I asked.

"Simplifying the complex? Hmm... let's see... I suppose I would have to say that for me, teaching learners mathematics and learning what it means to teach mathematics flow together."

And it was in this spirit and in this other space that Jennifer began the process of researching her teaching by encircling it within ecology.
Jennifer naively assumed that by placing her teaching within the realm of ecology, immediate answers to her questions would be revealed. However, as the days passed, it was only her impatience that became apparent. Frustrated, she picked up a book that she had been reading and turned the page. There in black and white was the reminder she needed.

Letting myself get written by a place. Bodily scars as the age lines in the droopy skin on the backs of my hands betray. Legibilities of having, once again, lived-through.
Wasting time. Doing nothing with great deliberateness. Collecting dry bones. Boredom: this is one great little demon we have banished from the discourse of authorship and expression and self-annunciation. Deliberately spending time in the old place, feeling through moist weaknesses.15

Perception of opportunities requires a sensitivity given through one’s own wounds. Here, weakness provides the kind of hermetic, secret perception critical for adaptation to situations. The weak place serves to open us to what is in the air. We feel through our pores which way the wind blows. We turn with the wind; trimmers. An opportunity requires... a sense... which reveals the daemon of a situation. The daemon of a place in antiquity supposedly revealed what the place was good for, its special qualities and dangers. The daemon was thought to be a familiaris of the place. To know a situation, one needs to sense what lurks in it. (Hillman, 1987, p. 161)16

Although Jardine's description details how he readies himself to write, this practice of dwelling and keeping watch was exactly what Jennifer needed to do. It was obvious to her now that she did not know what aspects of her mathematics teaching needed closer examination, so to search for a deeper understanding when she could not articulate what exactly it was that she was seeking a deeper understanding for became a ridiculous endeavor. Jennifer decided it was best if she wandered back to that other place where teaching and learning and what it means.
to teach mathematics flowed together. There she waited patiently, dwelling, and all the while, keeping watch for what lurked in it.
Surfacing & Noticing

So Jennifer, what came out?

I wouldn’t say that anything really came out... it was more like bubbles making their way to the surface of the water and then bursting in the moment of recognition. You know, like, there’s one... and another one! Once you’ve caught sight of one of them, all of a sudden, there they are! And you see them for the first time — but not because they magically appeared — they’d been there the whole time and were only invisible because you hadn’t been able to notice them before.

Do you mean that there were qualities about your teaching that you were unaware of?

Well, yes. Even though my focus was on ecology and the mathematics classroom, up until this point, it wasn’t specifically about my teaching.

Does it need to be?

I think at some point it has to be. You see, it was only when I began questioning the kind of place I was creating for my students’ learning of mathematics that I recognized the need to move deeper — when I say deeper, I mean delving into my teaching — not just being attentive to what’s emerging in my teaching but also to what’s already developed and become the roots of my mathematics teaching. Had I not realized this, any change that I might make would most likely be superficial because I hadn’t considered the whole of my teaching — I hadn’t dwelt long enough to notice what was there. So like Joel mentioned, my ecological ways of being would be at best, add-ons.

Okay, just a minute. You figured out that you needed to look at these aspects of your teaching but you still hadn’t figured out what you needed to be noticing, right?

Exactly. More dwelling. What I did know was that by exploring my teaching in these ways, I might be able to articulate why I was so uneasy about the sense of place I was creating with my students in the mathematics classroom... why it didn’t feel right. But still I had no idea of what specifically I in my teaching I should examine.
So what did you do?

Well, this time, I took the whole of my mathematics teaching back to where I'd been — you know, into the space of language and languaging.

Then what?

As I immersed my thinking into theoretical literature on language and the pervasive nature of language, I began by asking myself, so how do these concepts inform my understanding of teaching mathematics?

And... ?

Certain key ideas began to pool together. In fact, some were direct quotations from the books and articles I'd been reading.

Such as?

Well, like:

Language THINKS US as we think with and in language.  
- Martin Heidegger

"METAPHOR is not a mere embellishment; it is the basic means by which abstract thought is made possible."
- George Lakoff and Rafael Núñez

The map IS NOT the territory.  
- Alfred Korzybski

"...[language] does much more than construct concepts about the events and things in the world; it TELLS us what sorts of concepts we ought to construct."
- Neil Postman

And these ideas pooled together because they were all...
They all had to do with the metaphorical nature of how we think.

