Teaching Mathematics as Storytelling

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INTRODUCTION

We like to tell stories. We tell stories about mathematics, about mathematicians, and about doing mathematics. We do this firstly because we enjoy it. We do it secondly because the students like it. And we do it thirdly because we believe that it is an effective instructional tool in the teaching of mathematics. We are not alone in this. There is ample literature to support the enjoyment of storytelling on the part of both the story teller and the story listener. There is also an abundance of anecdotal data that suggest “telling a story creates more vivid, powerful and memorable images in a listener’s mind than does any other means of delivery of the same material” (Haven, 2000, p. xvii). Aside from the educational value, however, there is also beauty. There is beauty in a story well told, and there is beauty of a story that can move a listener to think, to imagine, and to learn.

But, what are the benefits for students? Marie Shedlock, a teacher who became a famous storyteller, puts this nicely.

> One might as well try to show with a pint measure how the sun and the rain have affected a plant, instead of rejoicing in the beauty of the sure, if slow, growth. The, again, why are we in such a hurry to find out what effects have been produced by our stories? Does it matter whether we know today or tomorrow how much a child has understood? For my part, so sure do I feel of the effect that I am willing to wait indefinitely. […] The teachers of general subjects have a much easier and more simple task. Those who teach science, mathematics, even, to a certain extent, history and literature, are able to gauge with a fair amount of accuracy by means of examination what their pupils have learned. The teaching carried on by means of stories can never be gauged in the same manner. (Shedlock, 1924, pp. 130-131)

With this spirit in mind we wrote this book. We believe it is of interest for those who teach mathematics, or teach teachers to teach mathematics. It may also be of interest to anyone who ever listened to a story and learned something from it. It may be of interest to those who like stories or like mathematics. It also may be of interest to those dislike either mathematics or stories, but are ready to reconsider their position.

In this book we share with the reader many stories that we have told and that we keep telling our students. We tell stories in the mathematics classroom to achieve an environment of imagination, emotion, and thinking. We tell stories in the mathematics classroom to make mathematics more enjoyable and more memorable. We tell stories in the mathematics classroom to engage students in a mathematical activity, to make them think and explore, and to help them understand concepts and ideas.

Furthermore, we share with the reader several techniques for storytelling that makes telling more interactive and more appealing to the learner. We also present a framework that may help potential storytellers create their own stories, as well as ideas as to how existing stories can be enriched and adapted for the needs of any particular audience. By such means we hope that more teachers and more colleagues will story-tell in their classrooms, and, like Marie Shedlock, be patient enough to wait for long-term benefits.

CHAPTER 1

A STORY

PRELUDE

Once upon a time, long ago in a land far away there lived a tribe of farmers called the Zalla. The Zalla were hard working people who were happy for all that they had. They were happy for the dark rich soil in which they grew their crops, they were happy for the rains that nourished their fields, but most of all they were happy for their sheep. They had lots of sheep. For the Zalla, the sheep were the source of warm coats, of carpets for the chilly floors of their caves and bungalows, and of tasty meat for dinner and fresh milk for breakfast. Needless to say, the sheep were very important to the Zalla.

Amzula, the chief shepherd of the Zalla tribe, was responsible for watching over the sheep. Every morning Amzula woke up before the sun rose and let the sheep out of their pens and into the pastures. The sheep explored the hills and the meadows, grazed on the green grass, breathed the crisp air, and satisfied their thirst with the fresh cold water of mountain fed streams. At dusk Amzula would call the sheep back to the safety of the pen before the fall of darkness. His most important job was to make sure that all the sheep had returned, for if any sheep were left out they might fall prey to the wild animals that prowled the hills and the meadows at night.

But here was the problem. Amzula could not count. In fact, no one in the tribe of Zalla could count. And no one in all the neighboring tribes of Agraba could count either. Not that these people were not smart, for they were. They just lived a long, long time ago, before humans knew how to count. Still, Amzula's most important job was to ensure that all the sheep returned for the night. Remember, these sheep were very important to the people of Zalla. What would you do? Is it possible, without counting, to make sure that the same number of sheep returned in the evening as left in the morning?

This question can be posed to students from elementary school to University, and from the variety of their solution attempts the idea of one-to-one correspondence will eventually emerge. That is, for every sheep exiting the pen we can put a pebble in a bowl, tie a knot, or make a mark on a shepherd's stick. Then, for every sheep that returns we can take a pebble out, untie a knot, or erase a mark, and if we are lucky the return of the last sheep will correspond with the removal of the last pebble or the erasing of the last mark.

So, what next, you would ask. It depends. With very young students a story like this may spark an awareness of the ingenuity of counting that they may have taken for granted. With older elementary school students, we would continue the story with Amzula the VI, our Amzula's great-great-great-grandchild who *could* count, but only up to ten. In helping him to assure that all the sheep got home, the students could reinvent the decimal numeration system. With students in grades 8 and 9, the ancient idea of one-to-one correspondence could lead to the concept of a function, one of the most important concepts in mathematics in general, and of the high school mathematics curriculum in particular. With University students, continuing the story of one-to-one correspondence in comparing finite sets extends naturally to the comparison of infinite sets, with some surprising results, including the fact that the set of even numbers is no smaller than the set of natural numbers.

A few of these exciting possibilities are explored in this book. For now, we just wanted tell a story in order to pave the way for the discussion of stories in the mathematics classroom.

WHAT IS A STORY?

When dealing with mathematics we may not be used to thinking in terms of telling a story. In fact, the task of learning mathematics seems quite remote from anything to do with stories. It might be that we occasionally tell a ‘mathematical’ story in the course of teaching children, but it is only an incidental accessorizing of the ‘real work’, which is often perceived as practicing the mathematical algorithms themselves. This, however, is a very shortsighted and impoverished view of the potential of stories in the teaching and learning of mathematics. It is our aim in the chapters that follow to show that through stories we can make mathematics more accessible to students, as well as more engaging.

But first let us look at what stories are and what makes them special. Words like narrative, account, yarn, legend, chronicle are listed among the synonyms that describe the meaning of story. A dictionary definition suggests that a story is “a factual or fictional account of an event or series of events”. However, neither definitions nor synonyms help us understand the power of stories and their purpose. Despite the feeling that “no definition is necessary”, or that “every child knows what a story is”, there have been attempts to articulate what makes a story distinguishable from any other kind of a narrative. We consider two of such articulations below.

