In this book, the reader is invited to enter a strange world in which you can tell the age of the captain by counting the animals on his ship, where runners do not get tired, and where water gets hotter when you add it to other water. It is the world of a curious genre, known as "word problems" or "story problems". It originated in the ancient civilizations of Egypt, China, and India, and is the subject of daily rituals among students and teachers in mathematics classrooms all around the world. An international group of scholars with a shared interest in this phenomenon explore multiple aspects of this world from multiple perspectives. These discussions take us deep into philosophical issues of the relationships between words, mathematical systems, and the physical and social worlds we all inhabit. Empirical investigations are reported that throw light on how students and their teachers experience and interpret this activity, raising profound questions about the nature and purposes of mathematics teaching/learning in general and how it could be improved.
Words and Worlds
Words and Worlds

Modelling Verbal Descriptions of Situations

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PREFACE

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INTRODUCTION

Making Sense of Word Problems: Past, Present, and Future

PAST

In 1992, the first two editors attended a workshop on arithmetic at the Max-Planck Institute in Munich and began a conversation on word problems that has continued to this day. With our colleagues, we did a considerable amount of research and analysis about arithmetic, particularly on the conceptual fields of additive structures (e.g., Verschaffel & De Corte, 1993, 1997) and multiplicative structures (e.g., Greer, 1992). Much of that work was framed in terms of the situations for which the four basic arithmetic operations provide models (Greer, 1987; Verschaffel & De Corte, 1993, 1997). At the meeting in Munich, we were becoming aware of many compelling, and largely independent, observations with the common theme that they apparently showed students in mathematics classes abandoning their sense-making capabilities, and in particular, carrying out arithmetic calculations that did not make sense in relation to the situations described. Our reaction was to embark upon an extended investigation of the phenomenon, continued in this book.

Of course, these were not original ideas. For example, in The Psychology of Arithmetic, Thorndike (1922, pp. 100-101) listed a number of examples of what he termed “ambiguities and falsities”, including the following (with his comment):

6. If a horse trots 10 miles in one hour how far will he travel in 9 hours?

7. If a girl can pick 3 quarts of berries in 1 hour how many quarts can she pick in 3 hours?

(These last two, with a teacher insisting on the 90 and 9, might well deprive a matter-of-fact boy [sic] of respect for arithmetic for weeks thereafter.)

Even earlier, the mathematician Charles Dodgson, better known by his pseudonym Lewis Carroll, composed an enlightening satire on typical “rule-of-three” problems, by analysing this example:

If 6 cats kill 6 rats in 6 minutes, how many will be needed to kill 100 rats in 50 minutes? (Fisher, 1975; and see discussion in Verschaffel, Greer, & De Corte [2000, pp. 132–134]).

In his analysis, humorously phrased yet making a very serious point, Dodgson takes a modelling perspective and shows how, in distinction from routine application of
“rule-of-three” procedures, the answer to the question depends on the assumptions made (for similar points, see Säljö, 1991).

The most startling body of work was inspired by a letter satirising mathematics, from the novelist Gustave Flaubert, in which he wrote:

Since you are now studying geometry and trigonometry, I will give you a problem: A ship sails the ocean. It left Boston with a cargo of wool. It grosses 200 tons. It is bound for Le Havre. The mainmast is broken, the cabin boy is on deck, there are 12 passengers aboard, the wind is blowing East-North-East, the clock points to a quarter past three in the afternoon. It is the month of May. How old is the captain? (Cited in Wells [1997, p. 234])

Research carried out in France and elsewhere used a simplified version:

On a boat there are 20 sheep and 6 goats. How old is the captain?

and other questions of the same general nature. The findings, that many children would respond to such questions by, for example, adding the number of sheep and goats, became a cause célèbre in France (Baruk, 1985) and considerable further research has attempted to explicate the children’s behaviour (e.g., Selter, 1994; this volume).

However, our interest focused on a distinctly different kind of case, where the question asked was not nonsensical in the style of “How old is the captain?” but, rather, students’ responses were (to us) failing to make sense because of unthinkingly applying arithmetical operations suggested by the situation described, under the apparent assumption that the situation could be unproblematically mapped onto these operations. Thus, in Greer (1993) and Verschaffel, De Corte, and Lasure (1994), we gave students (respectively, 13-14 years old and 10-11 years old) pencil-and-paper tests consisting of matched pairs of word problems. In each pair, one problem (called a standard problem, or S-problem) was such that the application of an arithmetical operation was reasonable (in our judgment); the other problem (called a problematic problem, or P-problem) required consideration of more subtle aspects of the situation described.

For instance, in the study by Verschaffel et al. (1994) the following S-item and P-item were paired:

Pete organised a birthday party for his tenth birthday. He invited 8 boy friends and 4 girl friends. How many friends did Pete invite for his birthday party?

Carl has 5 friends and Georges has 6 friends. Carl and Georges decide to give a party together. They invite all their friends. All friends are present. How many friends are there at the party?

In Verschaffel et al. (1994) ten such pairs of items were used. Responses were categorised as “realistic” either if the answer given implied some awareness of the calculation not being straightforward or if a comment accompanying a straightforward, non-realistic answer implied such awareness. Results indicated that S-problems are overwhelmingly answered appropriately, whereas percentages of
“realistic responses” (as defined above) are uniformly low (almost all in the range 5-20%).

Replications, using translations of essentially the same items (translated as necessary), have been carried out in many countries, including Belgium, China, Germany, Hungary, Japan, Northern Ireland, Switzerland, The Netherlands, and Venezuela (for an overview of these replication studies, see Verschaffel et al., 2000; see also Xin, this volume). The findings were strikingly consistent with the initial results obtained by Greer (1993) and Verschaffel et al. (1994), sometimes to the great surprise and chagrin of these other researchers who had anticipated that the “disastrous” picture of the Northern Irish and Flemish pupils would not apply to their students. While variations on the accompanying instructions have been used in the various replications, the minimal indication that P-problems might be indeed problematic has been a warning to this effect, and the respondents have been invited to comment on anything unusual they noticed about the item.

Besides replication work, other aspects were taken up by a number of collaborators, for example the contributors to a special issue of *Learning and Instruction* (Greer & Verschaffel, 1997). These perspectives included the nature of the task setting and how it is presented (Reusser & Stebler, 1997), teachers’ conceptions and beliefs about word problems (Verschaffel, De Corte, & Borghart, 1997), and sociolinguistic analysis (Wyndhamn & Säljö, 1997). A related body of work was developed in another ongoing research program centred in Leuven, probing the phenomenon of inappropriately assuming linearity, a pattern that is found at all ages and across many branches of mathematics (De Bock, Verschaffel, & Janssens, 1998; Van Dooren, De Bock, Hessels, Janssens, & Verschaffel, 2005; Van Dooren & Greer, in press). Some of the P-problems developed by us and others, and others such as those from Thorndike (1922) cited earlier, illustrate the making of answers that apparently assume linearity in cases where it is not reasonable.

The interim culmination of this activity was the book *Making Sense of Word Problems* (Verschaffel et al., 2000). We began this book by summarising the many examples showing that children frequently manifest what looks, at first sight, to be “suspension of sense-making” when solving school word problems (Schoenfeld, 1991). At second sight, and in response to interviews of students by ourselves and others, we came to agree with Schoenfeld, who stated that:

> Taking the stance of the Western Rationalist trained in mathematics, I characterized student behavior … as a violation of sense-making. As I have been admonished, however, such behavior is sense-making of the deepest kind. In the context of schooling, such behavior represents the construction of a set of behaviors that results in praise for good performance, minimal conflict, fitting in socially etc. What could be more sensible than that? (p. 340)

Accordingly, our attention in Part II of *Making Sense of Word Problems* turned to the nature of mathematical schooling, in general, and to the ways in which word problems are used, in particular. We analysed the effects of traditional mathematics education in this respect, and considered a number of design studies and innovative curricular projects in which attempts were made to use more realistic and challenging
In Part III, we considered philosophical questions about the nature of mathematical modelling, the use of language in and with mathematics, the sociocultural contexts in which mathematical schooling takes place, and we proposed a reconceptualisation of word problems (to be more precise, those word problems that putatively describe real-world scenarios) as “exercises in mathematical modelling”.

At the end of the book, we flagged numerous aspects of (genuine) mathematical modelling only partly addressed in the book, and others needing deeper analysis, many of which are developed further in the present volume, the assembly of which was prompted by our knowledge of many researchers sharing an interest in the curious genre that goes by the term “word problems” or “story problems”.

PRESENT

In this volume, we have gathered together related work from many countries that extends the empirical investigations and philosophical analyses of earlier work. The book is divided into five parts, each with a reaction.

Part I: Theoretical Perspectives

The contributors to the first part develop further a central topic discussed in Verschaffel et al. (2000), namely characterisation of the form of reality that is supposedly evoked in word problems and how that can be modelled (and to what purpose). We may say that there are two aspects of modelling that are implicated. First is the mathematical modelling of aspects of our world, both physical and social, that is pervasive in modern life (Davis & Hersh, 1986; Gellert & Jablonka, 2007; Skovsmose, 2005). Second, there is the transformation of that activity into the particular activity system that is school (Lave, 1992), a process that goes by various names, such as “recontextualisation” (Bernstein, 1990; Gellert & Jablonka, this volume), “didactical transformation” (Chevallard, 1985), and “alchemy” (Popkewitz, 2004). A constant theme throughout the (first part of the) book, and from earlier work, is that the presentation of word problems as routine exercises in mapping descriptions of scenarios onto arithmetical operations is detrimental to the kind of critical understanding of modelling that is needed in contemporary society. Many traditional treatments of word problems in mathematics education have been based on rationalist/empiricist assumptions of the transparency of representations, of one-to-one matchings or mappings of language, numbers, and geometric figures onto learners’ experiences of life – and vice versa.

In the first chapter, Palm’s aim is to systematically analyse the features of school tasks that make them more or less authentic simulations of the corresponding “real” activities. His starting point is that many teachers, textbook writers, and assessment developers are indeed struggling with the development of more “realistic” mathematical school tasks that resemble “out-of-school” task situations. He asks the question: What are the features of more “authentic” word problems, and
how do they affect students’ word problem solving? After discussing the terminology that has been used in the research literature to describe the “realism” of word problems, he presents a comprehensive framework for what might constitute word problems that closely emulate out-of-school task situations and shows how that framework has been used to critically analyse teaching and testing materials. (For an example of such an application, see the chapter by Depaepe, De Corte & Verschaffel in Part III of this volume).

The other two chapters in this part ask profound philosophical questions about the nature of reality relative to descriptions, simulations, and models of it. In Chapter 2, Gerofsky provides a wide-ranging review, drawing on theorists such as Baudrillard, Lyotard, Lacan, Derrida, Zizek, Bakhtin, and McLuhan. She addresses powerful and profound notions of language, simulation, and knowledge in our contemporary techno-cultural environment. Such theorists of postmodernity trouble the sense of a transparent, self-evident reality and its representations in many ways. In particular, Gerofsky adopts this perspective to deconstruct common perceptions that word problems embody simple and transparent representations, simulations, and straightforward applications of mathematical knowledge (or can be so treated for the purposes of teaching mathematics).

Gellert and Jablonka discuss in Chapter 3 the relationships between “words and worlds”, which are represented in mathematical word problems, in terms of relationships between “domains of practice”. These relationships consist in the recontextualisation of out-of-school practices for didactical purposes, including an inescapable relocation and appropriation of the corresponding discourses by pedagogic discourse. The students in the mathematics classroom face these issues as the demands of recognising different forms of discourse and of producing legitimate text. After analysing illustrative examples of the dilemmas that students face when confronted with these types of task, they discuss categories of expression and content of word problems from the perspective of recontextualisation.

The reader will notice, not only in this first part but throughout the whole book a tension – constructive, we believe – between those authors who argue that the detrimental aspects of word problems as currently deployed can be identified and addressed, and those that argue that the endeavour is fundamentally flawed. Whatever the degree of change called for (and there is unanimity on the need for reform of practices around word problems worldwide) there is a corresponding responsibility to propose how the use of word problems in mathematics education – if they are to be retained in some form – could be improved.

In his overview of the three chapters, Roth digs deeper into issues of language, and the nature of the social interaction that takes place when a person, or group of people, are presented with a word problem as a task, whether in school or as part of an experiment. What are the implicit rules, the “didactical contract” in class between teacher and students, or the “experimental contract” between child and researcher? How does someone in this situation construe the nature and purpose of this particular kind of social interaction?

Indeed, by asking these questions, Roth’s critique not only bears on the chapters of the first part but, in profound ways, foreshadows key aspects of the succeeding parts, as will be seen. In particular, from the perspective of cultural-historical activity
theory, he argues that the analysis of what happens around word problems, and of mathematics education in general, must be treated within the activity system that is schooling, and in relation to its sociopolitical effects (see Varenne & McDermott, 1998). Several aspects of this argument are addressed in the next part.

Part II: Sociocultural Perspectives

Word problems are cultural artefacts. As a humorous, but revealing, illustration consider a cartoon by the American humorist Gary Larson. It depicts a man who is clearly in Hades as indicated by the devil and flames shown in the background. He is contemplating bookshelves laden with thick volumes with titles “The big book of word problems”, “Word problems galore”, and so on. The caption for the cartoon reads “Hell’s Library”.

In Verschaffel et al. (2000) we noted that word problems, as a genre, have been around for a very long time, and in many cultures. In Chapter 4, Swetz provides a great deal more detail on their history, showing that word problems appear to have always been a part of the mathematical teaching and learning process. From inscriptions on clay tablets, renderings on bamboo strips and papyrus sheets to the message of the printed page, such problems have historically been a source of mathematical communication, learning, and in many cases, student frustration. In this respect, they supply intellectual footprints of how mathematics was used and valued, but also insights into the social and economic climate of their times. His contribution surveys the construction and employ of word problems in an historical and societal context, thus providing part of what is needed for a cultural-historical perspective, as urged by Roth. Swetz’s review also reminds us that academic mathematics and school mathematics emerged out of a rich diversity of mathematical practices, as particular kinds of mathematical practice (Lave, 1992).