So it was just these specific authors’ works?

Yes and no. It was these four ideas that kept making their way into my mind-space of language and language but these weren’t the only ideas I was thinking with. They’d emerged from other authors’ work such as Sfard, Abram, Capra, M. C. Bateson, van Manen, Jardine, Orr, Maturana, Varela, Lakoff and Johnson... but yes, it was these specific meta-voices that really helped to pinpoint my position of noticing — so that I could begin to examine my mathematics teaching.

And so what was your position or perspective of noticing? Can you explain it to me?

Let me see. Well for starters, it was directed towards the way in which metaphors become embodied in our forms of knowing, our actions, and our identities. Remember when we were talking about language?

Yes.

By metaphorical language I mean, the spoken, written, spatial — how phenomena are portrayed to exist, temporal — how time is conceptualized, and symbolic forms of communication that distinctively structure one’s teaching. So briefly, there it is. My point of noticing was to examine the metaphorical furnishings, if you will, of teaching mathematics and directly related to this, the kind of place that I was bringing forth in the classroom.

Do you think that metaphorical forms of communication really impact one’s mathematics teaching in such a profound manner?

Yes I do. Just take a moment to think about it: Theoretical FOUNDATIONS, instructional UNITS, conceptual FRAMEWORKS, learning JIGSAWS, cognitive STRUCTURES, SCAFFOLDING, BUILDING knowledge, and so on. Metaphors. We are constantly reading them, hearing them, using them, and thinking with them. So usual they come to be just as Joel wrote — permanent fixtures in one’s mind and over time, we no longer notice their presence. Unquestioned, these metaphors become embedded in our taken for granted language, language used to conceptualize mathematics teaching and
learning — language that directly impacted my teaching, my students’ learning, and the kind of mathematics that emerged.

Wait a minute. Go back to what you just said... are you saying that metaphors were affecting the kind of mathematics that emerged? Surely, math is math. Metaphors have nothing to do with the nature of mathematics.

Oh, but they do. My previous mind-space predetermined the mathematics content that was to be taught and learned. Because of this, I wasn’t aware of or didn’t pay any attention to emergent kinds of mathematics, integrative mathematics, or individual-within-collective mathematics.

So what?

Well, it does matter. If you’re not aware of how metaphorical language is thinking you as you think with and in it, it’s difficult to understand how it affects our thoughts and how it becomes an enabling and disabling feature in our ways of being.

Okay. So make sense of it for me at the classroom level. I still want to know what surfaced for you when you interrogated your teaching. Let’s go back to my original question, what came out? What surfaced as a result of your dwelling in this theoretical realm and in this other space of yours?

Well, what came to the surface of my consciousness were metaphors that carried with them very vivid meanings of how I thought, taught, and identified my role in the mathematics classroom. For the first time, I began to understand how unconscious these metaphors were.

What do you mean?

Well, because I hadn’t realized how pervasive and taken for granted these were, the metaphors existed below the surface of my consciousness. They were definitely there but up until this point, I didn’t have a theoretical way to examine my teaching and so, it was impossible for me to notice these metaphors. They were simply matter-of-fact ways of conceptualizing and enacting my teaching.
TRADITIONS, TENSIONS, AND TRANSITIONS

For example, how you mechanistically separated and organized the mathematics content for each grade level into teachable units of instruction?

Yes.

And so it was these embedded metaphors that you began to notice rising to the surface?

Exactly. And how it happened was just as I described for you at the start of our conversation — like bubbles making their way to the surface of the water and punctuated by them bursting as soon as I spotted them. What I also learned as I searched to identify my taken for granted metaphors was that they arose from the theoretical languages of my undergraduate mathematics education and Ministry documents, teacher texts, and mathematics literature I was working with. You see, it was here that the relationship between my activities in reading and writing and how I envisioned my work in the classroom... that is, planning, teaching, and assessing children’s understanding of mathematics became clear. Almost instantly, the metaphors that’d been totally invisible were now so obvious. Because they were visible, I could see the metaphorical images embedded in everything from the way I imagined the mathematics class to my conceptualization of mathematics curricula.