> What is a story? A compact answer is to say that it is a narrative unit that can fix the affective meaning of the elements that compose it. That is, a story is a unit of some particular kind; it has a beginning that sets up a conflict or expectation, a middle that complicates it, and an end that resolves it. The defining feature of stories, as distinct from other kinds of narratives – like arguments, histories, scientific reports – is that they orient our feelings about their contents. (Egan, 2004, 2008)

> What is a story? In essence, a narrative account requires a story that raises unanswered questions or unresolved conflicts; characters may encounter and then resolve a crisis or crises. A story line, with a beginning, middle and end, is identifiable. […] It is generally agreed that stories are a powerful structure

> for organizing and transmitting information, and for creating meaning in our lives and environments. (Green, 2004)

The common features in both descriptions are those of a conflict and a structure. (We elaborate on this in the next chapter). However, while Green's attention is mainly on the information embedded in a story, Egan's additional focus is on affect, on orienting feelings. Egan describes a story as a particular kind of narrative unit that orients our emotions to the events presented through the narration. That is, stories make us feel. And in this, stories are unique. We ascribe emotional meaning to events, and to people, and to our own lives by plotting them into partial or provisional stories. We orient ourselves emotionally to our environment by involving it in our stories. The value of the story to teaching is precisely its power to engage the students' emotions and also, connectedly, their imaginations in the material of the curriculum.

This feature of stories is perhaps easily understood in terms of the newspaper editor who asks a reporter, "What's the story on this?" The editor is obviously not asking the reporter to create fiction, but rather is asking the reporter to shape the events to bring out their emotional force. That is, the reporter is not inventing details, but rather selecting and organizing them in order to bring out the affective meaning of the topic. It is this latter sense that we are mostly concerned with.

In our description of how to teach mathematics, we are not concerned with fictional stories about the topic, but rather we are concerned with how we can shape the topic to enhance its attraction to students. In doing this, we will not be falsifying anything, or giving precedence to entertaining students over educating them. Instead, we will be engaging them. We see engaging students with mathematical activity as a crucial aspect of successful education as, and it is the real vividness and importance of this subject in which we want to engage students.

In summary, the great power of stories, according to Kieran Egan (1986, 2004, 2008), is in their dual mission: they communicate information in a memorable form and they shape the hearer's feelings about the information being communicated. Furthermore, Egan's reference to a *hearer* emphasizes that a story is being told and listened to, rather than decoded from a written text. In our discussion of story telling we first describe different shapes that a *story* may take and then focus on the *telling*.

"Telling a story is a way of establishing meaning" (Egan, 1986, p. 37). And establishing meaning should be, we believe, a central thread in teaching mathematics – a subject that is too often perceived as the manipulation of symbols, the meaning of which is often far from clear to students. Unfortunately, the telling of stories is an infrequent and seemingly off-task activity in a mathematics classroom. More often we hear stories *about* mathematics classrooms (how boring they were), *about* mathematical experiences (how humiliating they were), or *about* mathematics tasks (how pointless they were). What story can one possibly tell about long division? We do not suggest here that every lesson or every topic can be learned with a story. However, we believe that introducing stories *in* mathematics classrooms will change the stories *about* the mathematical experiences of learners mentioned above.

STORIES IN THE CLASSROOM

Communicating information and orienting feelings are feature of all stories – fairy tales, news reports, bedtime stories, fables or stories of historical events. Using stories in a classroom can serve many additional and different purposes. Stories may spark interest, assist in memory, and reduce anxiety. They can create a comfortable and supportive atmosphere in the classroom, and build rapport between the instructor and the students.

Creating interest with a story is an important initial step. Describing a chain of events may engage students, create excitement, mystery or suspense, and motivate thinking about a particular problem. Stories may convey passion and enthusiasm. They may also introduce ways of thinking and acting like their heroes, create empathy, and make the material more accessible and memorable. Stories in which students identify with the heroes may also make the lesson more relevant and more vivid. Stories that involve specific examples may help students relax as they provide something to hold to when moving to general theory or technical detail. In general, a classroom story can serve as a purposeful break from the routine, creating a refuge to return to and to seek more stories.

What is mentioned above about the different purposes of stories is true for any subject matter, including mathematics. What is special about the use of stories in the mathematics classroom is that they can assist in understanding difficult concepts and ideas, and assist in solving problems. According to Egan (2008), “The engaging quality of stories seems tied up with the fact that they end. Unlike history or our lives, in which succeeding events compel us constantly to reassess our feelings about earlier events, the story fixes how we should feel, and this provides us with a rare security and satisfaction.” In Mathematics classroom, however, a distinguishing feature of some of the stories is that there is no traditional ‘ending’.

Sparking students’ initial interest with a story is not that difficult to achieve. Our goal, however, is to sustain this interest, to sustain students’ engagement and not let it evaporate as the story ends. That is why some of our stories never end, some do not adhere to the ‘classical’ structure of beginning, middle, and end. The ‘end’, or conflict resolution, is often turned into student activity; the story evolves into exploration and problem solving. We exemplify this in the following chapters. In the next section we introduce several types of stories used to enhance the learning of mathematics.

STORIES OF DIFFERENT KINDS

Stories emerge in different form and kind. Some are real while others are fictional, some are written in prose while others are in verse, some make us think and others make us wonder, some make us cry while others make us laugh. There are stories that we enjoy and wish they would never end, and there are other stories that we rush to finish, eager to know how they end.

Literary genre sources distinguish between different narratives: folk tales, fairy tales, tall tales, bed-time stories, myths, legends, essays, fables, parables, and ballads. There are stories of the past and stories of current events. Initially stories developed as part of the oral culture, as the medium for preserving heritage and recording history. Later, with the development of print culture, books and newspapers became the main sources of stories. More recently, stories are presented as movies and TV programs, or even video games.

However, the discussion of literary genres, on all their varieties, is not our concern here. We are concerned with teaching and learning of mathematics, and making this subject come alive in the classroom. As such, we use stories of all the possible kinds presented in all the available media. Though ‘story’ is defined at times as a ‘sequence of events’, it is not the events that are of our interest here. We distinguish the kinds of stories not by their structure but by the kind of engagement with mathematical content the story brings.