Cooper and Harries (Chapter 5) take up an issue flagged in Verschaffel et al. (2000), namely the differential ways in which currently applied forms of assessment affect diverse students. Greer, Verschaffel, and Mukhopadhyay (2007, p. 96) suggested that:

If a decision is made to mathematize situations and issues that connect with students’ lived experience, then it brings a further commitment to respect the diversity of that experience across genders, classes, and ethnicity.

Following Bernstein (like Gellert & Jablonka), Cooper and his colleagues have conducted an extensive program of research on how social class and gender differences impact performance on standardised assessments. In the chapter here by Cooper and Harries, they employ one particular item from a national annual programme for testing mathematical achievement in the U.K. to illustrate some relations between social class, gender, and validity. The item deals with data about the traffic outside a school, and the key point is whether students attend to the data presented, or base responses on their knowledge of the world. Cooper and Harries draw on interviews with children as they demonstrate how easy it is to modify items so that validity interacts differently with such factors as social class and
gender. A body of sociological analysis, including that of Bernstein and Bourdieu, indicates how the use of such contexts might introduce differential validity by social class into the testing process (as commented by Roth, this volume).

Frankenstein (Chapter 6) starts by addressing concerns about the current push to include real-life mathematics word problems in the curriculum, particularly the non-neutral “hidden curriculum” that results from particular selections of real-life data used to create contrived and/or context-narrow word problems. The focus of her chapter is a discussion of examples, in various categories, of what she terms real real-life math word problems that are studied in a broad enough context so that students can see how understanding numbers and doing calculations can illuminate meaning in real life. From this discussion, she goes on to develop guidelines on how to construct such problems and then generalises some key quantitative understandings that emerge and deepen through working on real real-life mathematics word problems. These understandings underlie the kinds of questions she argues that we need to ask in order to grapple with various economic, political, and social issues.

Boaler approaches the three chapters in this part through the question “Can mathematics problems help with the inequities in the world?”. Current rhetoric emphasises equity in mathematics education, but is it possible to go beyond that and pursue equity through mathematics education by providing students with the tools to critique and act upon issues of importance in their lives, in their communities, and for humankind in general (Gutstein, 2006)?

Each of the three chapters, in different ways, bears on the question posed by Boaler. She illustrates how the plentiful examples from many cultures and times furnished by Swetz illuminate aspects of inequity in society, for example in relation to gender. Drawing on her own experience, Boaler cautions about the challenges and difficulties of following Freire’s dictum that education is politics, since (p. 134) “such examples make many students and teachers extremely uncomfortable, if not angry”. Frankenstein’s point, however, is that the supposedly neutral curriculum is inherently political, and that her purpose is to expose that. Boaler has also contributed significantly to the domain of concern addressed by Cooper and Harries, in particular in relation to how female students are more likely to engage with contextualised questions (e.g., Boaler, 1994). While reminding us that equity can also be pursued through the ways in which classroom relationships and discourse are guided, she concludes that the authors in this part do indeed show ways in which mathematics education could contribute to a more equitable society.

Part III: Probing Students’ Conceptions

In this part and the next, studies are reported that investigate how students and teachers, respectively, approach word problems. The methodologies used include not only pencil-and-paper tests, but go beyond these (as is essential for deeper understanding) to activities such as solving and posing problems in small groups and individual interviews.
Verschaffel, Van Dooren, Chen, and Stessens (Chapter 7) address the relationship between solving and posing realistic word problems, specifically problems involving division with remainder (DWR), i.e., where the appropriate “core calculation” is division of one positive number by another that does not divide exactly, and where the appropriate interpretation of the result of this calculation is an issue. One of the early observations that piqued our interest was reported by Davis (1989). In a lesson introducing division, students were given five balloons to be shared between two people. One boy cut a balloon in half, leading to the question raised by Davis (1989, p. 144): “Was this boy really thinking about solving the actual problem … or was he trying to accommodate himself to the … culture of the American school?”. This situation was adapted for matched S- and P-items in Greer (1993) (with slightly different versions in Verschaffel et al. [1994]):

If there are 14 pizzas for 4 children at a party, how should they be shared out?

If there are 14 balloons for 4 children at a party, how should they be shared out?

Among the P-items used in the study by Verschaffel et al. (1994) and its replications, this one has generally elicited a high percentage of realistic responses (50 to 80% “realistic reactions”), but in a fascinating twist, a Community College student gave the answer “4”, but added “the answer is really 4.5, but you can’t have half a balloon” (Greer, Verschaffel, & De Corte, 2002, p. 278, emphasis added).

In the study reported here by Verschaffel et al., Flemish students in grades 4–6 were given both problem-solving and problem-posing tasks relating to DWR problems. The quantitative and qualitative analyses of the results reveal that children from all age groups showed non-realistic perspectives in their problem-posing as well as in their problem-solving endeavours (with the results for problem posing being considerably weaker), that both the problem-solving and the problem-posing skills improved with pupils’ age and mathematical ability level, and that there was a significant positive relationship between children’s abilities to pose and solve realistic problems.

In Chapter 8, Xin addresses the question how Chinese students perform on P-items by reviewing and discussing the available research, especially his own research, on Chinese students’ realistic problem solving and how it is influenced by prevailing and alternative instructional settings. His conclusion is that, like their Western counterparts, Chinese pupils generally do not take real-world considerations into account when responding to P-problems. He invokes the theoretical construct of “cognitive holding power”, as developed by Stevenson and Evans (1994), who distinguish between settings possessing first and second order cognitive holding power. The former pushes students towards the direct acquisition and utilisation of procedures. In such an environment, students merely follow a teacher in learning how to accomplish specific tasks. Settings possessing second order cognitive holding power push a student into the utilisation of procedures that achieve more general purposes by operating on specific procedures to enable the interpretation of new situations, solving of problems, and learning of new skills. Such settings poses unfamiliar goals, and elicit the execution of second order
procedures. On the basis of the research and his knowledge of Chinese mathematics education, Xin suggests that, in order to promote students’ realistic problem solving, a series of measures should be taken, including supplementing P-problems in daily teaching and examinations, designing effective instructional approaches, and establishing learning settings with high second order cognitive holding power.

Whereas the first two chapters from Part III presented studies using pencil-and-paper tests, it is essential to probe further by interviewing, and otherwise interacting directly with, students, and the other two chapters in this part take this approach.

In the study by Säljö, Backlund, Riesbeck, and Wyndham (Chapter 9) the issue of realistic modelling has been investigated in the context of how upper elementary school pupils, working in groups of three, struggled to model a “realistic” word-problem about fair sharing in order to come up with an answer. The results show that the groups use different models, some of which are quite complex, but, in spite of this, their reasoning is not successful because they are hindered by the assumption that there is one mathematical model that can provide an exact numerical answer to the problem. It is argued that solving this kind of problem requires learning to reason at a meta-level about the situation and the mathematical model. It is interesting that this meta-level reasoning is conspicuously absent in the students’ problem-solving activities, even among those who are very good at mathematics (as judged by the usual criteria). What this study points to is an alternative perspective on human knowing and skills in which the discursive nature of our knowledge is emphasised.

Inoue (Chapter 10) questions previous interpretations of students’ responses as representing unrealistic answers to real-world problems with the assumption that the students had not seriously considered familiar aspects of reality. His study used interviews to afford the students the opportunity to provide justifications for responses on paper-and-pencil tasks that would be categorised as “non-realistic” by the criteria adopted in earlier studies. In contrast to a rather similar approach by Selter (1994, this volume), who worked with young children, Inoue interviewed university students. He suggests that examining a variety of “unrealistic” responses in mathematics classrooms could serve as an invaluable opportunity for us to discover effective ways of filling the gap between the theoretical and applied approaches to mathematical problem solving. According to Inoue, considering different justifications of the calculational answers and examining different sets of assumptions for solving word problems can provide rich opportunities for students to learn how to use their mathematical knowledge beyond school-based problem solving. This approach could help the students conceptualise word problem solving in terms of meaningful assumptions and conditions for modelling reality, rather than the assumptions imposed by textbooks, teachers, or other authority figures.

In her review of these four chapters, Leung stresses the limitations of ascertaining experiments that reveal and document failings in the mathematical performances of students (at least as viewed from the perspectives of the researchers). In general, she makes the point that students cannot be expected to do things spontaneously, such as problem posing, for which they have not been instructionally prepared (see Hatano, 1997, for similar comments). Thus, to interpret more fully how students behave, it is essential to have some knowledge of their instructional
histories. As Leung points out, moreover, the nature of the interaction in all four studies was, to a greater or lesser extent, likely to cue the students towards playing the “word problem game” (De Corte & Verschaffel, 1985; Verschaffel et al., 2000). Consequently, there is a need for intervention studies, now often called design experiments, to study the effects of prolonged exposure to a different pedagogy (such as Leung’s own work on classroom interventions to promote problem posing). Many such efforts were reviewed in Verschaffel et al. (2000), and others are reported in Part V of this book. Leung also suggests an enhancement of Polya’s (1945) model of phases in problem solving to take account of points in the process at which realistic considerations might lead the solver to reconsider and backtrack to an earlier phase. She also argues that there are important differences between well-structured and ill-structured problems, a distinction that is very relevant to the distinction between S-problems and P-problems.

Part IV: Probing Teachers’ Conceptions

Much research has examined the nature of word problems in published mathematical curricula and student responses to word problems as well as to more complex, authentic mathematical modelling problems. Less attention has been paid to how teachers think about and attempt to implement connections between school mathematics and the real world. Attention to teachers is of fundamental importance, however, because what teachers think and do essentially governs whether and how students will encounter real-world connections for the mathematics they learn in school. More generally, teachers are crucial in any attempt at innovation (Atweh, 2008). Moreover, as argued by Leung (this volume), a student’s instructional history influences profoundly how she/he will respond.

It is generally agreed that characterising word problems as exercises in modelling, necessarily involving flexible as opposed to merely routine expertise (Hatano, 2003), requires heavy intellectual investment (Hatano, 1997). It is also generally accepted that trying to persuade teachers to change their practices radically carries major responsibilities for engaging them as partners in the process (Atweh, 2008). Consequently, extended studies of teachers engaged in such processes are essential.

According to Chapman (Chapter 11), there is little research available on experienced mathematics teachers and their conceptions and use of contextual problems in their teaching. Understanding the nature of teachers’ conceptions and the relationship between these conceptions and their teaching could enhance our understanding of instruction and of the types of professional development that could help teachers to change or improve their thinking and use of contextual problems in their teaching. Chapman’s research indicates that teachers may conceptualise contextual problems in terms of computation, text, object, experience, problem, and tool. She further suggests that a teacher may hold one of three contrasting philosophical positions, that she terms objectivist, utilitarian, and humanistic. On the basis of in-depth interviews, she suggests how these conceptualisations and philosophical positions, and other factors such as expertise and experience, may relate to teachers’ pedagogical styles.
Depaepe, De Corte, and Verschaffel present in Chapter 12 an in-depth analysis of the word problems in a textbook series that is widely used in Flanders. Moreover, based on a seven-month observation period they investigated which problems from the textbook were used by two teachers during that period and which self-generated problems they added. They used the theoretical framework proposed by Palm (2002; this volume). Overall, both textbook-developed and teacher-developed materials seem to simulate quite well a number of aspects that are assumed to be important in designing realistic tasks, but for other aspects of Palm’s scheme, the results are considerably less positive.

In Chapter 13, Gainsburg reports on a study investigating mathematics teachers’ decisions about real-world connections (RWCs) in their teaching – how they use connections, why, and the factors that influence that use. Building on a prior, exploratory, study (Gainsburg, 2008), Gainsburg’s new study included responses of mathematics teachers to a questionnaire, observations of their lessons, and follow-up interviews. The findings are discussed in terms of frequency of classroom use of types of RWCs, teachers’ reasons for use and non-use of RWCs (and resources supporting use), differentiation of use of RWCs across types of student or class, teacher beliefs about the pedagogical power of RWCs, and the impact of school and district contexts on use of RWCs. Her findings may inform future research and teacher-education efforts concerning real-world connections in mathematics teaching.

As Ponte points out in his overview of this part, the ways in which teachers do or do not incorporate RWCs in their teaching reflect, on the one hand, their beliefs, conceptions, knowledge, and “craft knowledge” and, on the other hand, the constraints under which they operate, in terms of curriculum, textbooks, assessment demands, school practices, and so on. Indeed, the factors influencing what a teacher does are situated in the classroom, the school, the educational system, and the culture at large (see Greer et al., 2002, p. 285).

Ponte notes that educators and researchers, under the broad mantle of constructivism, have urged fundamental changes in the role of the teacher, from deliverer of knowledge to facilitator of learning. The extent to which this has actually happened is both limited and highly variable (and this variation needs to be considered in any attempt to generalise findings).

Part V: Changing Classrooms

In order to change in the direction broadly agreed upon, necessary steps include engaging in the ideological argument, demonstrating results through exemplary initiatives, and systemic change. In Verschaffel et al. (2000), several systematic attempts to design and implement learning environments were described and, in this book in general, and this final part in particular, the authors make suggestions about what might be changed, and how.

Bonotto (Chapter 14) analyses a number of teaching experiments consisting of a series of classroom activities in upper elementary school, using suitable cultural artefacts, interactive teaching methods, and aiming to create and sustain new
sociomathematical norms in order to create a substantially modified teaching/learning environment. The focus is on fostering a mindful approach toward realistic mathematical modelling, problem solving, and also a problem-posing attitude. Based on results obtained in these teaching experiments, Bonotto argues that it is not only possible but also desirable, even at the primary school level, to introduce early on fundamental ideas about modelling, especially “emergent modelling”. She also argues for modelling as a means of recognising the potential of mathematics as a critical tool to interpret and understand reality, the communities children live in, and society in general.