What kinds of metaphors? Give me some examples.
The **AUTONOMOUS** child

**COMMUNICATING**

**PROBLEM SOLVING**

**REASONING**

**MATHMATICAl POWER**

**REPRESENTATIONS**

**INSTRUMENTAL understanding**

**RELATIONAL understanding**

**FORMAL understanding**

**REALITY IS CONSTRUCTED** by the individual

**COMMUNICATING**

**PROBLEM SOLVING**

**REASONING**

**MATHMATICAl POWER**

**REPRESENTATIONS**

**INSTRUMENTAL understanding**

**RELATIONAL understanding**

**FORMAL understanding**

**PROCEdural understanding**

**CONCEPTUAL understanding**

**SAFOfOLDING**

Stage 3 **Symbolic Language**

Stage 2 **Concrete-Representational Language**

Stage 1 **Concrete-Representational Language**

Stage 0 **Concrete-Representational Language**

(based on Resnick, 1989)
Okay, for instance, take the mathematics class. As I imagined it, the class was composed of the students and myself as autonomous individuals. Anything outside the individual was considered to be part of the external environment.

Mathematics was connected yet fixed. It itself existed as a connected yet fixed body of knowledge. It was made up of separate strands of algebra, geometry, numbers and operations, measurement and so on.

I envisioned mathematical understanding to develop in a uni-directional and hierarchical manner: beginning at an informal, concrete stage and then moving towards higher levels of more formal, abstract stages of understanding.
Based on the premise that knowledge was about building structures, teaching mathematics for me became an activity of directing my students' thinking towards predetermined learning outcomes in regards to what they should know and facilitating the ways in which they should construct such knowledge.

**teaching mathematics**

MOVING STUDENTS ON A COURSE towards PREDETERMINED learning OUTCOMES

**curricula**

jigsaw puzzles

Mathematical skills

Mathematical concepts

And as well, curricula were jigsaw puzzles for teachers to assemble by piecing together concepts and skills set out by the B.C. Ministry and other standard mathematics documents.
I agree that these are very strong metaphors... mechanistic, linear, and hierarchical ones to be sure. But did they really affect the way you taught mathematics? And if so, how?

Having made them visible, I asked myself this same question: Were these metaphors simply figures of speech or were they more than that? Were these metaphors truly powerful forms of language? Language that was not only shaping how I perceived mathematics teaching and learning to be but also, profoundly impacting how such events came to be. So I turned my attention — a little apprehensively, I must admit — towards examining if and how, these metaphors I had identified existed in my forms of teaching.

And...?!

Well, rather abruptly I came face to face with the notion that all knowing really is doing and all doing really is being. What's more, how unthinkingly natural it all is.

You see, because I viewed mathematics to be a connected yet fixed body of knowledge and curricula were puzzles to be assembled, my goal in creating an integrated mathematics program was to connect the different pieces of mathematics together to produce a logical and coherent picture for the students. In thinking so, it made sense for me to insert their lessons in — in a piecemeal fashion for an hour each day between recess and lunch. And in keeping with the view that mathematical learning was sequential and hierarchical, because I taught a multi-age class, it made it necessary to sort the students according to their grade level and teach two different lessons. My role as their teacher was to guide each student's learning in a manner that enabled the student to construct sturdy frameworks of understanding; ones that began with concrete foundations of experiential knowings upon which more formal, symbolic representations were built. I even remember being asked on several occasions as to how I defined myself as teacher.

And, how did you describe yourself?

I was the children's facilitator... the initiator of learning opportunities or to stay with the metaphors, the provider of building materials.

After all of this, what was your reaction in realizing that your teaching was indeed enactings of the mechanistic, linear, and hierarchical metaphors that you were thinking with?
If you’d asked me this question before finding this all out, I would’ve said that I’d probably be shocked, disappointed... even horrified if I discovered that my teaching contradicted the ecological perspective that I thought I was embracing.

What do you mean? You weren’t shocked, disappointed, or horrified? Really, I’d think that burying one’s head in the sand, so to speak, would be a common reaction.