In Chapter 4 we will consider stories that frame or provide the background for a mathematical activity. For example, if our hero has to solve a problem to save a princess, any problem appropriate for the given group of students can be embedded in this story. Then, in Chapters 5 and 6, we note that some stories end where mathematical engagement starts and some stories continue alongside mathematics. Thus, we distinguish between stories that introduce, and stories that accompany and intertwine with mathematical activity.

Mathematics is often perceived as a collection of facts and skills to be learned, and often these facts and skills are counterintuitive to the learner. When this happens a common reaction is to seek refuge in the meaningless memorization of rules. Experienced teachers can easily point to such places, places in which encounters with mathematics are most puzzling and counterintuitive for their students. Instead of reciting rules we suggest in Chapter 7 explaining rules with stories. This introduces a new kind of story – a story that explains. Division by zero, division by a fraction, and the manipulation of negative integers are but a few examples of concepts that students find hard to understand and we believe that explaining with stories will be helpful. We also demonstrate how variation on a story can help in solving a problem or gaining a better understanding of a solution.

Another kind of story is a story that asks a question. In schools today we are more familiar with the distant cousins of such stories, often called ‘word problems’. These word problems start in kindergarten with apples that Jack and Jill put together and continue with tenth grade trains that leave stations at different times and aim to arrive sometime somewhere, periodically changing the speed of their travel. However, a closer look at such stories reveals that they are not really stories at all. They have been stripped of the details and emotions that help to orientate a listener’s feelings. What is left is an empty shell of a story with emphasis on the question mark at its end. Ironically, such problems evolved from true stories that presented a riddle or a puzzle. It is our intention to return to these roots, at least in some problems, and to re-create the story of the word problem. We develop this theme in Chapter 8.

Furthermore, we consider a joke as a short, at times very short, story and in Chapter 9 we exemplify several jokes relevant to specific mathematical content. We discuss how jokes can help both as a pedagogical tool, and as an assessment tool.

Chapter 10 outlines a specific framework for incorporating stories in planning for instruction. Chapter 11 exemplifies how this framework was used by one creative teacher and his students. Finally, in Chapter 12 we attend to how existing stories can be shaped and modified to serve a designated group of students.

We have mentioned different kinds of stories to tell in the mathematics classroom: stories that provide a frame or background, stories that introduce, stories that accompany or intertwine, stories that ask a question, and stories that explain. Of course, very often these distinctions are overlapping as the same plot of a story may result in a different engagement of learners. This classification is neither complete nor disjoint; there are stories that do not belong to any of these categories and there are stories that comply with more than one. Furthermore, the nature of a story may change, taking on different shapes in different contexts. Stories that accompany are usually told or read by a teacher. Other kinds of stories can be started by a teacher and then completed, or at least participated in, by the learner.

We believe that telling is no less important than the story itself, and telling – to which we devote full attention in Chapter 3 – is one of the ways to shape the story. In the following chapters we explore the components of a story and the art of storytelling.

CHAPTER 2

WHAT MAKES A STORY

What is it about a story that is so engaging? Before we address this question we present the classic story of Karl Friedrich Gauss as we would tell it to our students.

> *Although Karl Friedrich Gauss (1777-1855) would eventually grow up to be a brilliant mathematician (some would even say the greatest mathematician), as a child he was more than a handful for his teachers. You see, at heart Karl was a happy boy who liked nothing more than to tease and play tricks on his friends. One day, while he was still quite young Gauss was being particularly jovial in class. He had finished his work early and had proceeded to disturb his classmates with his mischievous antics. One of his favourite tricks was to imitate quite ordinary, but annoying sounds. He was very good at imitating the sound of creaking wood – as in a creaking floorboard or a creaking chair. This was especially appropriate given that the school that Karl attended was very old and looked just like the kind of place that would creak and groan when people moved about. In reality, however, the school was very well built and did not at all creak. But, year after year he would torment his teachers with these sounds. As they walked around the room or sat down in a chair their every move would be accompanied by a cacophony of creaks and groaning wood joints. Some teachers took this better than others, but no one took it worse than Mr. Schmidtsenburgersnoff. Mr. Schmidtsenbergersnoff was Karl's teacher when Karl was 9 years old. He was an overbearing man who was a stickler for discipline and made no bones about dishing out severe punishments for even the slightest misbehaviours. On this particular day Mr. Schmidtsenbergersnoff was in a worse mood than usual. So, annoyed with Karl's joking around, he walked down the aisle to the back of the classroom where Karl was sitting and set a task for him that he was sure would occupy young Karl for the remainder of the day. "For your pestilence, Karl, I will ask you to add up the numbers from 1 to 100!" he barked. All of Karl's classmates were stunned into silence. This was by far the most severe punishment the teacher had ever given out. Poor Karl they all thought, this time the teacher had surely broken him. As the teacher turned to walk back to his desk at the head of the class every child in the room stared at this humourless and evil man. Just before he got to the front of the class the creaking sounds started. Mr. Schmidtsenburgersnoff spun on his heals and stared at Karl. Surely young Karl couldn't be done already. The entire class stared, their collective breath held in anticipation. "The sum is 5050!" The*

classroom erupted into laughter. Once again Karl had gotten the best of the teacher.

This was too much! Mr. Schmidtsenburgersnoff stared at Karl and in a very icy voice said, "You are wrong, and as punishment for your pestilence you will come up to the board and work out the sum in front of the whole class." So, Karl simply walked up to the blackboard and while Mr. Schmidtsenburgersnoff strutted around in front of the classroom, Karl wrote out the following:

$$1+2+3+4+5+\ldots+\ldots+98+99+100$$
$$1+100=101$$
$$2+99=101$$
$$3+98=101$$
$$\ldots$$
$$50 \text{ pairs} \times 101 = 5050$$

The laughter in the classroom erupted again. Mr. Schmidtsenburgersnoff turned to the board. At this point he was blinded by his rage and he did not see the details of what was written there. He saw only gibberish. The intolerant behaviour of one student was bad enough, but the whole class behaving as they were was too much! He turned to the class and roared, "For your intolerable behaviour I assign you all the following tasks." And he walked to the board, erased what Karl had written there and wrote up the following three tasks.