Chapter 15 by Selter is a theoretically founded description of several activities that he and his team developed in order to encourage students to reflect on (solving) word problems. It is the main goal of this approach to teaching to give children more responsibility and to see them as experts for their own learning. By constantly using their own productions, their reflection on word problems was encouraged in classroom discourse and small-group work – more and more independently from the teacher, and not always as expected. His experiences lead him to the conclusion that it is a realistic option to gradually give children freedom that they can use effectively, even if a difficult topic such as word problems has to be tackled – sometimes with surprising results. This approach exemplifies an idea suggested by Greer (2009) that, as well as talking about mathematics in classrooms in order to try to “enter the child’s mind” (Ginsburg, 1997), educators and researchers should share with students what we know, or think we know, about the nature of learning and teaching mathematics. In the case of Selter, children become partners in problematising the genre of word problems.

The final chapter by Lesh and Caylor greatly enlarges the scope of the discussion, by placing the critique of word problems in the context of inquiry activities in science education, case studies and problem-based activities in professional schools that are heavy users of mathematics, science, and technology, and model-eliciting activities from Lesh’s extensive research program. According to the authors, it is productive to try to tease out distinctions, not in order to criticise one type of activity compared with another, but to identify testable hypotheses that may advance thinking about each. They end their chapter by arguing that in both mathematics and science education, as well as in fields where mathematical thinking is used, a great deal more research and development will be needed to keep pace with changes that are taking place beyond schools and in modern learning sciences, and by providing a list of trends whose implications need to be investigated in research in mathematics and science education.

In her overview, English draws attention to the widespread calls for mathematical activities to be more relevant and meaningful. Likewise she points to the need for people to be able to engage with more complexity in terms of both interpersonal communication and co-operation and in relation to technological advances.

There has been considerable attention paid in cognitive psychology and mathematics education for many decades now to problem solving. English comments on the generally disappointing results in terms of building on Polya’s analysis to improve students’ problem solving. As she points out, there is a need to clarify the various relationships between “word problems”, “problem solving”, “problem
INTRODUCTION

posing”, and “modelling”. Indeed, as Jonassen (2004) commented, problems (ironically) are caused by the use of the word “problem” to mean different things. Along with Lesh and his colleagues (e.g., Lesh & Zawojewski, 2007), English (e.g., 2006, in press) has worked towards a more integrated approach that theoretically relates problem solving, in the tradition of Polya, with mathematical modelling, with heavy emphasis on ill-structured problems. One suggestion, prompted by such work, is that beyond routine and adaptive expertise, as characterised by Hatano (2003) there is need for an even more advanced form of expertise, namely “creative expertise” which is needed to pose and solve ill-defined problems within complex situations – and English proposes a long list of ways in which classroom experiences could support the development of such expertise.

FUTURE

Given the severe criticisms of standard school word problems, why do we think it is still important to study and analyse them, and devote this book to the topic? Perhaps the most important reason is that, as is generally the case, there is a large gap between the enlightened state of researchers and scholars (or at least what they take to be enlightened) and practice in actual classrooms, in textbooks, tests, and in public discourse. Further, the problematising of word problems does not rule out the possibility that their faults could be rectified and their virtues retained – throwing out the bathwater while keeping the baby, so to speak. In particular, many propose that the most effective reform is through assimilating word problems within an approach to mathematics education that puts major stress on modelling.

In any case, this book shows clearly how many issues are thrown up by looking at word problems in mathematics classrooms and the practices around them, but by no means exhausts the topic. We believe that word problems should continue to engage our interest and research, providing, as they do “a microcosm of theories of learning” (Lave, 1992, p. 74).

While this volume has extended and deepened many of the issues identified in our previous book, there are many others discussed or identified by the authors that remain to be fully developed. For example, one topic that has received limited attention is affect (Roth, this volume). At the end of Making Sense of Word Problems, Verschaffel et al. (2000, p. 181) stated that:

The evidence laid out in the early chapters of this book affronts our conception of how children should come to regard mathematics and we are not alone in this reaction. Freudenthal (1991, p. 70) commented:

Wouldn’t it be worthwhile investigating whether and why this didactic breeds an anti-mathematical attitude and why the children’s immunity against this mental deformation is so varied?

It is arguable that the complex of practices in which word problems are embedded provides a prototypical example of what is wrong with mathematics education in general. The cost of continuing to teach mathematics, and word
problems in particular, according to current practices may be a population in which the majority of people remain alienated from mathematics.

As is implied in the last-but-one sentence of the above quotation, we take the view that the study of word problems has implications that go beyond word problems as such, and this view is reflected in many of the chapters. Hatano (1997) took a sceptical position about conventional word problems being useful for teaching the modelling perspective and argued that the P-problems used “are too trivial for students to recognise the significance of ‘high-cost’ modelling activity, which requires a great deal of time and effort to perform” (p. 384). Our response is that many important principles of modelling can be introduced through relatively simple word problems (see Usiskin, 2007), the most central being the realisation that it is important to consider whether a description of a scenario is appropriately, approximately, or inappropriately modelled by an arithmetic operation.

Indeed, in several chapters, this discussion goes even deeper, into how we experience and talk about what we call “reality”. Which relates to further questions such as: How does the teaching of modelling in school mathematics relate to the modelling that takes place, increasingly, in our society? Could school mathematics be put to the task of better preparing people for life in a world that is complex and technological, and in which many aspects of our lives are mathematised and demathematised (Gellert & Jablonka, 2007; Skovsmose, 2005)?

Hatano is certainly right, in our view, to stress that teaching in this more complex way carries a heavy cost in terms of extra hard work for teachers, curriculum and textbook writers, and students (Verschaffel, 2002). The cost of not doing so is, potentially, to produce students that believe that mathematics has no connection with reality, but is an activity carried on solely within its own rules. Further, there is the potential of producing adults who are unable to evaluate critically the simplest of mathematical arguments about issues that are important, as argued by Franken-stein (1989; this volume). Skovsmose (2005, p. 140) suggests that, in a world where more and more aspects of our lives are controlled by hidden mathematical models, people may be divided into certain roles, that he labels constructors, operators, consumers, and “the disposable” and he asks (p. 143) “How could mathematics education counteract the tendency to establish groups as ‘disposable’? How could mathematics education help to ensure citizenship (active and critical) for everybody?”.

Making a distinction between “understanding by schema application” and “understanding through comprehension activity” (Hatano, 1996), Hatano (1997, p. 386) makes two suggestions for increasing the return for the high cost of comprehension activity:

First, we can make a problem or its solution critically important for people’s lives. Alternatively, we can establish a culture that enjoys and highly evaluates comprehension activity in the target domain.

Greer et al. (2002, p. 285) asserted that “while it is an appropriate starting point to take a classroom as the unit of analysis, form a wider perspective, students’ and teachers’ beliefs about word problems (as for mathematics in general) are embedded
in the school setting, the educational system, and most broadly in society…” and later conjectured (p. 287) that the highly consistent results obtained in replications in many different countries of the original study by Verschaffel et al. (1994) could be attributed to homogeneity in practices of school mathematics, masking cultural differences in this context.

Hatano makes the (entirely reasonable, in our view) claim that the kind of teaching that we have been advocating “provides a majority of students with an opportunity for enduring modelling activity only after they have developed a social and intellectual atmosphere supporting it” (p. 386). That is precisely the point behind the design experiments reported in Verschaffel et al. (2000), and in the work of many of the authors represented in this book.

To summarise, there are a number of themes running through our collective work on this topic – past, present, and future – that apply, not just to the practices surrounding word problems, but to the teaching of mathematics in general:

– We assume that the child learning mathematics (or anything else) comes with an innate drive for sense-making that should not be violated, and believe that attention should be given in teaching mathematics to make connections with children’s lived experience.

– Teachers of mathematics and others involved in managing this teaching – teachers of teachers, administrators, curriculum developers, test constructors – should not do so mindlessly, “because that is how it has always been done”, rather need to reflect deeply about what they are doing, and why.

– In contrast to typical mathematics teaching in schools, that arguably constitutes a training in simplistic forms of thinking – word problems being an extremely potent case in point – an understanding of the very idea that mathematics can be used to model aspects of reality, and that this process is complex, and has many limitations and dangers, is essential to effective and responsible citizenship.

REFERENCES


INTRODUCTION


Van Dooren, W., & Greer, B. (Eds.) (in press). Students’ behaviour in linear and non-linear situations [Special issue]. *Mathematical Thinking and Learning*.


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PART I: THEORETICAL PERSPECTIVES
1. THEORY OF AUTHENTIC TASK SITUATIONS

INTRODUCTION

This book is about word problems and how students interact with them. Word problems have been defined differently in different publications. Here I will use the definition provided by Verschaffel, Greer, and De Corte (2000) in the preface of their book. They define them as “textual descriptions of situations assumed to be comprehensible to the reader, within which mathematical questions can be contextualised”. They also note that word problems “provide, in convenient form, a possible link between the abstractions of pure mathematics and its applications to real-world phenomena” (p. ix). This is a broad definition that includes pure mathematical tasks “dressed up” in a real-world context that for their solution merely require that the students “undress” these tasks and solve them. It also includes tasks requiring that students be involved in the full mathematical modelling cycle. The “situations” that are described may be of different kinds such as fairy tales, vocational practice, or everyday life. The latter two types can be further divided, which the framework of the OECD’s International Programme for Student Assessment (PISA) does, into tasks with contexts that “have been chosen to make them look superficially like real-world problems” and tasks that someone “in an out-of-school setting is likely to be called upon to address” (OECD, 1999, p. 51).

In many countries, there are requests to link school mathematics more closely to the real world outside mathematics. Such requests are not new but have led to recent curricula and assessment reforms in several countries (Palm, 2005). There are several reasons for the inclusion of applications and modelling (including word problems) in mathematics education (for a review of the arguments presented in the literature on mathematics education, see Blum & Niss, 1991). The reasons include the possibilities of the use of applications and modelling to (1) facilitate the learning of necessary skills for being able to use (and critically examine the use of) mathematics outside the mathematics classroom, and (2) facilitate the development of an experience of school mathematics as useful and powerful for solving meaningful task situations in life outside the mathematics classroom. The latter could provide motivation for, and establish relevance of, mathematical studies as well as facilitate the development of a comprehensive picture of mathematics that includes applications and modelling. That students obtain such a picture of school mathematics is a goal expressed in different curriculum documents from professional organisations and national ministries (e.g., Finnish National Board of Education, 2004; NCTM, 2000, p. 279).

To attain the goals inherent in such reasons, many researchers have argued that “realistic” or “authentic” word problems be included in the set of word problems
the students are given. Burton (1993) argues that: “[…] realistic problems are equally important to ensuring that learners perceive that mathematics does contribute to working at and resolving issues of living” (Burton, 1993, p. 12). Niss (1992) describes “an authentic extra-mathematical situation as one which is embedded in a true existing practice or subject area outside mathematics, and which deals with objects, phenomena, issues, or problems that are genuine to that area and are recognised as such by people working in it” (p. 353). PISA does not preclude the inclusion of virtual contexts but its authors emphasise the use of real-world problems with authentic contexts describing out-of-school settings and problems that someone in such settings would be called upon to address (OECD, 1999). Not marginalising such tasks may be important since, as Niss suggests, focusing only on “as if” situations, “leads to the perception that applications and modelling is a sort of game […]. Furthermore, it leads to the perception that school mathematics is not powerful enough to treat authentic extra-mathematical situations and problems. And from there it takes only a small step to arrive at the perception that school mathematics is useless.” (Niss, 1992, p. 354).

However, many researchers (and teachers and students) have argued that too many word problems are “pseudo-realistic” and require the students to think differently than in out-of-school task situations (e.g., Boaler, 1993; Nesher, 1980; Verschaffel et al., 2000). Verschaffel et al. argue that “Rather than functioning as realistic and authentic contexts inviting or even forcing pupils to use their common-sense knowledge and experience about the real world in the different stages of the process of solving mathematical application problems, school arithmetic word problem are perceived as artificial, puzzle-like tasks that are unrelated to the real world” (p. xv). Indeed, the general conclusion from a number of studies in the 80’s and 90’s is that students have a tendency not to make proper use of their real-world knowledge and to suspend the requirement that their solutions must make sense in relation to the out-of-school situation that is described in the task (for an overview see Verschaffel et al., 2000). In another line of research Cooper and his colleagues (see Cooper & Harries, this volume) observed that student solutions that do include “realistic” considerations sometimes include the task developers’ intended specific realistic considerations, but sometimes they include general realistic considerations, which are not always accorded full credit in assessment tasks. They also showed that a major reason for this difference in the type of realistic considerations taken into account is a different interpretation of what the tasks demand and that the likeliness of each type of interpretation is dependent of social class.

Thus, in the literature, one can detect hopes for what “really real”, authentic word problems could offer mathematics education, but at the same time a lot of critique that the available word problems do not live up to the characteristics that are argued would facilitate the attainment of important educational goals. Although there are publications that do give some evidence of positive effects of using more “authentic” word problems (e.g., Bonotto, 2004; Palm, 2008; Verschaffel & De Corte, 1997) the field lacks a firm body of convincing empirical evidence for the effects (in any direction) of the use of more “authentic” word problems. In addition, the terms used for the concordance between school word problems and
the, possibly, “real-world” situations indicated by the textual descriptions are ill-defined. The meanings of terms like “really real”, “realistic”, and “authentic” tasks differ between authors, are sometimes vaguely defined, and are sometimes not clarified in a publication at all (Palm, 2002). There is also a lack of frameworks to guide research and synthesise research results, which may be one of the reasons for the lack of synthesised research results in this area.