Strangely enough, it was more of an affirmation... finally being able to recognize the metaphors that had existed for so long beneath the surface of my consciousness and then to see the embodiment of them in my teaching — relief through affirmation... yes, that’s what it was. For some time, I’d had a hunch, a gut feeling, that the connected sense of place I was trying to create in the math classroom didn’t exist... but I couldn’t put my finger on it as to why.

But you had, had you not? You had identified rituals in your teaching that were linear, mechanistic, and hierarchical?

Sure, I was able to point to teaching actions that I’d unthinkingly inherited and see them as problematic... such as planning a program by dividing and ordering the mathematical concepts and skills for each grade into a September through June sequence. But doing so only indicated forms of teaching I deemed as undesirable. It still didn’t provide me with any kind of understanding as to what was giving rise to them or how I might go about creating a more ecological sense of place for my teaching and my students’ learning.

Yes, that’s right.

It was only when I moved deeper into my teaching and examined my metaphors... Hmm... how can I describe the process to you... For me, this process was very much like, fingerling the contaminated wound? — explicating my metaphors and then watching them fester — how the metaphors were being enacted in my mathematics teaching. Yes, that’s an accurate image.

How horrible that sounds!

Well, yes. The image impreses my experience as incredibly uncomfortable — even painful. And in some ways, it was. Finding out that you’re doing exactly the opposite of what you wish to do is definitely distressing and
uncomfortable but at the same time and in a different way, I by no means considered the study of teaching as pathologic.

Although that’s how I think most people would interpret your description.

I know... but no. To come to this place in my thinking was critical. The uncomfortableness of it all wasn’t a prompt for identifying and remedying a problem so much as it was an opening for me to arrive at a new place of understanding. You see, I considered the dis-ease of these events to be integral and vital to my growth in teaching mathematics. It was because of dwelling in this mind-space that I was able to grab hold of what I could before only express as a hunch or a gut feeling and now, I was able to put words to it and finally say, here it is!

for sure, making the invisible visible. When you describe your metaphors and explain how they gave rise to your forms of teaching, it clarifies the point you’ve been trying to make; that the metaphors and metaphorical patterns with which a person thinks has everything to do with that person’s teaching of mathematics and the sense of place that’s created in the classroom. What also becomes clear is that even though you wanted to create a connected sense of place through creating an integrated mathematics program, the metaphors you unconsciously rooted in your mind critically disabled the possibility for an organic or ecological kind of integrative mathematics to come about. The metaphors only allowed for mechanical piecemeal forms of teaching and learning — definitely not those that are dynamic, flowing, or unpredictably open. It’s exactly as you expressed earlier in our last conversation — that teaching learners mathematics and learning what it means to teach mathematics really do flow together. That all said and going back to Joel’s question, now that you had figured out what furniture didn’t suit your ecological living space,
how did you go about finding furnishings that might?
Deeper Yet

With her enactments of deeply rooted metaphors now exposed, Jennifer realized that not only were her conceptions and her mathematics teaching necessary parts of each other, they were in fact, each other. It was impossible for her to consider metaphors as merely figures of theoretical speech. She now understood the invisible yet ever-present manner in which these metaphors specified her patterns of thinking and her teaching of mathematics. In a very real way, the metaphors and metaphorical manners were Jennifer's rituals for place-making in the classroom and awkwardly, she knew that her taken for granted ways of teaching mathematics did not engender the ecological forms she wished to enact.

Settled into this other conceptual space, Jennifer's new understandings brought about another realization; that in the moments of making sense of the constraints of her metaphors and knowing that she wanted to enact ecologically coherent ones, she could not simply change by exchanging what she was thinking or doing in the classroom for something else.

AGAIN, I am reminded that learning what it means to teach mathematics is not an automatic process. It is not smooth, it is not straightforward, and it certainly does not appear on demand.

KEEPING WATCH WHILE DWELLING REQUIRES PATIENCE.

And before she could re-imagine metaphors that were ecologically sound and work towards rooting them into her classroom praxis, Jennifer knew that she needed to exercise a mindful kind of patience in order to move even deeper into this other space so that she could question, assess, and provoke shifts in her thinking. It was here in the act of dwelling that Jennifer found herself wondering about
conceptions of mathematics, mathematical understanding, and the different ways in which they are brought into being.