Find the sum of the first 200 whole numbers.

Find the sum of the whole numbers from 201 to 300.

Find the sum of the first 1000 whole numbers.

Can you figure these out?

This example of a story introduces a powerful strategy, often referred to as "Gauss' pairing method." As a story, it adheres to many of the general elements that all good stories have. There is a plot, there is a discernable beginning (but not a discernable end – more on this later), there is conflict, resolution of conflict (although not really – more on this later), imagery, human meaning, wonder, and humour. In addition to these general characteristics there is also the more specific element of pattern. In what follows we will attend to each of these elements in turn, discussing why they are so fundamental, how they were used in the example of the story presented above, and most importantly, how they can be used elsewhere. In discussing these elements we are informed by work on imagination, story design and literary technique (e.g. Baker & Greene, 1987; Bauer, 1993; Egan, 1986, 1997; Pellowsky, 1977).

PLOT

A plot seems like a rather obvious element of a story. In fact, it is hard to imagine a story without a plot. Our goal here is not to argue the obvious need for a plot but rather to discuss in greater detail exactly what the plot is, and how it contributes to the stories we are promoting.

What a plot *is*, is often confused with what a plot *does* (Johnson, 2000). What a plot *does* is move the listener (or the reader) through the events of a story, guiding not only their feelings, but also their thoughts. It prompts the listeners to ask questions, questions that will necessitate their constant attention to the story through to its conclusion. What a plot *is*, however, is a conscious effort on the part of the author to create a story line that will fulfil his or her purpose. In our case that purpose is to situate the rather mundane topic of summing arithmetic sequences in the context of human meaning (to humanize it if you will) and thus spark an interest in the students. However, it is not our intention to simply capture the students' attention through adventure, drama, comedy, or tragedy. We also intend to focus that attention on the mathematics contained within the story. For us the story is a means to an end, not an end unto itself. What is important is not that the students hear a story that contains mathematics but that they *engage in* the mathematics that emerges out of the story. The plot is our most powerful tool for achieving this.

To capture a student's attention in one context and then to transfer that attention to a different context is not an easy task. Anyone who has been frustrated by the loss of attention that occurs as they move from an introductory concrete activity to a more formalized (and perhaps abstract) lesson can attest to this difficulty. It is easy to mis-place and to mis-read a student's attention. Take for example the use of physical humour to capture students' attention – *as you walk into a classroom carrying the day's work you intentionally trip over an extension cord and fall in a brilliant display of physical acting accompanied by loud noises, displacement of furniture, and bits of paper and books flying everywhere.* There is no doubt that the fall attracted the students' attention. If you continue on with the 'act' continuing to bumble and fumble your way through the lesson there is also no doubt that you will keep their attention. If, however, you recover from your unfortunate entry into the classroom and proceed with your intended plan of teaching a riveting lesson on long-division, there is little doubt that the rapt attention you had during your initial calamity will wane. This is perhaps a bit dramatic, but no different in essence than the wane of attention that occurs as one attempts to move students from the enjoyable (and concrete) activity of listening to a story to working with pencil and paper.

So, what is it about these two scenarios – Gauss' story and tripping over extension chord – that make them different? Why does attention wane in the second scenario? Well, attention is a delicate thing – although it is easily captured it is not easily held. Constant stimulus (whether external or internal) is necessary. As Foghorn Leghorn says, "You have to be a magician to keep a kid's attention for more than five minutes these days!" This is perhaps a little bit too cynical, but nonetheless true in essence. The second scenario presented above, whereby some ‘trick’ is used to capture attention, suffers from the fatal consequence that it then tries to shift that attention to something that is not stimulating, or less stimulating. The first scenario does not suffer this same flaw. Instead it adheres to a fundamental principle – we want to capture a student’s attention in the same direction in which we want to hold it (Dewey, 1913).

So, what are the implications of this principle for our plot? Although it is more difficult to capture a child’s attention with mathematics than swordplay or fairytale princes, it is well worth it when it is time to hold their attention. As such, we construct the plot in such a way that the mathematics is featured prominently in the development of the story – it is not simply an *add-on* that we *add on* towards the end. This is not to say that other aspects of the plot cannot also be developed (such as character development, the ebb and flow of conflict, etc.) but these should not be featured to the exclusion of the mathematics. The story presented above stands as an example of this. It could be argued that the mathematics is on par with the characters in the story. It is part of Karl’s character description (*“Although Karl Friedrich Gauss (1777-1855) would eventually grow up to be a brilliant mathematician (some would even say the greatest mathematician)”*), it is the challenge that Karl must meet (*“This was by far the most severe punishment the teacher had ever given out”*) and it is the weapon with which Karl conquers his adversary *(“The classroom erupted into laughter”*). Mathematics is always there, and always in the foreground. The plot even sets the stage for the transition from hearing (and/or seeing) mathematics to *doing* mathematics *(“For your intolerable behaviour I assign you all the following tasks”*). Thus, the plot accommodates the transition from *capturing* the students’ attention to *holding* the students’ attention.

To demonstrate these ideas further let us look at two very different versions of the same story – the story of Archimedes’ death.

Version 1

> *Archimedes was not only a great mathematician, he was also a great engineer. In particular, he was very good at designing ‘engines of war’ – that is, machines that could be used in battle. He was so good at it, in fact, that it can be said that when Marcellus’ Roman army attacked Syracuse Archimedes alone held the advancing Roman army at bay. The ‘engines of war’ that Archimedes had designed thwarted Marcellus’ Roman army at every turn. So effective were these ‘engines’ that even Marcellus held Archimedes in high regard. So, when Syracuse finally fell to the Roman army Marcellus gave the order that Archimedes was to be captured … but not harmed. The soldier who eventually found Archimedes, however, did not heed these instructions. Incensed by Archimedes’ refusal to abandon his work and follow him to Marcellus, the Roman soldier killed Archimedes on the spot.*

This version, despite adhering to the facts of the events, puts Archimedes the man at the foreground. If properly developed, this telling could move a listener to appreciate the sheer genius of Archimedes. Unfortunately, such a telling does not deal with mathematics at the level we intend to foster in this book. So, let us now look at a second telling of the same story.