In this chapter I will suggest a local theory of authentic task situations and a framework specifying one way of looking at the notion of authentic tasks. These developments may contribute to a theoretical base for the study of word problems that are “realistic”, or “authentic”, in the sense of concordant with the real-world situation that is indicated in the task. One purpose is to capture the idea of concordance between in- and out-of-school situations in a fine-grained operational framework that can be used to guide, structure, and synthesise research. In addition, it can support development and analysis of tasks as well as a discussion of the meaning of, and characteristics of, authentic tasks among both practitioners and researchers. I will also argue that sometimes it may be productive to view theory as a combination of a framework, a set of claims, and how they fit together. One reason for this is that the framework can sometimes be used for a number of different research issues other than those within one specific theory. Thus, such a focus on the components of the theory facilitates the possibilities to discuss the usefulness, validity, and necessary refinements of the theory as well as of the framework alone when the framework is used for other purposes than as a part of a specific theory. For example, the framework suggested in this chapter, specifying the notion of authentic tasks, may be used and judged for its usefulness and appropriateness as a part of the theory of authentic task situations with claims about student task-solving behaviour. But it can also, for example, be assessed for its appropriateness and value as support for teachers’ and researchers’ development of word problems with high concordance with real-world situations indicated by the task descriptions. Before the theory of authentic task situations and the framework describing authentic tasks are described I will briefly discuss theories and research frameworks in general.

THEORIES AND RESEARCH FRAMEWORKS

Although the role and use of theory in the discipline of mathematics education has grown considerably the last 30 years (Silver & Herbst, 2007), many leading researchers are now emphasising the importance of developing theories to advancing this field of inquiry (e.g., Lester, 2005; Niss, 2007; Silver & Herbst, 2007). Silver and Herbst (2007) observe that such theories can appear in many guises and at many levels, and identify at least three different types of theories: grand theories of mathematics education, middle-range theories that concern subfields of study, and local theories that help mediate specific connections among practices, research, and problems. Thus, theories include, for example, general philosophical theories, such as theories of learning, but also theories having the role of providing terminology and distinctions to research (Niss, 2007).
But, although theory is a key entity in developing research in mathematics education Niss (2007) notes that “it is neither clear what ‘theory’ is actually supposed to mean, nor what foundations theories have” (p. 1308). He suggests that:

A theory is a system of concepts and claims with certain properties, namely

– A theory consists of an organised network of concepts … and claims about some extensive domain, or a class of domains, consisting of objects, processes, situations, and phenomena.

– In a theory, the concepts are linked in a connected hierarchy … in which a certain set of concepts, taken to be basic, are used as building blocks in the formation of the other concepts.

– In a theory, the claims are either basic hypotheses, assumptions or axioms, taken as fundamental … or statements obtained from the fundamental claims by means of formal … or material … derivation … (p. 1308).

Lester (2005), not believing in a grand “theory of everything”, suggests that:

…we should focus our efforts on using smaller, more focused theories and models of teaching, learning and development. This position is best accommodated by making use of conceptual frameworks to design and conduct our inquiry. I propose that we view the conceptual frameworks we adopt for our research as sources of ideas that we can appropriate and modify for our purposes as mathematics educators” (p. 460).

He describes a research framework as:

…a basic structure of the ideas (i.e. abstractions and relationships) that serve as the basis for a phenomenon that is to be investigated. These abstractions and the (assumed) interrelationships among them represent the relevant features of the phenomenon as determined by the research perspective that has been adopted. The abstractions and interrelationships are then used as the basis and justification for all aspects of the research (p. 458).

Eisenhart (1991) differentiates between three types of research frameworks: theoretical, practical, and conceptual. Theoretical frameworks guide research by reference to formal theory, and practical frameworks are informed by practitioners’ experiences. “A conceptual framework is an argument that the concepts chosen for investigation, and any anticipated relationships among them, will be appropriate and useful given the research problem under investigation” (Lester, 2005, p. 460). Adapting a model for thinking about scientific research by Stokes, Lester argues that educational research can be focused on the pursuit of fundamental understanding, considerations of use, or a blend of the two motives, use-inspired basic research (Lester, 2005). Frameworks can be developed to guide research driven by either one or both of these purposes.

It may be noted that, when using the definitions of theory by Niss (2007) and research framework by Lester (2005), both notions include an organisation of concepts and ideas, but that a framework does not need to include claims about phenomena. Thus, a theory may sometimes be thought of as including a framework and claims.
In the following I will describe a local theory of authentic task situations. First, a conceptual research framework inherent in the theory will be outlined. It is intended for use-inspired basic research, and is also presented in Palm (2006) and described in more detail in Palm (2002). As noted above, the concordance between word problems and real-world task situations has been given considerable attention in the literature, but there is a lack of descriptions that capture this relation by specifying, on a fine-grained level, the task characteristics of word problems that emulate out-of-school task situations well. In the following I will use the term authentic for this relation and suggest such characteristics through the framework. After the conceptual framework has been outlined the claims made by the theory are stated. Then the use of the theory for different purposes and the validity of the framework and the theory are discussed. However, first I will provide four examples of word problems that will be used to illustrate the aspects and help exemplify how the framework can be used to analyse tasks.

Example 1.

In a bakery you see a 20 cm long cylinder-shaped Swiss roll. A dissection straight through the cake produces a circular shape with a diameter of 7 cm. The points of time in a day when the Swiss rolls are all sold are normally distributed with mean 5.30 p.m. and standard deviation 15 minutes.

a) What is the volume of the Swiss roll?
b) What is the probability that the Swiss rolls are all sold before 6.00 p.m., when the bakery closes?

Example 2 (from National Pilot Mathematics Test Summer 1992, Band 1-4, paper I, see Cooper, 1992).

This is the sign in a lift at an office block:

This lift can carry up to 14 people

In the morning rush, 269 people want to go up in this lift. How many times must it go up?

Example 3 (Carpenter, Lindquist, Matthews, & Silver, 1983).

360 students will go by bus on a school trip. Each bus can hold 48 students. How many buses are required?
Example 4.

All students in the school will, on the 15th of May, go on a school trip together. You have decided that everyone will go by bus, and that you shall order the buses. You have seen in the student name lists that there are 360 students in the school. Your teacher said that you can order the buses from Swebus, and that each bus can hold 48 students.

Fill in the note below, which you are going to send to Swebus to order the buses.

<table>
<thead>
<tr>
<th>Swebus – Bus order</th>
</tr>
</thead>
<tbody>
<tr>
<td>Your name:……………………………………………………………</td>
</tr>
<tr>
<td>School:………………………………………………………………</td>
</tr>
<tr>
<td>Date of the trip:………………………………………………………</td>
</tr>
<tr>
<td>Number of buses to order:…………………………………………</td>
</tr>
<tr>
<td>Other requirements:…………………………………………………</td>
</tr>
<tr>
<td>…………………………………………………………………………</td>
</tr>
</tbody>
</table>

Framework for Authentic Tasks

The framework is concerned with the meaning of a concordance between word problems and real-world tasks situations. The point of departure is that the enterprise of developing tasks with such concordance may be viewed as a matter of simulation. Comprehensiveness, fidelity, and representativeness are fundamental concepts that will be used in relation to the concept of simulation. The framework is based on the assumption that “if a performance measure is to be interpreted as relevant to ‘real life’ performance, it must be taken under conditions representative of the stimuli and responses that occur in real life” (Fitzpatrick & Morrison, 1971, p. 239). Comprehensiveness refers to “the range of different aspects of the situation that are simulated” (Fitzpatrick & Morrison, 1971, p. 240). Criterion situations are “those in which the learning is to be applied” (p. 237). Fidelity refers to the “degree to which each aspect approximates a fair representation of that aspect in the criterion situation” (p. 240). Representativeness refers to the combination of comprehensiveness and fidelity (Highland, 1955, cited in Fitzpatrick & Morrison, 1971, p. 240) and will be used as the technical term for the resemblance between a school task and a real-world task situation.

The framework comprises a set of aspects of real-life situations that are reasoned to be important to consider in the simulation of real-world situations. A restriction of comprehensiveness is always necessary. It is not possible to simulate all aspects involved in a situation in the real world and consequently it is not possible to simulate out-of-school situations in such a way that the conditions for the solving of the task will be exactly the same in the school situation. However, the characteristics of the school tasks and the conditions under which they are to be solved can affect the magnitude of this gap, and this gap can affect the similarities in the mathematics used. The proposed aspects were chosen on the basis that a
strong argument can be made that the fidelity of the simulations of these aspects clearly has an impact on the extent to which students, when dealing with school tasks, may engage in the mathematical activities attributed to the real situations that are simulated. The match in mathematical activities here refers not only to methods and concepts used in manipulating mathematical objects within the mathematical world in order to obtain mathematical results, but also to the competencies required in the process of creating a mathematical model based on a situation in the “real world”, as well as the competencies required for interpreting the obtained mathematical results in relation to the original situation.

The Framework: Aspects of Importance

The aspects of real-life situations considered to be important in their simulation (see Table 1) are:

Table 1. The aspects of real-world situations considered to be important in their simulation.

<table>
<thead>
<tr>
<th>A. Event</th>
<th>F. Circumstances</th>
</tr>
</thead>
<tbody>
<tr>
<td>B. Question</td>
<td>F1. Availability of external tools</td>
</tr>
<tr>
<td>C. Information/data</td>
<td>F2. Guidance</td>
</tr>
<tr>
<td>C1. Existence</td>
<td>F3. Consultation and collaboration</td>
</tr>
<tr>
<td>C2. Realism</td>
<td>F4. Discussion opportunities</td>
</tr>
<tr>
<td>C3. Specificity</td>
<td>F5. Time</td>
</tr>
<tr>
<td>D. Presentation</td>
<td>F6. Consequences</td>
</tr>
<tr>
<td>D1. Mode</td>
<td></td>
</tr>
<tr>
<td>D2. Language</td>
<td></td>
</tr>
<tr>
<td>E. Solution strategies</td>
<td>G. Solution requirements</td>
</tr>
<tr>
<td>E1. Availability</td>
<td>H. Purpose in the figurative context</td>
</tr>
<tr>
<td>E2. Experienced plausibility</td>
<td></td>
</tr>
</tbody>
</table>

A. Event. This aspect refers to the event described in the task. In a simulation of a real-world situation it is a prerequisite that the event described in the school task has taken place or has a fair chance of taking place. For example, picking marbles from an urn and noting their colours (a common event in probability word problems) is not something people do in out-of-school life and therefore does not have a corresponding real event. The events in the examples can all be considered to have a fair chance of happening (a person sees a Swiss roll in a bakery, a number of people want to go up in a lift in the morning, a number of people involved in a team will travel by bus to a game).

B. Question. This aspect refers to the concordance between the assignment given in the school task and in a corresponding out-of-school situation. The question in the school task being one that actually might be posed in the real-world event described is a prerequisite for a corresponding real-world situation to exist. The question in Example 1a is a question that probably would not be asked in the described event, while the questions in the other word problems might be. The owner of the
bakery may want to know that an adequate number of Swiss rolls are baked each day. The people in the lift queue may want to know when it may be their turn. The people in Examples 3 and 4 may need to know how many buses to order.

C. Information/data. This aspect refers to the information and data in the task and includes values, models, and given conditions. It concerns the following three sub-aspects:

C1. Existence. This subaspect refers to the match in existence between the information available in the school task and the information available in the simulated situation. In Example 1 the mean and standard deviation are given, which is information that would not be available in the corresponding real-world situation. This results in a large discrepancy between the mathematics applicable in the school situation (students trying to solve the b-question in Example 1) and the mathematics applicable in the corresponding out-of-school situation (in this situation these statistical measures would not have been used).

C2. Realism. Since students’ solution strategies are partly based on judgements of the reasonableness of their answers, and an important reference is reality (Stillman, 1998), the realism of the values given in the school tasks (in the sense of identical or very close to values in the situation that is simulated) is an aspect of importance in simulations of real-life situations.

C3. Specificity. This subaspect refers to the match in specificity of the information available in the school situation and the simulated situation. This match is sometimes important for the possibilities of the students’ reasoning to be similar in the in- and out-of-school situations since a lack of specificity can produce a slightly different context and since strategy choice and solution success is dependent on the specific context at hand (see Baranes, Perry, & Stiegler, 1989; Taylor, 1989). For example, the difference between sharing a loaf of bread and sharing a cake can make students reason differently (Taylor, 1989). In addition, if the price of a specific sort of candy is the issue in the out-of-school situation and it is not known in the school situation that the price is about this object then the students will not have the same opportunities to judge the reasonableness of their answers.

D. Presentation. The aspect of task presentation refers to the way the task is conveyed to the students. This aspect is divided into two subaspects:

D1. Mode. The mode of the task conveyance refers to, for example, if the word problem is communicated orally or in written form to the students and if the information is presented in words, diagrams, or tables. Since, for example, all students do not cope equally well with written communication (e.g., Newman, 1977), and mathematical competencies required to handle graphical representations are not the same as those required to handle verbal representations (e.g., Nathan & Kim, 2007) simulation of this aspect can influence the mathematics required or possible to use.
D2. Language use. Linguistic analyses show that in many word problems the semantic, referential and stylistic aspects of these texts are different from texts describing real life situations. Such school tasks require different competencies in interpreting the tasks than the corresponding out-of-school tasks, and thus such language use impedes the possibilities of the same use of mathematics in the in- and out-of-school situations (Nesher, 1980). In addition, an impeding impact of difficult terms (Foxman, 1987; Mousley, 1990), and sentence structure and amount of text (Mousley, 1990), have been reported. Thus, in simulations, it is of importance that the language used in the school task is not so different from a corresponding out-of-school task situation that it negatively affects the possibilities for the students to use the same mathematics as they would have used in the situation that is simulated. The term “dissection” in Example 1a may be a term that impedes understanding in the school word problem.

E. Solution strategies. To be simulated, a task situation includes the role and purpose of someone solving the task. This aspect is divided into two subaspects:

E1. Availability. The availability of solution strategies concerns the match in the relevant solution strategies available to the students solving school tasks and those available to the persons described in the tasks as solving the corresponding tasks in real life beyond school. If these strategies do not match, then the students do not have the same possibilities to use the same mathematics as they would have used in the situation that is simulated. In Example 4 the students are supposed to take the role of themselves, while in Example 3 it is not known in what role the students are solving the task.

E2. Experienced plausibility. This subaspect refers to the match in the strategies experienced as plausible for solving the task in the school situation and those experienced as plausible in the simulated situation. For example, when a textbook section starts with a description of a particular method for solving tasks, followed by a set of tasks, this may be experienced as a request to use this method and that other methods applicable in the out-of-school situation will not apply to these tasks.

F. Circumstances. The circumstances under which the task is to be solved are factors in the social context (Clarke & Helme, 1998), and are divided into the following subaspects:

F1. Availability of external tools. External tools refer to concrete tools outside the mind such as a calculator, map, or ruler. The significance of this aspect may be visualised by thinking about the difference between the mathematical abilities required to calculate the monthly cost of a house loan using specially designed software (which would be used at a brokers’ office) and by doing so only having available a calculator.
F2. **Guidance.** This subaspect refers to guidance in the form of explicit or implicit hints. Hints in school tasks such as “You can start by calculating the maximum cost”, would clearly (if not also given in the simulated situation) cause a vast difference in what the students are expected to accomplish in the two situations.

F3. **Consultation and collaboration.** Out-of-school task situations are solved solely by oneself, through collaboration within groups, or with the possibility of assistance. In simulations, those circumstances have also to be considered since input from other people can affect which skills and competencies are required to solve a task (Resnick, 1987).

F4. **Discussion opportunities.** This subaspect refers to the possibilities for the students to ask about and discuss the meaning and understanding of the task. A lack of concordance between in- and out-of-school situations in this subaspect can cause differences in the mathematics used since this communication has been shown to have the power of affecting the experienced meaning of the task and the solution strategies applied (Christiansen, 1997).

F5. **Time.** Time pressure is known to impede task-solving success. In simulations, it is therefore important that time restrictions are such that they do not cause significant differences in the possibilities of solving the school tasks compared with the situations that are simulated.

F6. **Consequences of task solving success (or failure).** Different solutions to problems can have different consequences for solvers. Pressures on solvers and their motivations for the task affect the task-solving process and are therefore an aspect to consider in simulations. This aspect may include efforts to promote motivation for word problem solving (people in situations encountered in life beyond school are often motivated to solve those problems). It could also mean putting the products into real use. This could, for example, be done by publishing the results of a statistical survey in the local newspaper or by confronting local politicians with the results. The students could also mark the reduced prices (when working with percentages) when selling self-made products for the purpose of collecting money for people in need (which many schools in, for example, Sweden do). A large project with real consequences is described in Tate (1995). In this project the students used mathematics in their efforts to get liquor stores relocated away from their school neighbourhood.

G. **Solution requirements.** The notion of solution is to be interpreted broadly, meaning both solution method and the final answer to a task. Judgments on the validity of answers and discussion of solution methods (in textbooks and assessment marking schemes) or phrases in the task text (such as “using derivatives solve the following task”) can constitute requirements for the solutions to school tasks. In a simulation, these requirements should be consistent with what is regarded as an
appropriate solution in a corresponding simulated situation, and the students should be aware of this.

Example 2 (the lift task) may be solved by dividing 269 by 14 and rounding off the answer upwards to 20. However, in a real-world situation, a realistic assumption would be that people arrive at different points of time, or that some people working on the lower floors would take the stairs, resulting in the lift going up a different number of times than 20 (a similar argument about the links to reality in this task was made by Cooper, 1992). To prevent students from being forced to think differently than they would in corresponding out-of-school situations, calculations and answers based on such assumptions must also receive credit.

H. Purpose in the figurative context. The appropriateness of the answer to a task, and thus the necessary considerations to be made, sometimes depend on the purpose of finding the answer. In other tasks, the whole solution method is dependent on the purpose (see Palm, 2002). Thus, in simulations it is sometimes essential that the purpose of the task in the figurative context is as clear to the students as it is for the solver in the simulated situation (for an example of difference in clarity of the purpose see the short discussion of Examples 3 and 4 under the heading “Validity of the theory” below).

Claims by the Theory

Based on the organised network of concepts in the framework the theory includes a number of claims in the forms of hypotheses. Firstly, in general, the theory hypothesises that there is a positive correlation between the representativeness of simulations, as experienced by the students, and the similarity between the students’ behaviours in the in- and out-of-school task situations. As a consequence, the higher the representativeness of a simulation is, the larger will be the proportion of students that makes proper use of their real-world knowledge when working with a word problem, and that will not suspend the requirement that their solutions must make sense in relation to the out-of-school situation that is described in the task. However, in addition to the task text and the task-solving environment, the characteristics of the students themselves affect the way they interact with a task. Thus, individual students may be differently affected by a simulations’ representativeness. Furthermore, different aspects of real-world situations affect students’ behaviour differently, and the way they affect them may also vary between situations. Some aspects will also be more important for student behaviour than others and the degree of importance will also vary between situations. From this it follows that similarity in student performance between the in- and out-of-school situation will be dependent not only on the number of aspects that are simulated with high fidelity, but also on which aspects that are well simulated. What these most important aspects are will vary from one task situation to another. However, for each word problem, analyses of the task situation may reveal what the most important aspects are and why.
The reason underlying the above claims is the well-known fact that the conditions under which students are set to solve tasks affect their task solving (e.g., Lave, 1988). Thus, conditions similar with respect to important aspects may lead to similar task solving behaviour. For example, Lave (1998) compared arithmetic calculations in the context of shopping in a supermarket, in a simulation of supermarket shopping, and in solving pure arithmetic tasks from a paper-and-pencil test. The students’ solution strategies and task solving success differed between the contexts. The tasks encountered under the first two conditions displayed the highest solution rates.

Secondly, a repetitive encounter with word problems that are simulations with a high degree of experienced representativeness and include figurative contexts that are experienced as meaningful affects students so that they increase their engagement in the figurative contexts, and a larger proportion of the students will use their knowledge of the real-world situations described in the tasks in their word problem solving.

The reasons underlying this second claim originate in the important role that beliefs play in mathematical task solving (Schoenfeld, 1985). The students’ beliefs must allow them the freedom to act in school as if it was an out-of-school situation. To accomplish this experience for students, the rules of school word-problem solving would have to come closer to the rules of the simulated situations. The students must, for example, believe that their solutions are going to be judged according to the requirements of the real-world situation, and not have to think about what different requirements the teacher might have. But such beliefs are not consistent with the intentions carried by the generic form of the genre of traditional word problems, described by Gerofsky (1999), and since new beliefs are normally integrated in the existing belief system gradually, it is likely to take a frequent encounter with word problems with high representativeness for students to develop beliefs that support serious considerations of the real-world situations described in the tasks. Furthermore, studies on motivation show that significant engagement in a task is enhanced when students experience the task as being meaningful (Ryan & Deci, 2000).

Use of the Theory for Different Purposes

The research framework inherent in this theory of authentic task situations is intended for use-inspired basic research. The theory and its framework may be used to guide further research about representativeness by suggesting aspects to be taken into consideration. These aspects can also help organise and synthesise research findings in order to attain a more coherent picture of the acquired knowledge of the influence of the representativeness of a simulation. With such a more coherent picture more well-founded predictions and explanations of students’ word problem-solving behaviour could be made. Furthermore, it could facilitate discussions amongst both teachers and researchers about the properties of school tasks that are intended to emulate out-of-school situations, and could be useful in the
development and critical analysis of contextualised tasks intended for classroom instruction, textbooks, and assessments, as well as for research purposes.

Validity of the Theory

A theory in education can very seldom be judged by adopting a view of predictions as deterministic in the sense it is used in the physical sciences. However, it is still essential to assess the validity and usefulness of the theory through investigating the theory’s claims by conducting empirical studies and analysing how the patterns of the empirical evidence fit with the claims of the theory. This process may lead to refinements of both the claims and the framework inherent in the theory.

Empirical arguments for or against aspects of the theory can be based on studies that have used the framework in their research design. The validity of the theory can also be examined by applying the framework in an analysis of tasks used in studies that did not include the framework in their research design and relate this task analysis to other observed variables in those studies, such as student behaviour. The research framework inherent in this theory of authentic task situations has, so far, been used in the research design of two empirical studies (but see the chapter by Depaepe, De Corte, and Verschaffel in this volume for a third example). The first study investigated the impact of authenticity on students’ use of real world knowledge in their solutions to word problems, with the restriction that a higher degree of authenticity has to be accomplished within the frames of the practicalities of normal classroom procedures (Palm, 2008). Word problems from the studies focusing on students’ “suspension of sense making” in the ‘80s and ‘90s summarised in Verschaffel et al. (2000) were used. The framework was used to guide a revision of these word problems to attain versions of them with higher representativeness. The use of the framework provided a structured set of lenses by which the selection of task properties to change could be made and to attain some control of the differences between the task versions. Two tests, one including the more authentic task versions and one including the less authentic task versions, were randomly administered to 160 fifth-grade students. The students provided written responses, but interview data from all students were also collected. Consistent with the theory the students who were faced with the more authentic task variants both provided written solutions that were consistent with the realities of the “real” situations described in the tasks and activated their knowledge of the “real” situations, whether or not it affected their written solutions, in a significantly higher proportion of the tasks. An example of a pair of tasks included in the study is Examples 3 and 4. Example 4 is considered to simulate the aspects in the framework to a larger extent than Example 3. For example, the consideration in the task development of the aspect Purpose resulted in a clarification of the purpose of the task solving in Example 4 by making the students order the buses by filling in an ordering form. Of the students who were faced with Example 3, 75% provided the answer “8 buses” (and not, for example, a fraction of buses such as 7.5 buses) or gave a realistic comment to why their answer made sense. Of the students who dealt with the more authentic Example 4, 95% provided the answer 8 buses, which
was considered to be consistent with the described situation in the task. The difference was statistically significant. DeFranco and Curcio (1997) carried out a similar study on this bus task. In one of the two variants of the task the students were made to believe they were actually ordering the buses through the use of a teletrainer with a person answering at the other end. In this way, the difference between the clarity of the purpose in the in and out-of-school situation was made smaller. Also, in this study, the proportion of students providing “realistic” answers was higher on the more authentic version of the bus task.

The second study is an analysis of Finnish and Swedish national assessments for upper secondary school in which the framework was used as a tool for describing in what way, and to what extent, the word problems in the assessments could be considered authentic or not (Palm & Burman, 2004). The analysis showed, for example, that about 50% of the word problems were considered to both describe an event that might occur in life beyond school and include a question that really might be posed in that event. About 25% of the tasks also possessed the quality that the information/data given in the task was similar to that in some corresponding real-world task situation to such a degree that it required the same mathematics for its solution as would have been required to solve the task in the simulated situation. When also the similarity in the availability of external tools between the assessment situation and the real-world task situations was considered, 20% of the tasks were judged to simulate these four aspects with such fidelity that the same mathematics would be required by the assessment tasks and the simulated real-world situations. The results of this study show that the framework can be useful in describing the characteristics of word problems in relation to their authenticity.

SUMMARY AND DISCUSSION

Mathematical word problems may serve several purposes. The study of issues related to some of the purposes, such as providing support for thinking about abstract mathematical concepts, is not the focus of this chapter and may require other theoretical underpinnings than are suggested here. The focus here is on purposes such as facilitating an experience of mathematics as useful in out-of-school situations and practicing solving problems that require respecting circumstances that need to be considered in out-of-school task situations. The importance of attaining these goals has been emphasised by many authors and curriculum documents. To attain these goals, many scholars have promoted the use of tasks that emulate out-of-school task situations, but many have also criticised the word problems most often used today for not living up to such characteristics.

It is of great significance that claims about the impact (or non-impact) of authenticity can be corroborated by empirical evidence. The available body of research on the impact of authenticity is far from sufficient, which influences practitioners working in mathematics education. Developing tasks that simulate important aspects of meaningful task situations with fidelity takes a great deal of time, effort, and money. Today many teachers, textbook writers and professional assessment developers spend a lot of work trying to develop such tasks, motivated by the belief that their effort will be worthwhile. Others do not share this belief and therefore not
this direction in their work. Due to the lack of evidence, decisions are many times forced to be based on assumptions. A more extensive body of research about the consequences of the authenticity of mathematical word problems is needed so that practitioners will have better possibilities to base their decisions about task development on empirical evidence grounded in scientific research. This effort would allow a more efficient use of available resources and better opportunities for developing efficient learning environments.

The quality of scientific inquiries in mathematics education, and in all other disciplines, is greatly enhanced by the use of theories and frameworks of different kinds and at different levels to support the research being pursued. To contribute to the theoretical base for the study of authentic word problems and the influence of authenticity on students’ performance on word problems, a local theory of authentic task situations has been offered. This theory does not, as of now, make predictions about the learning of new concepts or about transfer of school-learned mathematics to new real-world situations, but is concerned with the prediction and understanding of students’ performance on school word problems. Inherent in this theory is a framework conceptualising the idea of concordance between school word problems and out-of-school task situations. This framework can also be used outside the theory, for example in studies of the influence of authenticity on students’ affective experiences, transfer, and learning. It can be used to assist in the development of authentic word problems for both instructional and research purposes. For example, if real-world situations involving rich and desired mathematical activities are identified the framework may be used to simulate these situations. It can also be helpful in structuring research studies and the information gathered from them to achieve a more comprehensive picture of the impact of authenticity of mathematical word problems.