Version 2

> *It can be said that Archimedes alone held the advancing Roman army at bay. Through his brilliance he had managed to construct 'engines of war' that had frustrated Marcellus' army for weeks. The source of this brilliance, some would say, was due to Archimedes' deep and unwavering concentration. If a problem interested him he could, and would, focus on that problem to the exclusion (even neglect) of all other issues, no matter how pressing. This neglect had of course led to the 'great bath the mathematician' fiasco only a few years earlier* (See chapter 5). *Ultimately, this deep concentration would be both his greatest advantage and his greatest disadvantage.*
>
> *When the Roman army finally broke through Syracuse's defences, Marcellus gave the order that Archimedes was to be brought before him – alive! This order, however, as clear as it was, was not enough to quell the outrage of the soldier who eventually found Archimedes. Had the soldier known about Archimedes' habit of slipping into moments of deep thought even at inappropriate times he would not have been as enraged by Archimedes' response to his advance. "Noli turbare circulos meos" ("Do not disturb my circles"), Archimedes yelled at him. Perhaps Archimedes was unaware that this was a Roman soldier approaching. Perhaps he was even unaware that Syracuse had been overrun by the attacking army. At that moment, the only thing that Archimedes was aware of was the problem that so firmly held his attention. It is said, that even as the soldier ran Archimedes through with a spear, Archimedes never took his eyes off his diagram.*

In this telling the focus of the plot is not on Archimedes. Instead, it is on Archimedes' thought process – on his deep concentration when trying to solve a problem. This may not seem like a mathematical topic, per se, but when one considers this in the context of what students need in order to succeed in problem solving it suddenly seems to be very relevant to mathematics.

So, what differentiates the second version of this story of Archimedes' death from the first version? It is not the factual details. Both versions tell the same details: Marcellus is the leader of the Roman army. The Roman army is attacking Syracuse. Syracuse is being defended by the use of 'engines of war' designed by Archimedes. When Syracuse fell Marcellus ordered the capture of Archimedes. Archimedes was eventually killed by a Roman soldier because he rebuffed the soldier's commands.

The difference lies in the way the plot moves the reader to experience the story. The plot, in each version, has been designed to move the reader's attention to different features of the story. In the first version that feature is Archimedes the man. In the second version that feature is Archimedes' ability to focus his mind. Although it is the plot that achieves this focusing of the listener's attention, it is us, the authors of the story, who orchestrate this focusing through the way we design it. That is, to the hearer the plot is the thread that leads them; to the writer or storyteller the plot is the weaving of that thread.

CONFLICT

Consider how young children linguistically grasp the phenomenal world around them. In dealing with temperature, they first learn 'hot' and 'cold', where 'hot' means "hotter than my body's temperature" and 'cold' means "colder than my body's temperature." These are the terms for one of the first, and most general, discriminations children make and learn to label. We recognize problems using this universal categorizing tool – good/bad, high/low, earth/sky, courage/cowardice, wet/dry, big/little, sharp/blunt, fast/slow, bitter/sweet and so endlessly on. It is as though we first have to divide things into oppositions in order to get an initial grasp on them. Holding onto such oppositions instead of recognizing the inherently complex nature of the world around us can be problematic (after all, the world is not black and white), but it can also be very useful. Somehow this initial binary classification helps us to orient our thoughts, to gain position, and to gain perspective on issues.

At a very young age (3 or 4), long before we are aware of the subtleties and nuances that life will require us to be sensitive to, we begin to sort through the daily happenings through a lens of binary opposites. Those who study young children will recognize how common such binary thinking is – the "manner in which [children] can bring some order into [their] world [is] by dividing everything into opposites" (Bettelheim, 1976, p. 74). As children get older, they come to understand that not every stranger is to be feared and that people are not all good or all bad, but they never lose this initial appreciation for binary classification.

So, how does this idea of binary classification schemes and the reliance on binary oppositions help us in creating 'good' stories? If we reflect on the kind of fantasy-stories young children enjoy so readily, we see that they are built on relatively stark binary oppositions between security/danger, good/bad, courage/cowardice, and so on. The fairy tales and stories we read to children do not create these perceptions; they capitalize on them. Story writers have long known the power of this literary tool of creating conflict through good versus evil, children versus adults, cats versus dogs, beauty versus hideousness, innocence versus corruption, and so on. From the cowboy in the white hat to the damsel in distress the inevitable struggle against their binary opposite forms the basis of popular fiction. But the idea of binary opposition is more than just clever marketing. It succeeds because we have a propensity for appreciating such conflicts. It would be imprudent, then, to ignore this powerful literary tool when creating stories for the purpose of conveying mathematics.

It is not enough, however, to simply announce that there is conflict. The conflict needs to be constructed in the course of the plot through the careful selection of opposing characteristics. In our example at the beginning of this chapter the main conflict is between Karl and Mr. Schmidtsenburgersnoff. There are several binary opposites that were used first to construct this conflict and then to amplify it – Karl is young while Mr. Schmidtsenburgersnoff is old; Karl is clever while Mr. Schmidtsenburgersnoff is not, Karl is jovial while Mr. Schmidtsenburgersnoff is serious, Karl is mischievous while Mr. Schmidtsenburgersnoff is strict. Each of these opposite character traits serves not only to create the conflict, but also to align the listener's allegiance with Karl. That is, the opposing qualities we choose to highlight serve not only to construct the conflict, but to orientate the listener's feelings.

To further accentuate this point we return to our hero from the previous section, Archimedes. Only this time the context is different.