REFERENCES


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INTRODUCTION

It is interesting to treat mathematical word problems historically as a microcosm of the philosophical debate around the nature and relationship of mathematics, language, and reality. Word problems have always been an instantiation of the use of language and story to pose mathematical conundrums – but the purposes of these conundrums, and their purported relationship to “real life” have not remained stable over time.

Elsewhere (Gerofsky, 2004) I have argued that mathematical word problems constitute a written and pedagogical genre which carries within its form echoes of related genres (for example, riddles, parables, puzzles, and competitions of wit), none of which bears any simple, necessary relationship to a presumed practical “reality”.

I have also hypothesised a historical change in the pretext for the use of mathematical word problems in Europe at the time of the Renaissance (beginning in the mid-15th century), when knowledge of algebra became widespread (Gerofsky, 2004, pp. 127-132). My conjecture is that, in pre-algebraic societies, mathematical word problems are the only way available to establish mathematical generality, through the “heaping up” of numerous examples in story form. Evidence from the earliest of these stories (from ancient Babylon, about 4500 years ago) to those published in the most recent of school textbooks gives plenty of examples of the fantastical, non-“real” nature of many of these stories (and by extension, the genre itself). With the introduction of algebra to Europe at the time of the Renaissance, mathematical word problems lost their function of expressing generality through exemplification (although it could be argued that for most school students, whose mathematical knowledge is pre-algebraic, word problems might continue to fulfil this function even today). I conjecture that a re-purposing of word problems took place as part of the Renaissance-to-Modernist project, including claims that mathematical word problems were of practical use in a mercantilised, scientised society and would prepare young people for participation in the worlds of trade, science, and engineering. Part of this Renaissance/Modernist claim involved a treatment of language as a transparent medium that matched the realities of that society in an unproblematic way.
It can be argued (for example, in Toulmin, 1992) that the Renaissance-to-Modernist project operated continuously as the paradigm for Western and other societies from the mid-15th century to the mid-to-late 20th century. Assumptions of this project include the presumed separation of, and preference for, rationality over emotion, mind over body, the individual over the collective, empiricism over intuition, consciousness over the unconscious, and a declarative over metaphoric uses of language. This rationalist project is associated with and supported by intellectual and technological advances in science, medicine, mechanical, and industrial processes. Clarity, a rejection of ambiguity, and a presumption of one-to-one mappings between concepts (including mathematical formulae) and practices (including technical and economic praxis) underlay many of these advances. Most of us who are now writing research and theory in mathematics education grew up in societies that favoured these Modernist assumptions.

I contend that our cultural/societal/technological world has changed drastically in the past twenty-five years, and continues to change in ways as extreme as those which distinguished Renaissance from medieval times. With McLuhan (2003), I link changes in our technological service environment, particularly the advent of personal computers, computer networks, and social networking, with changes in culture, perception, and the most fundamental ways of thinking, being, and relating in the world. These changes have been observed and theorised by postmodern and poststructuralist philosophers. If we are interested in making sense of the ongoing changes we are living daily, in education as in other areas of our lives, we would do well to engage with the ideas of these contemporary philosophers.

However, most researchers in mathematics education, including those of us working with mathematical word problems, have continued to work within Modernist paradigms that assume an unproblematic transparency of language, and one-to-one matching or mapping models of the relationship between mathematical representations and “reality” (meaning generally a scientific, engineering, or mercantile reality). In this paradigm, curriculum writers really ought to pay attention to eliminating ambiguity and increasing fidelity to “real life situations” in the word problems they write, and students ought to learn correct ways of interpreting the transparently obvious matchings that connect the language of word problems to their matching real-world situations. Unfortunately for proponents of this project, language is, by its very nature, never unambiguous (see, for example, Lakoff & Johnson, 2003); mathematical models are necessarily neither transparent nor obvious matchings with phenomena, and must by their nature stress and ignore particular features of live situations (Mason & Pimm, 1984); and human consciousness comprises only a small part of human minds, not to mention “reality” (Dehaene, 2001). These considerations were brushed aside as minimal “noise” in the Modernist project, but loom large in postmodern theorising, including scientific theorising in new fields like cognitive science.

Few mathematics education writers have addressed the very useful notions of reality, simulation, and knowledge in our contemporary technocultural environment elaborated by theorists like Bakhtin, Lacan, Zizek, Lyotard, Derrida, McLuhan, Baudrillard, and others (though see Brown, 2008 for a noteworthy exception). These theorists of postmodernity trouble the sense of a transparent, self-evident
reality and its representations in a number of ways. For example, Bakhtin, in his work on genre, counters a view of language as a transparent mode of representation, positing that all utterances operate within emergent, mutually referencing, generic environments. Lyotard claims that universalised narratives and absolute knowledge are no longer possible, even as a dream, in the postmodern condition, and all knowledge is contingent, local, and provisional. Lacan’s cultural interpretations of Freudian psychoanalytic theory set the inchoate, unknowable, unbearable Real outside the necessary defences created by human culture and psychology, and yet always present. Zizek extends Lacan’s concept of the Real to the realms of the arts, politics, and society. McLuhan encourages us to become aware of both the figure and the ground in dealing with culture and technology, pointing out that a narrow focus on content and meaning, or on moralising against changes that have already taken place, makes us wholly unaware of the enormous unintended effects of a changed service environment and its entailments for consciousness, representation, and our notion of reality. Finally Baudrillard’s work on simulation and simulacra is especially useful in problematising word problems, as it deals with modes of representation that range so far from the iconic as to supplant reality, creating a world of simulacra that bear no relation to any reality whatever, and interact with other simulacra to create a virtual Real.

In this chapter, I will draw upon some of the ideas of Bakhtin, Lacan, Zizek, McLuhan, and Baudrillard to explore postmodern conceptions of the relationship among language, culture, and reality, and use these concepts to interrogate mathematics educators’ claims that we should be able to succeed in matching word problems with real life.

**BAKHTIN: GENRES AND “REALITY”**

The work of Russian literary and linguistic theorist Mikhail Bakhtin (1895-1975) introduced many new ways of conceptualising speech and writing when it was “discovered” by the West in the 1970s and 1980s. Although Bakhtin came from a background in structural linguistics and literary theory, his ideas about dialogism, addresivity, heteroglossia, the carnivalesque, chronotopes, and speech genres have been taken up by poststructuralist and postmodern theorists, and his work has been considered comparable with Derrida’s and Foucault’s. Bakhtin’s work on genre is most useful in considering questions of reality in relation to mathematical word problems.

Bakhtin defines “speech genres” (which also include written genres like mathematical word problems) as “relatively stable types of utterances” within a particular sphere in which language is used (Bakhtin, 1986, p. 60). While particular literary and rhetorical genres had been studied since antiquity, Bakhtin reframed the concept of a genre to include not only literary forms like the epic or lyric poem, or rhetorical forms like political oration, but all forms of language. This was a radical move; in effect, it placed all linguistic activity in the category of generic speech or writing. While Bakhtin emphasised the extreme heterogeneity of speech genres (since the diversity of human activity is productive and ever-changing, and new activity types entail new spoken and written genres), he also asserted the importance
of observing and analysing the nature of utterance types or genres as part of any research that involves particular forms of human language and consciousness.

Bakhtin's insistence on the primacy of generic forms in language is closely related to his conception of “chronotopes” (literally, space-time configurations) which are created by writers and speakers working in developing genres, and which form the “givens” for any writer or speaker working in an established genre. A chronotope can be thought of as the distinctive time-space “world” created by a literary or speech genre – for example, the shadowy world of the castle of the Gothic novel, the fictional world of Dickensian London, or the lonely, isolated space station created in many science fiction stories and movies. A chronotope is a cluster of features that involves spatial settings, time schemes, typical plot developments, and character types, and types of language and imagery used in a genre. For example, here is Bakhtin’s characterisation of the chronotope of what he calls “adventure time” in a genre of ancient novel he calls “the adventure of ordeal” – the love story genre clearly recognisable in comic or tragic form in innumerable stories from Romeo and Juliet, to folk ballads and tales, and melodramatic, picaresque movies:

There is a boy and a girl of marriageable age. Their lineage is unknown, mysterious... They are remarkable for their exceptional beauty. They are also exceptionally chaste. They meet each other unexpectedly, usually during some festive holiday. A sudden and instantaneous passion flares up between them that is as irresistible as fate, like an incurable disease. However, the marriage cannot take place straightaway. They are confronted with obstacles that retard and delay their union. The lovers are parted, they seek one another, find one another; again they lose one another, again they find each other. There are the usual obstacles and adventures of lovers: the abduction of the bride on the eve of the wedding, the absence of parental consent (if parents exist), a different bridegroom and bride intended for either of the lovers (false couples), the flight of the lovers, their journey, a storm at sea, a shipwreck, a miraculous rescue, and attack by pirates, captivity and prison, an attempt on the innocence of the hero and heroine, the offering-up of the heroine as a purifying sacrifice, wars, battles, being sold into slavery, presumed deaths, disguising one’s identity, recognition and failures of recognition, presumed betrayals, attempts on chastity and fidelity, false accusations of crimes, court trials, court inquiries into the chastity and fidelity of the lovers. The heroes find their parents (if unknown). Meetings with unexpected friends or enemies play an important role, as do fortune-telling, prophecy, prophetic dreams, premonitions and sleeping-potions. The novel ends happily with the lovers united in marriage (Bakhtin, 1981, pp. 87-88).

Herbert J. Gans has outlined similar chronotopes that function in American journalism and determine the selection and narrative formation of “stories” for the news (Gans, 1979) – for example, the chronotopes around the concepts of “small town pastoralism” and “the preservation of social order”. (For an interesting recent discussion on the power of these and similar genres in framing news stories on
bioethical issues, see Braun, 2007a, and responses by Price, 2007, Chambers, 2007, and Braun, 2007b). Even in the news, which is our source for much of our sense of the larger realities of our world, it’s “narrative all the way down” (Chambers, 2007).

The term schema (plural schemata) has been used by other genre theorists like Swales (1990) for a concept very close to chronotopes. Schemata are culturally-mediated clusters of activities, features, and narrative lines that characterise particular types of events. For example, the North American schema for a birthday party might include games, balloons, singing “Happy Birthday”, blowing out candles, eating cake and ice cream, and opening presents, while the Chinese birthday celebration schema might include eating long noodles and eggs, decorating the house with red paper, and receiving gifts of money in red envelopes. The chronotope includes the temporal and narrative aspects of schemata as well as typical spatial, cultural, and social settings. Cultural background knowledge of schemata/chronotopes allows us to make predictions, draw implications, “read between the lines”, and read social cues from verbal and written utterances within familiar cultural contexts.

Bakhtin has gone so far as to claim that it is the chronotope that defines genres and generic distinctions (Bakhtin, 1981) – that is to say, a distinctive time-space world of settings, objects, events, character types, and narrative sequences is characteristic of each genre, and in fact becomes its defining feature. These chronotopes can be fresh and surprising in a newly-developing genre, or “fossilised” to reflect an earlier culture in a genre that is well-established and traditional. Later genre theorists like Jamieson (1975) show that the chronotopes of historical antecedents to a genre are carried within the form and referentiality of the genre, often without the writer or speaker being consciously aware of these generic resonances. Another contemporary genre theorist, Miller (1984), finds that a genre carries its typical chronotope not only in terms of temporal-spatial settings, but also in terms of the possible intentions that can be expressed and explored within that genre. She writes that “what we learn when we learn a genre is not just a pattern of forms or even a method of achieving our own ends. We learn, more importantly, what ends we may have.” (p. 165)

So if mathematical word problems form a written pedagogic genre within education, and if this genre is defined by its chronotope (presumably a fossilised one, given the four-and-a-half millennia elapsed since our earliest examples of the genre), what relationship can word problems have with reality?

Bakhtin’s conceptions of dialogism, chronotopes, and genres presumes a real world, but one that is only known through the mediation of language (and thus genre and culture). In Bakhtin’s theory, language can never be a transparent means of reflecting an unproblematic external reality, but a creative and productive means of knowing through dialogic interpretation. Solipsistic interpretations of aspects of reality are avoided through social dialogue (whether amongst people in conversation in the same time and space or between, for example, authors and readers, who may be separated by centuries and great distances), and through a living, evolving relationship and interpenetration between the “real” of experienced lives and
generic chronotopes. Bakhtin characterises his concept of the chronotope with an interesting analogy that compares the relationship between chronotope and reality with the relationship of a living organism with its separate but mutually affecting environment:

However forcefully the real and the represented world resist fusion, however immutable the presence of that categorical boundary line between them, they are nevertheless indissolubly tied up with each other and find themselves in continual mutual interaction; uninterrupted exchange goes on between them, similar to the uninterrupted exchange of matter between living organisms and the environment that surrounds them. As long as the organism lives, it resists a fusion with the environment, but if it is torn out of its environment, it dies. The work and the world represented in it enter the real world and enrich it, and the real world enters the work and its world as part of the process of its creation, as well as part of its subsequent life, in a continual reviewing of the work through the creative perception of listeners and readers… If I relate (or write about) an event that has just happened to me, than I as the *teller* (or writer) of this event am already outside the time and space in which the event occurred. It is just as impossible to forge an identity between myself, my own ‘I’ and that ‘I’ that is the subject of my stories, as it is to lift myself up by my own hair. The represented world, however realistic and truthful, can never be chronotopically identical with the real world it represents. (Bakhtin, 1981, pp. 254-256)

So in Bakhtin’s theory, *all* representations (including all talking and writing) are generic, and mediated by the language and the chronotopes that characterise the genre; but, at the same time, genres both nourish, and are nourished by, aspects of experienced reality which form their environment, and genres evolve and emerge through dialogic negotiation amongst people, and through contact with aspects of a reality that can never be identical to any representative genre.