> *There was a time in Archimedes' life when he turned his attention to the wonders of the circle. Archimedes knew that there existed a special relationship between the distance around the circle and the distance across the circle, that is, between the circumference of a circle and its diameter. And he spent long hours trying to figure out exactly what this relationship was.*[*]
>
> *As advanced as the Egyptians were, they had never seemed to come up with a very good way to measure a circle. They would construct a cylinder of the same circumference and then measure how much rope it took to wrap around it. This method was still in use in Archimedes' time. In fact, it was the preferred method of one of Archimedes' good friends Bartholomew. Bartholomew was well known throughout Syracuse as one of the best circle measurers in town and Archimedes often went to him when he had a particularly difficult circle to measure. But you could imagine that with such a good reputation for precision and such a high demand for circle measurement that Bartholomew was falling behind on his commitments. As a result, Archimedes needed to start measuring his own circles.*
>
> *Bartholomew laughed at his old friend when he heard of Archimedes' plans to measure his own circles. After all, it was well known that Archimedes preferred to work with pencil and paper rather than with hammer and chisel. Nonetheless, he set about trying to come up with a better method to measure the circumference of a circle. It is not clear in what circumstances Archimedes came up with his idea, although it is almost certain that it did*

[*] We return to this story and this special relationship in Chapter 5.

> *not include a bath or any nudity*[*]*, but an idea eventually came to him. Archimedes realized that if he drew a square inside his circle (not just any square, mind you, the biggest square possible with its corners touching the circle) as well as a square outside his circle (the smallest square possible with its sides touching the circle) then the circumference of the circle would be somewhere between the perimeter of the two squares. This was easy enough to do, but it generated too large a set of possibilities for the circumference of the circle. So, he switched to drawing polygons with more sides – pentagons, then hexagons, and so on. By the time he got up to twelve sided polygons, he found that he was producing answers as accurate as his friend Bartholomew. Bartholomew didn't laugh anymore. He eventually went out of business and had to take early retirement. He moved to Jamaica where he spent his time fishing and making wooden circles for the tourists.*

Okay, we never said that our stories had to be historically accurate. We trust that students would be able to distinguish historical detail from humorous intermezzo. Getting back to our story, the casting of the relatively onerous and clumsy way of measuring circles as exemplified by Bartholomew versus the elegance of Archimedes' solution sets up a nice conflict, not between Archimedes and Bartholomew, but between the theoretical and the practical. By accentuating Bartholomew's clumsiness and Archimedes' elegance the listener's allegiance is drawn towards Archimedes. In so doing, the elegance and the power of Archimedes' theoretical approach to measuring circles is more easily appreciated. This is no small feat considering that for an elementary school student the most obvious (and often practiced) way of measuring a circle is to lay a string around its perimeter and then to measure the outstretched string. This method is no different than that of Bartholomew.

This brings up another point regarding the selection and construction of characters through whom the conflict arises. In the introductory story of this chapter the characters are constructed so as to help listeners *align* themselves with Karl. This differs from saying that the listener's feelings are oriented towards Karl. Up until now we have only talked about orienting feelings or building allegiances. Alignment is slightly different. It has more to do with who the listener can relate to rather than who they are 'cheering' for. In the story of Karl everything from his age to the setting of a classroom has been designed to help the listener relate to Karl. In the most recent story of Archimedes and Bartholomew the story has been written, although to much lesser extent, to help the listener relate to Bartholomew. Here is a character who is hard working and has a very practical method of measuring circles that is not too dissimilar from the method that the listener may be familiar with. The hope is that the listener will align themselves with Bartholomew and then be moved to improve upon their own thinking when presented with Archimedes alternate method. Although this is sometimes difficult to do there is one very

[*] This story is revisited in Chapter 5.

important rule that needs to be followed to guard against failure – never vilify the character that you want the listeners to align themselves with. This is why Bartholomew is not portrayed as Archimedes' nemesis.

Once a conflict is created it seems only natural to expect that this conflict then be resolved. In the story of Gauss this is both true and false. It is true that the initial conflict between Karl and Mr. Schmidtsenburgersnoff is resolved with Karl getting the better of his foe. However, this is followed up with a new conflict – a conflict between Mr. Schmidtsenburgersnoff and the entire class. This is no accident. The conclusion of the story with an unresolved conflict is a deliberate strategy for facilitating the transition from listening to doing. This strategy is further enhanced if the final conflict can be constructed so as to help the listener align themselves with the protagonist of the story. In this case the protagonist becomes all of the students in Karl's class. The alignment here is obvious – the class of students who are listening to the story will easily relate themselves to the class of students in the story. In a way, the story introduces a mathematical activity and then intertwines with this activity. This issue is discussed in detail in Chapter 5.

IMAGES

One result of the development of language was the discovery that words can be used to evoke images in the minds of their hearers, and that these images can have as powerful emotional effects as reality might, and in some cases even more.
Images created in traditional oral cultures have the crucial social role of aiding memorization. So we find myths replete with vivid and often bizarre images that give them what we might categorize as powerful literary impact. The original purpose of this 'literary impact' was the urgent need to preserve knowledge in cultures without writing. They achieved this by stimulating a range of psychological effects, which continue today in quite different circumstances, long outliving the social purpose they were developed for. Similarly, language development in children leads to the capacity to evoke mental images of what is not present and to feel about them as though they were real. Recall, as most of us can quite vividly, images from some of the earliest stories you remember. Some images, no doubt, are influenced by pictures in books, but it is common to find that the most vivid and evocative images are those we generate for ourselves while listening to stories (Egan, 1986).

There are a number of techniques for systematically using images in teaching. One such technique, called *Guided Imagery*, involves the teacher taking the students verbally to some different time and place by describing the sights, sounds, and smells of this other time and place. *Guided Imagery* can be a powerfully effective technique in many circumstances. What we mean by the use of images here, however, is on a much smaller scale. It does not require relatively elaborate preparations or set-piece performances. Rather it requires the teacher to be more consistently conscious of the array of vivid images that are a part of every topic and to draw on them consistently in vivifying knowledge and concepts.

In the context of mathematics, images are almost completely useless as a vehicle for transmission of content. But as a vehicle for capturing the imagination of listeners it is an indispensable tool. In many ways the images used are is what makes the story worth listening to. Without them what is said may become a boring and loosely connected string of facts. The seemingly meaningless details that are woven into the fabric of the story add colour and drama. In the story of Karl, images were used to add detail to the story. The descriptions of the school and Karl's unique way of tormenting his teacher, although marginally relevant to the plot, are details added to make the story more palatable. It would have been sufficient to say that Karl was mischievous and at the time was behaving badly. The added details about the nature of his misbehaviour give personality to the characters as well as add humour to the situation. Together they combine to make the story more enjoyable and more memorable.

HUMAN MEANING

Scientific knowledge, especially as stacked in textbooks, has an aura of objectivity – it is secure, uninfluenced by what we might hope or fear, and a solid assertion of what is true. Or, at least, that is what we are supposed to think. Knowledge, once formed, tends to become disembodied from its human origins. That is, while knowledge is preserved in the form of books, formulas, proofs, theorems, and such, we must not forget that before all this the formation of knowledge was the result of human thought, effort, and desire. Knowledge is a product of human hopes and fears; our emotions are crucial to its development, and its meaning cannot be truly understood if seen as some bloodless and emotionless enterprise.

Text-books, in particular mathematics textbooks, have tended to disguise from us the simple truth that all knowledge is human knowledge. Too often textbook writers seem to forget this, and science and mathematics texts seem particularly 'inhuman'. This is especially true when considering that human emotions provide one of the easiest tools that students have available for understanding the material in texts. The educational trick is to display knowledge as the product of human ingenuity, energy, passion, hope, fear, and so on. People not too dissimilar from ourselves made it, invented it, discovered it, and formulated it for human purposes, and they did so with human motives. Instead of representing knowledge as a given – telling students the rules for the use of parentheses or for solving equations and giving them exercises until they get the rules right – we might make the knowledge memorable and meaningful by re-embedding it in the context of its original invention or human uses. This might be dramatically shown in mathematics. When students learn a mathematical relation by seeing who invented it and for what purpose, it is more easily learned, more clearly understood, and more likely remembered. The 'who' can be a single person (Karl Friedrich Gauss), a culture (ancient Greeks), or imaginary characters.

So while teaching mathematics we might sensibly remember that everything we teach has a human source – the decimal point and quadratic equations were invented by someone – and that by bringing to the fore the human emotions, ambitions, intentions, fears, and so on, we can expect to engage our students' imaginations in learning. The story of Karl is a perfect example of this. Although the historical truth of this story can be argued (especially given the creative liberties we have taken), what cannot be argued is the human element that is built into the story. We have already talked about the ability of this story to align the listeners first with Karl's character and then with Karl's classmates. The method for adding sequences of numbers that Gauss developed is presented, not as an abstract and disconnected algorithm, but as a method that grew out of a human need and personalized ingenuity. The algorithm has a human context, and as such takes on greater human meaning. Students can relate to Karl and his plight, they can appreciate his situation, and they can share in his accomplishment. In constructing the story in this way we are, in essence, constructing human meaning.

As a completely different example consider the renowned theorem of Pythagoras. If high school graduates remember only one theorem from their mathematical experience, it would most likely be this one. Through some method of repeated application the name for this one formula has stuck in their minds. What may not have stuck, however, are the details of the relationship ($a^2 + b^2 = c^2$) or where it is appropriate to use it to calculate a third side of a triangle when given the first two. What is missing for these graduates is a human connection to the theorem. For them 'Pythagoras' was something to be memorized and to be used in exercises dealing with abstract situations. If, perhaps, they had seen the great human need for constructing right angles that this relationship satisfied they would likely have a greater connection to the theorem. By tying knots at equal distances in a rope, or making triangles with sticks of different integer lengths measured by the same unit, the ancient engineers and carpenters noticed that only a few configurations result in right angles. Students can relive this experience and discover for themselves a few such configurations, nowadays referred to as Pythagorean triples.

Both of these examples – Gauss and Pythagoras – have something in common: they construct human meaning in the third person. That is, we can write (and have written) the stories so that the listener will see that the mathematics had meaning to some human at some time, even if that person and/or time are fictional. A more powerful way to create human meaning in mathematics is through the first person. That is, to make the mathematics meaningful to the listeners, and to have them engage in the mathematics themselves. This is not to say that every time someone is asked to 'do' some mathematics they will find deeply personal meaning in the act. What it does say, however, is that if they do not 'do' mathematics they will definitely not find personal meaning in it. As such, the orchestration of first person human meaning will necessitate a transition from listening to mathematics in the story to doing the mathematics in the story. As mentioned earlier, the story of Karl facilitates a move in this direction through the invitation to the class to participate in the conclusion of the story. This is discussed further in Chapter 8, Stories that ask a question, and Chapter 12, Using existing stories.

THE SENSE OF WONDER

It is easy to feel the emotion of wonder in the face of the dramatic features of the natural world – a mountain view, a gold and scarlet sunset, a vast waterfall, and the immensity of space. Wonder is a kind of emotional memory of what we have lost. The "overflow of powerful feelings" that accompanies wonder can, like the associations discussed above, be directed to almost any object. Everything we see around us can be re-seen in the light of wonder. Wonder can be an engine of intellectual inquiry. It is a part of literate rationality's persistent questioning.

There are many nuances to the term wonder. First and foremost is the act of wondering, that can become a pivot for mathematical instruction. To engage students in mathematics is to ignite the fires of curiosity, to get them wondering why things are as they are. "I wonder why…" is the beginning of scientific thinking. Nature becomes an object of wonder and inquiry. I wonder why the bathwater rises as I sink into it? I wonder how many worms there are in the garden? I wonder why the sky is blue? I wonder how many times is it possible to cut a string in half? A very direct way to get students to wonder about things in mathematics is to ask them 'why' something is as it is. Why do we need to have a zero? Why is $\pi = 3.14$ (or is it)? Why is 1 not a prime number? Why does column addition work? The answers to each of these questions, when explored and discovered by students, will lead to a greater understanding of mathematics.

There is a second, and equally powerful, meaning to the word wonder. This usage is to instil wonder, to be in awe of something, or to see it as wonderful. Stimulating wonder energizes the imagination. In our teaching of mathematics, then, we will be sensible to attend to how one can evoke a sense of wonder in relation to the topic we are dealing with. This will require the teacher to reflect on the topic and locate what is wonderful within it. Anything, seen in the right light, can be seen as wonderful. Even if we are learning how to deal with the everyday transactions of shopping, one can evoke some sense of wonder by embedding the task in a context that draws attention to the astonishing variety of goods brought from all corners of the world, the ingenuity that has gone into arranging food in hygienic containers with stunning efficiency, the UPC codes by which products are recognized, the work of generations of chemists and physicists that has gone into making such taken-for-granted products as toothpaste and other cleaners, fruit juices, frozen peas, and so on. This does not demand lengthy factual lessons on the background of each item, but rather a constant alertness to the wonders of the things around us. Unfortunately, it is hard for some people to pull back from their utilitarian routines. However, the task of stimulating interest in mathematics often involves locating mathematics in the wider context of wonder, and, of course, in being alert to the students' recognition of wonder. A part of good teaching that helps the transition to a richer understanding of mathematics is locating something wonderful in everything we teach; doing so will not only make learning easier for the student, but will also be more interesting and satisfying for the teacher.