Bakhtin’s conceptualisation stops short of the most radical postmodern theories of representation through its positing of the existence and influence of an external reality on culturally and linguistically mediated genres. (Note that it is interesting to compare Bakhtin’s account above with Baudrillard’s concept of “impossible exchange”, discussed later in this chapter.) But Bakhtin’s genre theory does already pose a very serious and fundamental challenge to those who claim to be able to represent reality through a simple matching process. If all representation (including, for example, news reports, biography, photographs and videotapes, drawings and graphs) is generic and operates within the evolving norms of its chronotope, then there is no “neutral” representation possible. Every human expression operates within generic universes of time, space, storyline, possible intentions and meanings, and these are inescapably linguistically and culturally mediated through dialogue and interpretation. We have no way of representing a “base-level” reality, although our storied, represented knowing is both nurtured by reality and affects that reality as well.
A similar, perhaps more comprehensive, theory of the relationship between language, culture, and “the Real” comes from the French psychoanalytic theorist, Jacques Lacan (1901-1981). Like Bakhtin’s theories, Lacan’s work is viewed as bridging structuralist and postmodern approaches. Although the bulk of Lacan’s work is a revisiting of Freud’s psychoanalytic theory, Lacan has been taken up as much by artists and cultural theorists as by psychoanalysts.

I will refer here as well to the work of Slavoj Zizek (born 1949), the Slovenian philosopher and film and cultural critic whose interpretations of Lacanian theory in terms of cultural phenomena (and vice versa) have won him international fame. Zizek’s illustrations of Lacanian concepts in terms of film and popular culture have resonance both in terms of understanding Lacan and making sense of contemporary arts and culture.

Other researchers in mathematics education have taken up ideas from Lacan and Zizek. Notably, Brown and collaborators (Brown & England 2005, Brown & McNamara 2005, England & Brown 2001) have used Lacanian psychoanalytic theory to investigate teacher and learner identity construction; Brown (2001, 2008) and Hardy (2007) have used Lacan’s work to critique mathematics education’s individualist, “neutral”, Modernist traditions; Walkerdine (1988) has explored desire and pleasure in mathematics from a Lacanian point of view. But to my knowledge, no one in mathematics education so far has taken up Lacanian notions of the Real to critique assumptions about representations and applications of mathematics, and it is in this way that I will use a facet of Lacan’s oeuvre here.

In Lacan’s psychoanalytic theory, there is a sharp distinction between what he terms “reality” and “the Real”. “Reality” refers to our socially, linguistically mediated world in the realm of the Symbolic, very similar to Bakhtin’s concept of reality as constructed and formulated by the chronotopes of genre. There is nothing basic or necessary about this “reality”; it is constructed out of language, cultural history, and narrative, and always owes a great deal to others who came before us and created these cultural traditions, schemata, and linguistic structures. “Reality” is the social order, regulated through law and socially taken-as-given traditions.

The Real is that which is outside language, symbolism, social structure, and narrative, primary to the Symbolic and not reducible to any kinds of symbols. The Lacanian Real induces anxiety and trauma because it is unmediated and non-categorisable – impossible, inchoate, excessive, unnameable. Lacan describes the Real as “the essential object which is not an object any longer, but this something faced with which all words cease and all categories fail, the object of anxiety par excellence” (Lacan, 1991, p. 164).

To go back a step: Lacan’s theory posits a three-part structure to the human psyche, reminiscent of, but not identical to, Freud’s id-ego-superego divisions. Lacan’s three divisions, the Real, the Imaginary, and the Symbolic, are theorised to develop sequentially in humans, although more than one of these may be present simultaneously.

The Real is the primary state of newborn infants (and, according to Lacan, of non-domesticated animals) where there is no separation of self from environment
or from the mother. There are no desires or fantasies, but only needs. Nothing is named or separated; all is one. There is nothing missing or lacking – the Real is what it is, complete, random, excessive.

The Imaginary develops with the child’s awareness of another – generally the mother. What was one becomes two, and with this separation, there is a sense of loss. Lacan’s Imaginary corresponds to Freud’s “mirror stage”, which is illustrated by the myth of Narcissus: the self externalises or projects its own image onto the other (the mother), and sees this image as either a figure of perfection or a monster. Lacan identifies the Imaginary with fantasy of all kinds, and claims that it is the Imaginary that is invoked when we fall in love; we strive to make two into one again, to achieve wholeness through union with an imagined other. With the Imaginary, the beginning of a sense of self separate from an other is born, and, with it, fantasy and feelings of lack, as the self demands to be like (and reunited with) its perfected fantasy mirror-image.

With the beginnings of language, the child enters the Symbolic realm, and with language, accepts the laws that regulate society and culture. Lacan calls this symbolic order “the big Other” or, alternately, “the Name of the Father”. With language comes the realisation that “two becomes three (or more)” – that there is more to the world than the child and its mother, and that there must be an acceptance of the symbolic order and law for community and culture to function. From this stage on, the person is bound up in language, and desires are formulated in terms of the web of language. Unfulfillable desires are a necessary entailment of the Symbolic order. Lacan goes so far as to include the unconscious as part of the Symbolic, writing that “the unconscious is structured like a language” (Lacan, 1977, p. 203).

Once we are caught up in the Symbolic order, it is impossible to experience the unmediated/unmediatable Real – and yet the Real continues to have influence in our psychic lives as the immutably material, the not-describable, the undeniable, the horrific. It is the Real that constrains our fantasies and desires and trips us up; the Real constitutes the gaps in our socially-constructed “realities”, where language and categories fail.

Zizek uses examples from novels and horror films to illustrate the idea of the irruption or intrusion of the Real into the Symbolic Order. For instance, he gives an example of the traumatic nature of a sidelong glimpse of the Real from Robert Heinlein’s science fiction novella, *The Unpleasant Profession of Jonathan Hoag* (Heinlein, 1983). In this story, an American couple learns that our universe is only one of many, designed by mysterious “universe-designing” beings, who had unfortunately overlooked some minor flaws in their design. The couple are advised that by the time they drive home in their car, these flaws will have been repaired, but they must by no means open the window of their car till they arrive home. Through the car window, they see an ordinary sunlit summer scene; and of course an irresistible reason arises that compels them to take the forbidden action of rolling down the car window to talk to a police officer:

She complied, then gave a sharp intake of breath and swallowed a scream. He did not scream, but he wanted to.
Outside the open window was no sunlight, no cops, no kids – nothing. Nothing but a grey and formless mist, pulsing slowly as if with inchoate life. They could see nothing of the city through it, not because it was too dense but because it was – empty. No sound came out of it; no movement showed in it.

It merged with the frame of the window and began to drift inside. Randall shouted, “Roll up the window!” She tried to obey, but her hands were nerveless; he reached across her and cranked it up himself, jamming it hard into its seat.

The sunny scene was restored; through the glass they saw the patrolman, the boisterous game, the sidewalk, and the city beyond. Cynthia put a hand on his arm. “Drive on, Teddy!”

“Wait a minute,” he said tensely, and turned to the window beside him. Very cautiously he rolled it down – just a crack, less than an inch.

It was enough. The formless grey flux was out there too; through the glass, city traffic and sunny street were plain, through the opening – nothing. (Heinlein, 1983, quoted in Zizek, 1991, p. 14)

This “formless grey flux” is, for Zizek, an example par excellence of the Lacanian Real, irrupting at the borderline of inside and outside, and at the borderline of familiar “reality” and those gaps or chasms where the realities of our culturally-constructed world fail.

Zizek’s citations of literary and cinematic versions of the Real include the 1999 film The Matrix and the 1997 film, Alien Resurrection. At the climax of The Matrix, the protagonist, played by Keanu Reeves, finds out that the world we consider reality is electronically generated and virtual. When he “awakens into the ‘real reality’ he sees a desolate landscape littered with burned ruins – what remained of Chicago after a global war. The resistance leader Morpheus utters the ironic greeting: ‘Welcome to the desert of the real.’” (Zizek, 2002, p. 15). In Alien Resurrection, Sigourney Weaver’s character, the biological clone of the protagonist in the original Alien film, insists on entering a forbidden room on the space station (as the characters in Heinlein’s novella insisted on taking the forbidden action of rolling down the car window). There she encounters the unnameable and unbearable horror of the Real: numerous bottled and pickled versions of herself, in foetal or monstrous form, the results of attempts at cloning gone horribly wrong, and finally a barely-surviving monstrous version of herself that looks her in the eye, speaks to her, and which she must kill (Zizek & Fiennes, 2006).

Let me return briefly to mathematical word problems, before addressing the theories of McLuhan and Baudrillard. When mathematics educators call for word problems that match real life situations, clearly what is intended by the term “real life” is not the horrific and inchoate Lacanian Real, nor could it be Bakhtin’s closely related concept of a “reality” which can never be equated to any chronotope. The real life referents in word problems are of the nature of Lacan’s culturally and symbolically constructed “realities”, or Bakhtin’s generic chronotopes. These “realities” or choronotopes are inventions of human language and society, created out
of a tangential or glancing contact with the Real (which is always present alongside the Imaginary and the Symbolic Order, or alternately, alongside culturally- and linguistically-mediated genres), but more importantly, invented in dialogic response to other examples of the same genre, other traditions and instantiations of a culture.

Film theorist Sobchack elaborates on the fact that instances of a genre (in this case, so-called “genre films”) are imitations and responses to one another and to other related chronotopes more than they are imitations of life:

[Genre films] are made in imitation not of life but of other films. True, there must be the first instance in a series or cycle, yet most cases of the first examples of various film genres can be traced to literary sources, primarily pulp literature [...] And once the initial film is made, it has entered the pool of common knowledge known by filmmaker and film audience alike. Imitations and descendants – the long line of “sons of”, “brides of”, and “the return of” – begins. (Sobchack, 1975/1986, p. 104)

I have argued elsewhere (Gerofsky, 1999, 2004) that word problems constitute a literary and pedagogical genre within mathematics education. Reading the concept of genre with regard to Bakhtin’s chronotopes, or Lacan’s “realities”, and Sobchack’s characterisation of the dialogic, “echoing” nature of genres, it seems very clear that word problems will always be responding primarily to other word problems, and to the related cultural traditions of parables, puzzles, and riddles. There is an insulating quality in this exchange; like an image caught in a series of reflections and distortions in a hall of mirrors, or a sound that enters an endlessly echoing chamber, the chronotopes of genre, or the symbolised “realities” constructed through language allow us to make minimal contact with the dangerous and disturbing Real. Instead, much of our creativity works within the culturally-mediated bounds of mutually-referencing human constructs, playing with and responding to genre, creating new genres in dialogue with earlier ones, creating stories that are in some ways the same, and in other ways different from, those that went before. But we should be clear that new kinds of word problems are far more like other word problems than they are different, and are made in imitation of other word problems far more than they are imitations of the Real.

MCLUHAN: LAWS OF MEDIA, CULTURE, AND TECHNOLOGY

Marshall McLuhan (1911-1980), the Canadian theorist of media, technology, and culture, offers ways to understand why we are now in the “postmodern condition” (Lyotard, 1984) and why the Renaissance-to-Modernist project has come to an end. McLuhan’s ideas were extended and elaborated by French postmodern theorist Baudrillard (1929-2007) to show that concepts of “reality” have changed radically under postmodern technological and cultural conditions, so that we have all now experienced hyperreality, virtual reality, simulations, and simulacra to some degree. It is with reference to these concepts of a changed experience of reality that I will discuss McLuhan’s laws of media.

McLuhan’s work comprised a study of the effects of technology (interpreted in its broadest sense) on human perceptions and cultures. One of McLuhan’s basic
premises was that any technology extends and externalises (“outers” or “utters”) part of the human body. For example, a hammer extends the forearm and fist; a bowl extends cupped hands; clothing extends skin; a wheel extends moving feet; number extends fingers (digits) (McLuhan & McLuhan, 1988). Those body parts are stressed or numbed to some degree in the new technological environment – think, for example, of the numbing, immobilising effects of technology on the bodies of people who spend a great deal of time riding in the extended “body” of a car.

Some technologies extend perceptual organs or the nervous system, and McLuhan asserts that these technologies have the effect of altering the balance of the sensorium – that is to say, they alter the relative importance of sight, hearing, touch, etc. in human life and culture, with seismic consequences for both individuals and societies. There are two classes of technology that have particularly significant ramifications in McLuhan’s work: the phonetic alphabet, and electric/electronic media.

McLuhan differentiates the phonetic alphabet from other writing systems like ideograms (for example, Chinese characters) or syllabaries (for example, Korean, Hebrew, or Arabic writing systems). Where each ideogram carries a gestalt of sounds, meanings, and associations, and a syllabary character carries a cluster of consonant and vowel sounds and morphemic associations, the phonetic alphabet is the only kind of writing system in which each character carries no meaning at all, but only a single, minimal sound. McLuhan argues that the introduction of the phonetic alphabet and phonetic literacy in a culture entails a violent and dramatic shift in the sensory balance and in modes of consciousness to accommodate this translation of spoken language into these sequential strings of meaningless bits. McLuhan says that when alphabetic literacy comes into a society that has an acoustic sensory bias, there is a rapid and dramatic shift to an overwhelmingly visual bias, and along with it, a shift from a collective, tribal, holistic consciousness to an individual, private, and atomistic one.

This may seem like a great deal to attribute to the single technology of the phonetic alphabet, but McLuhan builds an argument that is convincing in its details and in its explanatory and predictive force. McLuhan considers two significant historical moments when the phonetic alphabet was introduced to societies: Ancient Greece in the time of Plato, and Renaissance Europe, with the introduction of the Gutenberg printing press in the mid-15th century and widespread alphabetic literacy that followed. In both cases he observes that oral, tribal cultures (Homeric Greece and medieval Europe) are overtaken by visual, individualistic cultures. Plato denigrates the Homeric poets, with their orally transmitted “encyclopedia” of culture in the form of orally memorised, improvised, and recited epic poetry, and suggests the epics be abandoned in favour of written, connected, logical, sequential arguments. (Interestingly, Plato was a transitional figure, bridging the oral world of Socrates’ dialogues and the newly-alphabetised culture of books).

The Renaissance overthrew the collective, oral, and acoustic consciousness of the Middle Ages in favour of individual consciousness, silent reading, linear rationality, and the revival of the culture and learning the of (alphabetised) Ancient Greeks. McLuhan argues that it is the nature of alphabetic reading, with its extension
of the eye as the preferred sensory modality, that brought about a cultural shift to valuing a single point of view (and the innovation of perspective painting), a sense of removed objectivity (only available through an emphasis on vision), and the concepts of privacy, goals, linearity, specialism, rationalism, and a valuing of the continuous, connected, and static (see tetrads on visual space in McLuhan & McLuhan, 1988, pp. 204–205). Modern science and humanism are grounded in these side effects of a societal ocular bias.

McLuhan argues that what was done by the alphabetisation in the Renaissance-to-Modern period is now in the process of being undone in our contemporary world of electric and electronic networked media. McLuhan characterises these media as the extension and “outering” of our nervous systems, so that we are now living inside the nervous system of the planet. We are now all connected to everyone else at the level of the nervous system, so that there is no possibility of privacy, the lone individual, or the free-standing genius (as conceived in the Modernist project). In some ways, we are currently “living the Renaissance backward”, with a return to an acoustic-tactile collective consciousness and in-depth participation in place of individual consciousness and arms-length objectivity.

The Global Village was McLuhan’s term, coined in the 1960s, and what seemed far-fetched in a world connected by television seems obvious now in our world of the Internet and widely available digital technologies. McLuhan could see this process of the extension of the global nervous system beginning in the late 19th and early 20th century with the development of the telegraph, telephone, radio, and electric lighting, accelerated with television and communication satellites, and enormously accelerated with computers, computer networks, and other digital technologies (McLuhan, 2003, 2005). Marshall McLuhan died in 1980, at the very advent of the digital revolution we are still experiencing, but his work predicted many of the phenomena explored in the 1980s and 1990s by postmodern philosophers: the end of privacy, the obsolescence of alphabetic literacy, violence as a quest for identity by tribalised national groups, a rejection of universal grand narratives, and a return to inner landscapes, the performative on a global scale, and holistic, audio-tactile experience. Wired magazine recognised the importance of McLuhan’s work in 1993, recognising him as the magazine’s “patron saint” on its masthead and running special articles about him. McLuhan’s work has been a strong influence in the work of French postmodern philosopher, Jean Baudrillard, and it is in Baudrillard’s work that we find an articulated account of postmodern “realities” affected by the shift from visual to acoustic culture.

Jean Baudrillard (1929-2007) acknowledged the influence of McLuhan on his work, and along with most French contemporary philosophers, he was also strongly influenced by the work of Lacan and anthropologist Claude Levi-Strauss. Baudrillard’s work follows McLuhan in its connections among technology, media, and culture and the ruptures in human consciousness and relation to the world that come as a result of what McLuhan calls the “total service environment” that accrues about a new technology. Baudrillard takes his ideas to a higher philosophical, sometimes apocalyptic pitch than McLuhan, engaging as Zizek does in analyses of very current events in the news and popular culture and using these to build new philosophical arguments.
Baudrillard’s ideas about representing “reality” are discussed primarily in relation to his concept of simulations and simulacra in postmodern society, and in his concept of the impossibility of exchange in our contemporary world (and thence, the impossibility of equivalence or representation). Much of Baudrillard’s work is focused on the idea of *absence*, particularly the absence of a referent for signs and the absence of a transcendent reality to ground claims of truth and validity. Both these absences are important in our consideration of reality and mathematical word problems, since these problems consist of words and stories often taken to refer to “real-life situations”, and since their use in math education is often legitimised by claims to validity in the realm of a greater reality.

In his essay, *Simulacra and Simulations*, Baudrillard (1988) presents the idea of the “precession of the simulacra” in our contemporary globalised, networked, digitised society – the idea that simulations now *precede*, and in fact *supplant* reality, existing entirely without any corresponding or matching referent, and interacting primarily with other simulations:

> It is no longer a question of imitation, nor of reduplication… it is rather a question of substituting signs of the real for the real itself… A hyperreal sheltered … from any distinction between the real and the imaginary, leaving room only for the orbital recurrence of models and the simulated generation of difference. (p. 170)

Baudrillard’s “hyperreal” is best exemplified by the most exuberant excesses of American and now global culture (Las Vegas, various Disneyland’s) which establish environments based on simulated “nostalgic” or “historical” references to a history that has been altered and fictionalised (viz. Main Street USA, or the Luxor Hotel and Casino).

One step beyond simulation, simulacra arrive prior to any referent, create a virtual experience that is taken as real, and interact with other simulacra and simulations. We are all becoming casually familiar with simulacra through our interactions on the networked social software of Web 2.0. We throw sheep at one another on Facebook, participate in the viral proliferation of video genres on YouTube, and watch our universities use “real” cash to purchase virtual islands for online campuses on Second Life.

Our postmodern world of networked computers and digital media creates strange and hitherto-unknown simulacra that have effects beyond the virtual. An article in *Walrus Magazine* (Thompson, 2004) documents some aspects of the economy of virtual worlds in online fantasy games like EverQuest and Ultima Online. The *Walrus* article documents a young economist’s discovery of the economic and governmental systems of an online game, EverQuest (Castronova, 2001). He discovered a strange system where simulacra (virtual money and virtual goods) were traded for US dollars:

> The Gross National Product of EverQuest, measured by how much wealth all the players together created in a single year inside the game… turned out to be $2,266 US per capita. By World Bank rankings, that made EverQuest richer than India, Bulgaria, or China, and nearly as wealthy as Russia. It was the
seventy-seventh richest country in the world. And it didn’t even exist. 
(Thompson, 2004, p. 41)

Not only are there multi-million dollar businesses that trade in game points, game
levels, avatars, offshore banking, and currency trading amongst games, but gaming
sweatshops in China and Mexico have recently been documented. In these sweat-
shops, hundreds of low-wage employees are hired to spend long hours and days
playing games so that their on-line characters gain powers, levels, and virtual
possessions, which are then sold through brokers to wealthy buyers.

Simulacra of crime have sprung up as quickly as banks in these virtual worlds. 
(The Vancouver, British Columbia Police Department recently set up an avatar-led
recruiting office in Second Life). The [Sims Online] game had a chain of cyber-brothels, run by a character
named “Evangeline”. Evangeline had organised a handful of Sim women to
perform hot-sex chat inside the game for customers, who paid in Simoleans…
[It was] later discovered that some of Evangeline’s “girls” were underage
girls in real life, and that Evangeline herself was a seventeen-year-old boy
living in Florida… Soon The Sims Online was on the front page of the New
York Times. (Thompson, 2004, p. 46)

Real-life wars are fought using video games and virtual environments, to the point
where simulacra may take precedence in creating experiences of war, at least for
the privileged:

The US military has already licensed a private chunk of [online world] There
and created a simulation of the planet on it. The army is currently using
the virtual Baghdad in There as a training space for American soldiers.
(Thompson, 2004, p. 47)

For reasons like this, Baudrillard made the famous, highly controversial statement
that the Gulf War of 1991 had not taken place. Certainly the nature of warfare has
changed drastically when both training and missile launches take place in virtual,
video game environments and when battles are telecast live by satellite on CNN.

Following McLuhan, it could be argued that the world of technology-mediated
simulacra where we now live creates a total service environment that mitigates
against a definable real that can be separated from the virtual; the real and virtual
are inextricably entangled and mutually affecting.

Baudrillard goes beyond technological arguments to an even more fundamental
argument for the impossibility of any representation of the real in any secular
society. Using the anthropological concepts of exchange of Levi-Strauss and
Marcel Mauss as a fundamental to the circulation of commodities in a society,
Baudrillard argues that exchange has become impossible, and thus “reality” exists
only as simulacra:

There is no equivalent of the world. That might even be said to be its defini-
tion – or lack of it. No equivalent, no double, no representation, no mirror…
There is not enough room both for the world and for its double. So there can
be no verifying of the world. That is, indeed, why ‘reality’ is an imposture. Being without possible verification, the world is a fundamental illusion. (Baudrillard, 2001, p. 3)

Baudrillard’s argument deals with the world or universe as a whole, but also with systems within the world like law, politics, economics, aesthetics, even the field of biology. In any of these systems, it is possible to pretend to be able to represent reality at the micro level, but at the macro level, the entire system is without grounding, unless we posit a “higher reality” through religion or metaphysics (and this is not acceptable in a secular society). Taking politics as an example, Baudrillard writes:

Politics is laden with signs and meanings, but seen from the outside it has none. It has nothing to justify it at a universal level (all attempts to ground politics at a metaphysical or philosophical level have failed). It absorbs everything which comes into its ambit and converts it into its own substance, but it is not able to convert itself into – or be reflected in – a higher reality which would give it meaning. (Baudrillard, 2001, p. 4)

For “politics”, we could substitute “mathematics”, since Gödel’s Theorem has proved it impossible to devise a mathematical system that is both consistent and complete; or “physics”, since quantum mechanics and Heisenberg’s uncertainty principle have placed a radical uncertainty and inconsistency at the heart of this field and of our ideas of matter itself.

Baudrillard’s concept of impossible exchange leads to a conclusion very much like Lyotard’s assertion that, in our postmodern condition, no grand narratives are possible. Writing about economics, Baudrillard says:

That principle [of a grounding of the field in reality and rationality] is valid only within an artificially bounded sphere. Outside that sphere lies radical uncertainty. And it is this exiled, foreclosed uncertainty which haunts systems and generates the illusion of the economic, the political, and so on. It is the failure to understand this which leads systems into incoherence, hypertrophy and, in some sense, leads them to destroy themselves. For it is from the inside, by overreaching themselves, that systems make bonfires of their own postulates, and fall into ruins. (Baudrillard, 2005, p. 6)

Taking this big, universe-sized idea to our little world of mathematical word problems, there is a kind of unacceptable hubris in claims that there can be a precise equivalence, a transparent matching, an exchange between “reality” and these brief, generic pedagogic stories. To claim that mathematical word problems (or the theorems of physics, or the narratives of history, or novels in the style of “Realism”) have a relationship of identity with reality is to “make a bonfire of our own postulates”. Baudrillard’s concept of reality, like Lacan’s “Real”, cannot be captured in language or signs of any kind; it cannot be matched up with its equivalent, since it is constitutionally impossible to have an equivalent for reality. Positivistic science, a universe completely marked out with the grid lines of Newtonian physics, Laplace’s deterministic project to know all present, past and future eventualities by
extrapolation from a complete knowledge of this instant – all of these aspects of the Modernist projects have been foreclosed by the impossibility of providing a grounding or an exchange for reality, and we are left with an unresolvable uncertainty, perhaps mystery, at the heart of things.

CODA: (HOW) CAN WE WORK WITH WORD PROBLEMS?

Mathematics educators who are interested in working with word problems have a great many options available for exploration and development (many of which are described in greater detail in Gerofsky, 2004, pp. 133-154). For example:

- Word problems are closely related to ancestral or antecedent genres including parables, riddles and recreational mathematical puzzles, and educators can profitably explore the resonances among these genres, and experiment with treating mathematical word problems playfully, lingeringly, philosophically, “as if” they were riddles, recreations, or parables.
- Mathematical word problems have the potential to offer memorable imagery that can act as a touchstone for teachers and learners in building and discussing abstract concepts. The imagery that works best may often be anything but “realistic” in its usual interpretation (as matching transparently and unproblematically with the situations of adult work life); for example, learners may find more memorable images in fairy-tale stories of dragons than in stories about mortgage interest rates. The term “realisable” (i.e., imaginable) captures this sense of striking, imaginative imagery, and has been developed in this way by researchers at the Freudenthal Institute (see, for example, Van den Heuvel-Panhuizen, Middleton, & Streefland, 1995).
- Mathematics educators can learn from educators in other fields who work with revitalising generic forms through reframing and re-purposing these forms – see, for example, the inspiring work of language educators Davis and Rinvolucri (1988) in revitalising forms like “the dictation” and “the language lab”.
- Mathematical word problems are inherently ambiguous. If mathematics educators are willing to embrace this inevitable ambiguity (rather than holding up mathematics as a realm of absolute clarity and certainty), teachers and students can learn a great deal from the richness of interpretation available in even these brief story problems.
- Mathematics educators can draw learners’ attention to the generic nature of word problems and explore the mathematical implications when some features of the genre are held invariant and others are varied or pushed to the limit. Learners become aware of the generic nature of mathematical word problems very early in their experience of schooling, usually by third grade (see, for example, Puchalska & Šemadeni, 1987), but this awareness is seldom acknowledged or developed in school.

The approaches to word problems sketched above problematise form as well as content, and involve an awareness of the media of communication, genre, and forms of narrative along with the mathematical concepts these are meant to embody. Researchers and educators who choose to ignore contemporary theoretical
developments, and insist on treating mathematical word problems as simple and transparent one-to-one matchings with some purported universally-acknowledged “reality”, will simply doom their work to irrelevance. Students know that mathematical word problems are neither transparent nor “real”, and (rightly) do not accept word problems as object lessons in the usefulness of mathematics in “real life”. It is doubtful if a Modernist “reality-matching” approach was ever more than a pretext for the continued use of word problems in mathematics education. A consideration of contemporary theoretical constructs of the real should help educators grasp the need for more sophisticated consideration of the nature of mathematical word problems, and consequently their uses.

REFERENCES
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