Oh sure, one may ask, as students surely will, what is there about mathematics that is so awe inspiring as to elevate its status to that of wonder when, for the most part, mathematical topics traditionally encountered in school are taken for granted and do not offer much to wonder about. There is no answer to such a challenge. For like beauty, wonder is in the eye of the beholder. We can only say that for many, ourselves included, there is wonder in everything in mathematics. From numbers to shapes to algorithms the tightly knit web of connections that supports what we call mathematics is awe-inspiring. At every turn and in everything we do in relation to mathematics we are reminded of the connections that binds it together into a cohesive whole. However, we do not expect our students to experience the same feelings of wonder that we do. They have not seen enough of mathematics to stand in awe of its grandeur. Our goals for our students are much more modest. We want them to see the wonder in individual mathematical concepts. If wonder is an engine of enquiry, then teachers must find a way to stimulate and use learners' sense of wonder to help them wonder about things they learn or observe, and in so doing re-see things as wonderful.

Consider, yet again, our introductory story about Karl. With an appropriate pause in the telling technique – to be explored in Chapter 3 – students may wonder about how Karl could complete the seemingly impossible task imposed by the teacher. It is possible to turn the problem to the class before revealing the solution. Personal attempts to attack the problem may foster appreciation of the ingenuity of the Gauss' pairing method. Further, though there is nothing in the story that can be explicitly identified by a student as wonderful there are many implicit things that, with the help of the teacher, can be brought to the attention of the student. To begin with, the algorithm that Karl presents is highly flexible. Not only can it assist in adding up very large sequences of numbers (or sequences of very large numbers) it can also work on a large variety of sequences. Arithmetic sequences of multiples such as 3, 6, 9, … can just as efficiently be solved as sequences of non-multiples such as 2, 5, 8, … Decreasing sequences can be added as easily as increasing sequences and sequences involving both positive and negative numbers. The algorithm allows students to feel (perhaps for the first time) the power of control that they can have over a large number of situations with a relatively small collection of tools.

Besides flexibility, Gauss' pairing method for determining the sum of the sequence has great utility. It allows a student who holds its secrets to be able to solve a great number of problems. From stacks of soup cans in the grocery store to the great pyramids of the ancient world the algorithm enables them to calculate quickly how many objects (soup cans, blocks of stones) there are in these geometric constructs. Like the flexibility of the algorithm, the utility of the algorithm is something that is implicit within the story. It takes the teacher to bring them alive.

HUMOR

Humour, like imagery, can be used to make a story more palatable. It can colour our stories with details and engage our students' emotions in a way that mathematics typically does not. Depending on how it is used, humour can capture attention or focus attention. It can provide an access point for some students as well as an outlet for others. Through humorous anecdotes, comical characters, or laughable situations, humour can both ignite and delight the mind. We suggest that humour can be used as part of telling a story.

Humour, most simply described, is the incongruence between what is expected and what occurs. This definition can be used to describe a multitude of various uses of humour in our stories. From the silliness of Karl's misbehaviours to the outrageous retirement destination of Bartholomew, the seemingly useless details add colour to our stories. Primary school students, in particular, appreciate silly antics. Why not insert them into the story? Introduce a hilarious noise that is made every time a specific character sits, walks, blinks, sneezes … whatever. But better than describing it, imitate it – add a soundtrack to a story and the children will be engaged. For the more mature students a twist on the expected is a better bet. Introduce completely incongruent elements into the story – Eratosthenes has a cell phone, Archimedes has a pet buffalo, Gauss' favourite number is 147. Such seemingly frivolous details will make the students sit up and take notice. We added humorous detail in several stories discussed in this chapter. Recall Karl's favourite trick of imitating the sound of creaking wood, recall the place of retirement for Bartholomew.

A more difficult task is to integrate humour in such a way that it focuses attention on mathematics. The example above about Gauss' favourite number can achieve this. Upon hearing that it is 147 students will immediately ask why – what is it about this number that is so special? This is now an optimum situation for teaching. Attention has been focused on mathematics through a simple trick of incongruence.

PATTERNS

> *When I read a math paper, it's no different than a musician reading a score. In each case the pleasure comes from the play of patterns, the harmonies and contrasts.* (Rudy Rucker, 1999, p. 11)

Many have argued that mathematics is the science of patterns. At the same time, research has repeatedly shown that children have a seemingly natural disposition towards patterns and patterning activity. It makes sense then that we should want to make use of this very powerful tool in the teaching of mathematics. According to Mason (1996), patterns are the 'heart and soul' of mathematics and, as such, many mathematical activities can be structured around attending to patterns. Patterns have the power to engage students, to embody mathematics, and to activate imaginations. In fact, some of the important roles of patterns are similar to those of stories – they can be used to introduce concepts or activities, they can be used to explain mathematical ideas that learners find difficult, they can be used to ask a question, and they can be used to instil wonder. Ultimately, the question is not where can we use patterns in the teaching of mathematics, but where can we not?

Consider the story of Karl from the beginning of this chapter. The solution that Karl suggested involved a pattern. The challenge that was eventually set for the rest of the class can be resolved involving a pattern, or at least this is the intention. In fact, the means to solving the tasks set for the rest of the class lie in the recognition of the analogy that exists between the various questions and the solution strategy that Gauss came up with. The story of Archimedes and circle measurement can be developed to an activity of pattern recognition and serve to access a variety of different topics from ideas on convergence to the relationship between the circumference of a circle and its radius. We elaborate on this in Chapter 5.

SUMMARY

In this chapter we presented several stories that can be told in a mathematics classroom and discussed the elements that make these stories interesting, memorable and engaging. We considered these components as means towards the goal of students learning mathematics through engaging in a meaningful activity.